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Delay-dependent Asymptotical Stability for Neutral System with Time-varying Delays*

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Abstract :This paper concerns delay-dependent asymptotical stability for neutral systems with time-varying delays. Some new delay-dependent stability criteria are derived by taking a new Lyapunov-Krasovskii functional(LKF) and free weighting matrices. The relationship between the cross terms is derived in the Leibniz-Newton formula. Neither model transformation nor bounding technique for cross terms has been given to derive the criteria. The proposed stability criteria are given in terms of strict linear matrix inequality(LMI) and it is accordingly easy to check the robust stability of the considered systems. Numerical example cited in the final part of this paper demonstrates that the proposed criteria are effective and are an improvement over the previous papers .

Key words :asymptotical stability ;linear matrix inequality ;Lyapunov-Krasovskii functional

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1 Introduction

During the last decades, considerable attention has been devoted to the problem of stability analysis for time-delay systems or neural systems[1-8]. The existing stability results for time delay systems can be classified into two types according to their dependence on the information about the size of delays, namely delay-independent stability and delay-dependent stability. The delay-independent stability is independent of the size of delays and delay-dependent stability is concerned with the size of delays. In general, for small delays, delay-independent criteria are likely to be conservative. Various different techniques have been proposed to derive the delay-dependent criteria, for example, model transformation and decomposition techniques[2,5], Razumikhin theorem[3], and using free weighting matrices[4]. Recently, for a class of neutral systems with interval time-varying delays, the stability and robust stability analysis have been discussed in ref[1,6].

In this paper, a new delay-dependent robust stability criterion has been proposed for a class of uncertain systems with time-varying delay. By employing some free-weighing matrices, sufficient conditions are given in the form of linear matrix inequalities (LMIs) such that time delay system is robustly asymptotically stabile. Numerical examples are provided to demonstrate the effectiveness and applicability of the proposed method.

Through this paper, the superscripts" T "stand for the transpose of a matrix and the inverse of a matrix ;$\mathbf{R}^n$ denotes n-dimensional Euclidean space ;$\mathbf{R}^{n\times m}$ denotes the set of all real matrices with m rows and n columns ;$P>0$ means that P is positive definite ;I is the identity matrix of appropriate dimension ; $*$ denotes the matrix entries implied by symmetry.

2 Problem formulation and preliminaries

Consider the uncertain neutral system with time delays described by

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$$\begin{cases} \dot{x}(t) - C\dot{x}(t-\tau(t)) = Ax(t) + A_d x(t-d(t)) \\ x(t) = \varphi(\theta), \theta \in [-\max\{d,\tau\}, 0] \in \mathbf{R} \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, the discrete time delay $d(t)$ and the neutral time delay satisfy

$$0 \leqslant d(t) \leqslant d, \dot{d}(t) \leqslant d^* < 1, 0 \leqslant \tau(t) \leqslant \tau, \dot{\tau}(t) \leqslant \tau^* < 1 \tag{2}$$

A, A_d, B and C are constant matrices of appropriate dimensions.

In order to simplify the treatment of the problem $\Im: C([-\tau,0],\mathbf{R}^n) \to \mathbf{R}^n$ is defined to be

$$\Im(x_t) = x(t) - Cx(t-\tau(t)) \tag{3}$$

The necessary condition for the stability of the system (1) is that the operator $\Im$ is stable. The stability of the operator $\Im$ is defined as follows.

Definition 1[13] The operator $\Im$ is said to be stable if the zero solution of the homogeneous difference equation $\Im(x_t) = 0, t \geqslant 0, x_0 = \psi \in \{\psi \in C([-\tau,0]\ \Im\psi = 0)\}$, is uniformly asymptotically stable.

3 Stability analysis

In this section, a sufficient condition for the asymptotical stability of neutral system with uncertainties (2) is derived in the form of strict linear matrix inequalities.

Theorem 1 For given $d > 0, d^* > 0, \tau > 0$ and $\tau^* > 0$, the nominal system of (1) is asymptotically stable if the operator $\Im$ is stable and there exist positive-definite symmetric matrices P, Q_1, Q_2, S_1, S_2 and M_i $(i = 1,2,3,4)$ of appropriate dimensions such that

$$\begin{bmatrix} \Omega & -dM & W^{\mathrm{T}}(dQ_2 + S_2) \\ * & -dQ_2 & 0 \\ * & * & -(dQ_2 + S_2) \end{bmatrix} < 0, \Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ * & * & \Omega_{33} & \Omega_{34} \\ * & * & * & \Omega_{44} \end{bmatrix} \tag{4}$$

