DOI :10. 11721/cqnuj20130222

Accurate Calculation of the Differential Cross-Section of $p\bar{p}\to\pi^+\pi^-$ Reaction with N-$\bar{\text{N}}$ Renormalized Loop-chain Contribution*

ZHANG Jia-wei, LIU Chun-lan, CHEN Xue-wen
(Dept. of Physics, Chongqing University of Science and Technology, Chongqing 401331, China)

Abstract: In the work, we make a detailed discussion on the reaction, via nucleon-antinucleon propagator, within the SU (2)-invariant coupling model between charged mesons and nucleons ($\pi^\pm$N) of perturbation theory of quantum field theory. The renormalized loop-chain propagator of nucleon-antinucleon (N-$\bar{\text{N}}$) was strictly calculated and given an analytical expression with the large momentum integral limitation method. Furthermore, we obtain a simple and analytic expression of the differential cross-section of $p\bar{p}\to\pi^+\pi^-$ with nucleon-antinucleon (N-$\bar{\text{N}}$) renormalized loop-chain propagator, i. e. $\frac{d\sigma_{(\text{chain})}}{d\Omega}=\frac{G^4}{64}\frac{1}{(2\pi)^2}\frac{1}{p_0^2}\frac{\sqrt{p_0^2-\mu^2}}{\sqrt{p_0^2-m^2}}\frac{1}{(2p_0^2-\mu^2-2\sqrt{p_0^2-m^2}\sqrt{p_0^2-\mu^2}\cos\theta)^2}T(q^2)$. We also calculate and obtain numerical results of the differential cross-section of $p\bar{p}\to\pi^+\pi^-$ at tree, one-loop and chain diagram in different center-of-mass energy, respectively, and make a detailed comparison and discussion and show some important information about radiative correction. In large energy within 2.5 GeV $\leqslant p_0 \leqslant$ 100 GeV, these corrections are in the reasonable range, i. e. $R_{(\text{chain})}$ (*or* $R_{(\text{loop})}$) <2.0. The results provide a significant reference to understand in-depth of the SU(2)-invariant coupling model of strong interactions theory and to explore the reaction cross-section between mesons and nucleons (πN). In addition, it will provide effective reference to the applicability of the perturbation theory of quantum field theory in nucleon interactions, the renormalized calculation in different ways and approaches.

Key words: SU(2)-invariant coupling model; loop-chain propagator; renormalization; differential cross-section; radiative correction

中图分类号:O572. 24 **文献标志码**:A **文章编号**:1672-6693(2013)02-0096-06

1 Introduction

The pion-nucleon (πN) interaction is an important ingredient in many other hadronic reactions and in particular for the meson production in nucleon-nucleon (NN) collisions[1]. It is one of the most basic and fundamental processes in strong interaction physics. The study has a long history in hadron physics[1]. It is very important that the scattering reactions of different particles for the understanding interaction of elementary particles[2]. In a variety of phenomenological models, which describe the strong interaction of meson and nucleon, the SU(2)-invariant coupling model, which is based on the strong interaction symmetry in the isospin space, has been a remarkable success[3]. This model can describe effectively the interaction between nucleons and mesons. However, these studies are mainly concentrated in the low-order (tree-level) calculation using perturbative Quantum Field Theory, but it is very few at high-order correction, such as loop, chain-leve1 and so on. Therefore, taking into account the improvement of the experimental technologies and the collision energy, it is no doubt that in the future numerous da-

* **Recieved**:10-18-2012 **Accepted**:11-15-2012 **网络出版时间**:2013-01-18 15:05
Foundation:Research Foundation of Chongqing University of Science &Technology (No. CK2011B34); Natural Science Foundation Project of CQ CSTC (No. CSTC2012jjB40006)
First author biography:ZHANG Jia-wei, male, Ph. D. in physics, research field is theoretical physics, E-mail: fsk1c1@126. com
收稿日期:2012-10-18 **修回日期**:2012-11-15
资助项目:重庆市科委自然科学基金(No. CSTC2012jjB40006);重庆科技学院基金(No. CK2011B34)
作者简介:张家伟,男,博士,研究方向为理论物理,E-mail:fsk1c1@126. com
网络出版地址:http://www. cnki. net/kcms/detail/50. 1165. N. 20130316. 1337. 201302. 96_022. html

ta require more accurate theoretical predictions, especially the interactions of elementary particles.

Usually, it is really difficult to deal with high-order corrections in the perturbative Quantum Field Theory, especially, to obtain the analytical differential cross section excepting tree-level. The divergence will emerge, when the high-order corrections are taken into consideration. However, the divergent term can be eliminated by a renormalization scheme. The contribution of the renormalized finite quantity (radiative corrections) is always very small, whereas it is very important for further research.

The renormalization theory has become a well-established theory and is widely used in the perturbative high-order corrections[3], which is based on the spirit of renormalization from Dyson who is one of the main founders of the renormalization theory of Quantum Field Theory[4]. The calculation on renormalization loop propagator will be worth for development of the whole quantum theory, and it will be also a significant work for revealing and discovering the intrinsic quality of particle reaction.