where $\Omega_{11} = Q_1 + S_1 + PA + A^{\mathrm{T}}P + M_1 + M_1^{\mathrm{T}}, \Omega_{12} = M_2^{\mathrm{T}}, \Omega_{13} = M_3^{\mathrm{T}} + PC, \Omega_{14} = M_4^{\mathrm{T}} - M_1 + PA_d, \Omega_{22} = -(1-\tau^*)S_1$, $\Omega_{24} = -M_2, \Omega_{33} = -(1-d^*)S_2, \Omega_{34} = -M_3, \Omega_{44} = -(1-d^*)Q_1 - M_4 - M_4^{\mathrm{T}}$, and $M^{\mathrm{T}} = [M_1^{\mathrm{T}}\ \ M_2^{\mathrm{T}}\ \ M_3^{\mathrm{T}}\ \ M_4^{\mathrm{T}}]$, $W^{\mathrm{T}} = [A\ \ 0\ \ C\ \ A_d]^{\mathrm{T}}$.

Proof. Considering (1), we define the functional

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) \tag{5}$$

where $V_1(x_t) = x^{\mathrm{T}}(t)Px(t), V_2(x_t) = \int_{t-d(t)}^{t} x^{\mathrm{T}}(s)Q_1x(s)\mathrm{d}s + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)Q_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta, V_3(x_t) = \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s)S_1x(s)\mathrm{d}s + \int_{t-\tau(t)}^{t} \dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)\mathrm{d}s$。

Differentiating $V(x_t)$ with respect to t, we have

$$\dot{V}_1 = 2x^{\mathrm{T}}(t)P\dot{x}(t) = 2x^{\mathrm{T}}(t)P(C\dot{x}(t-\tau(t)) + Ax(t) + A_dx(t-d(t)))$$

$$\dot{V}_2 = x^{\mathrm{T}}(t)Q_1x(t) - (1-\dot{d}(t))x^{\mathrm{T}}(t-d(t))Q_1x(t-d(t)) + d\dot{x}^{\mathrm{T}}(t)Q_2\dot{x}(t) - \int_{t-d}^{t}\dot{x}^{\mathrm{T}}(s)Q_2\dot{x}(s)\mathrm{d}s$$

$$\dot{V}_3 = x^{\mathrm{T}}(t)S_1x(t) - (1-\dot{\tau}(t))x^{\mathrm{T}}(t-\tau(t))S_1x(t-\tau(t)) + \dot{x}^{\mathrm{T}}(t)S_2\dot{x}(t) - (1-\dot{\tau}(t)\dot{x}^{\mathrm{T}}(t-\tau(t)S_2\dot{x}(t-\tau(t))$$

$$\zeta(t) = [x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-\tau(t))\ \ \dot{x}^{\mathrm{T}}(t-\tau(t))\ \ x^{\mathrm{T}}(t-d(t))]^{\mathrm{T}}$$

Furthermore, we add the following zero equations $\alpha = 2\zeta^{\mathrm{T}}(t)M(x(t) - x(t-d(t)) - \int_{t-d(x)}^{t}\dot{x}(s)\mathrm{d}s) = 0$, where $M^{\mathrm{T}} = [M_1^{\mathrm{T}}\ \ M_2^{\mathrm{T}}\ \ M_3^{\mathrm{T}}\ \ M_4^{\mathrm{T}}]$.

Hence, we have $\quad \dot{V} \leqslant \zeta^{\mathrm{T}}(t)\Omega\zeta(t) - \int_{t-d}^{t}\dot{x}^{\mathrm{T}}(s)Q_2\dot{x}(s)\mathrm{d}s - 2\zeta^{\mathrm{T}}(t)M\int_{t-d(x)}^{t}\dot{x}(s)\mathrm{d}s \leqslant$

$$\zeta^{\mathrm{T}}(t)\Omega\zeta(t)-\int_{t-d(t)}^{t}\dot{x}^{\mathrm{T}}(s)Q_2\dot{x}(s)\mathrm{d}s-2\zeta^{\mathrm{T}}(t)M\int_{t-d(t)}^{t}\dot{x}(s)\mathrm{d}s\leqslant\frac{1}{d(t)}\int_{t-d(t)}^{t}(t)[\zeta^{\mathrm{T}}(T)\quad\dot{x}^{\mathrm{T}}(S)]\begin{bmatrix}\Xi & -dM\\ * & -dQ_2\end{bmatrix}\begin{bmatrix}\zeta(t)\\ \dot{x}(s)\end{bmatrix}\mathrm{d}s$$

where $\Xi=\Omega+W^{\mathrm{T}}(dQ_2+S_2)W$, and $\Omega=\begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14}\\ * & \Omega_{22} & \Omega_{23} & \Omega_{24}\\ * & * & \Omega_{33} & \Omega_{34}\\ * & * & * & \Omega_{44}\end{bmatrix}$, $W^{\mathrm{T}}=[A\quad 0\quad C\quad A_d]^{\mathrm{T}}$, $\Omega_{11}=Q_1+S_1+PA+A^{\mathrm{T}}P+M_1+M_1^{\mathrm{T}}$, $\Omega_{12}=M_2^{\mathrm{T}}$, $\Omega_{13}=M_3^{\mathrm{T}}+PC$, $\Omega_{14}=M_4^{\mathrm{T}}-M_1+PA_d$, $\Omega_{22}=-(1-\tau^*)S_1$, $\Omega_{24}=-M_2$, $\Omega_{33}=-(1-d^*)S_2$, $\Omega_{34}=-M_3$, $\Omega_{44}=-(1-d^*)Q_1-M_4-M_4^{\mathrm{T}}$.

Using Schur complement, (4) holds guarantees $\begin{bmatrix}\Xi & -dM\\ * & -dQ_2\end{bmatrix}<0$, and then $\dot{V}<0$ holds. Therefore, nominal system of (1) is asymptotically stable.

Remark 1 From the proof process of Theorem 1, one can clearly see that neither model transformation nor bounding technique for cross terms is involved. Therefore, the stability criteria are expected to be less conservative.

Remark 2 When $C=0$, similar to the proof of Theorem 1, we have

$$\begin{bmatrix}\Omega & -dM & dW^{\mathrm{T}}Q_2\\ * & -dQ_2 & 0\\ * & * & -dQ_2\end{bmatrix}<0,\Omega=\begin{bmatrix}\Omega_{11} & \Omega_{12}\\ * & \Omega_{22}\end{bmatrix}\tag{6}$$

where $\Omega_{11}=Q_1+PA+A^{\mathrm{T}}P+M_1+M_1^{\mathrm{T}}$, $\Omega_{12}=M_2^{\mathrm{T}}-M_1+PA_d$, $\Omega_{22}=-(1-d)^*Q_1-M_2-M_2^{\mathrm{T}}$ and $M^{\mathrm{T}}=[M_1^{\mathrm{T}}\quad M_2^{\mathrm{T}}]$, $W^{\mathrm{T}}=[A\quad A_d]^{\mathrm{T}}$.

Remark 3 It should be noted that the conditions (4) and (6) are given as strict linear matrix inequalities (LMIs) which can be easily solved by using the Matlab LMI toolbox.

4 Numerical example

In this section, an example is provided to demonstrate the effectiveness and the less conservation of the proposed design algorithm.

Example Consider the nominal neutral system (1) with

$$\dot{x}(t)=\begin{bmatrix}-0.9 & 0\\ 0.1 & -0.9\end{bmatrix}x(t)+\begin{bmatrix}-1.1 & -0.2\\ -0.1 & -1.1\end{bmatrix}x(t-\tau)+\begin{bmatrix}-0.2 & 0\\ 0.2 & -0.1\end{bmatrix}\dot{x}(t-\tau)$$

Tab. 1 **Allowable upper bound of *d***

method	[10]	[9]	[11]	[12]	Theorem 1
d	1.371 8	1.784 4	1.785 6	1.826 6	1.904 2

According to Theorem 1, the upper bound on the time delay to guarantee the system which is asymptotically stable is listed in Tab. 1.

5 Conclusions

A new delay-dependent asymptotical stability condition for neutral systems is considered in this paper. Sufficient conditions are given in terms of strict linear matrix inequalities. Numerical example is given to illustrate the advantages of the theoretic results obtained, and show that our results are much less conservative than some existing results in the literature.

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变时滞中立系统的时滞依赖渐近稳定性分析

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摘要 本文讨论了一类变时滞中立系统的时滞依赖渐近稳定性问题。通过利用 Lyapunov-Krasovskii 泛函（LKF）和自由权矩阵方法，得到了该系统渐近稳定性的时滞依赖新判据。交叉项间的联系由 Leibniz-Newton 公式给出。定理的推导没有利用模型转换和交叉项有界方法。由于结果以严格线性矩阵不等式形式给出，所以很容易验证所考虑系统的稳定性。最后的数值实例验证所给判据的有效性和对已有结果的改进。

关键词 渐近稳定性；线性矩阵不等式；Lyapunov-Krasovskii 泛函

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