For the radiative correction problem, Feynman and Brown have done some valuable researches[5], where they considered the contributions limited to finite order renormalized calculation. In this paper, we present strictly calculation for n-$\bar{\mathrm{n}}$ renormalized chain propagator, and obtain the precise results of differential cross section of $p\bar{p}\to\pi^+\pi^-$ with n-$\bar{\mathrm{n}}$ renormalized chain, one-loop and tree propagator, and discuss the relevant radiative correction.

2 The calculation to differential cross section with $p\bar{p}\to\pi^+\pi^-$

In the SU(2)-invariant coupling models of strong interaction between meson and nucleon, the strong interaction Lagrangian[3], which is also known as Yukawa coupling, is the following,

$$\begin{aligned} L_{\pi N}(x) = \mathrm{i}\sqrt{2}G[&\bar{\psi}_p(x)\gamma^5\psi_n(x)\varphi^+(x) + \\ &\bar{\psi}_n(x)\gamma^5\psi_p(x)\varphi^-(x)] + \\ \mathrm{i}G[&\bar{\psi}_p(x)\gamma^5\psi_p(x) - \\ &\bar{\psi}_n(x)\gamma^5\psi_n(x)]\varphi^0(x) \end{aligned} \tag{1}$$

Where $\psi_N(x)$ and $\bar{\psi}_N(x)$ (N stand for neutron (n) or proton (p)) are the spinor field operator and its conjugate operator for nucleon (n or p), respectively. $\varphi^{\pm}(x)$ and $\varphi^0(x)$ are scalar operator for mesons, and G is the coupling constant of the πN strong interaction. From this expression, we can see that it satisfies the charge conservation.

The interaction Lagrangian functionequation (1) described the internal complex process of πN strong interaction, which has been confirmed by a large number of theoretical calculations and experimental observations [1,6-7]. In this paper, it will apply to take relevant the calculation.

2.1 Accuratecalculation of the n-$\bar{\mathrm{n}}$ renormalized chain cropagator

According to Lagrangian functionequatuion (1), we can get Feynman diagram of the process $p\bar{p}\to\pi^+\pi^-$ with n-$\bar{\mathrm{n}}$ renormalized chain propagator as the Fig. 1, which satisfies strictly the charge conservation. By using Feynman rules [3], the n-$\bar{\mathrm{n}}$ chain propagator can be expressed as a series that is a 4×4 matrix function

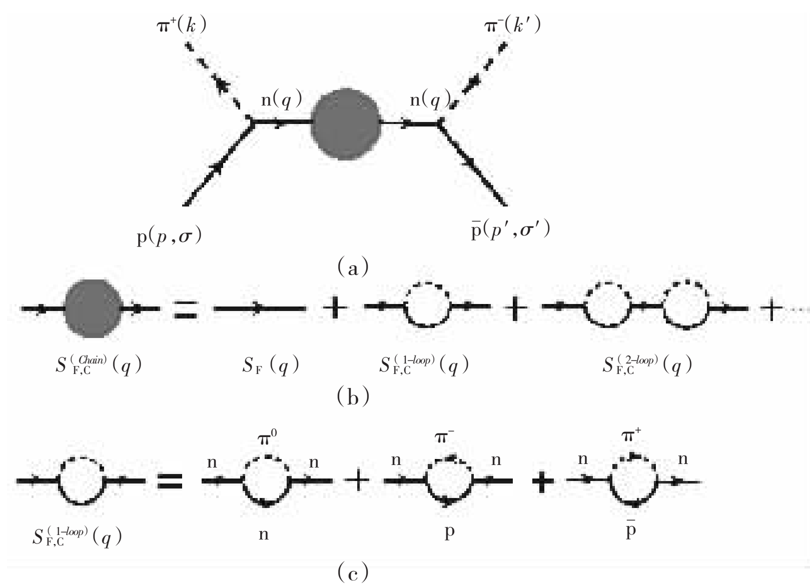

Fig. 1 (a) Feynman diagram for the process $p(p,\sigma)+\bar{p}(p',\sigma')\to\pi^+(k)+\pi^-(k')$ with n-$\bar{\mathrm{n}}$ renormalized chain propagator; (b) n-$\bar{\mathrm{n}}$ renormalized loop-chain propagator with momentum q; (c) n-$\bar{\mathrm{n}}$ renormalized one-loop propagator contain three forms with n-π^0, p-π^- and $\bar{\mathrm{p}}$-π^+.

$$\begin{aligned} S_{F,C}^{(\mathrm{chain})}(q) = &S_F(q) + S_F(q)\cdot[\Pi_{C-3}(q)\cdot S_F(q)] + \\ &S_F(q)\cdot[\Pi_{C-3}(q)\cdot S_F(q)]^2 + \cdots \end{aligned} \tag{2}$$

with

$$S_F(q) = \frac{\mathrm{i}(\gamma\cdot q+m)}{(q^2-m^2+\mathrm{i}\varepsilon)}, \Pi_{C-3}(q) = \Pi_C^{(n-\pi^0)}(q) + \Pi_C^{(p-\pi^-)}(q) + \Pi_C^{(\bar{p}-\pi^+)}(q) = 5\Pi_C^{(n-\pi^0)}(q)$$

Here, $\Pi_C^{(n-\pi^0)}(q)$ represents a function of the renormalized finite quantity of n-π^0 loop. Using the scheme of momenta normalization and renormalization in Ref. [6], we can obtain $\Pi_C^{(n-\pi^0)}(q)$ that has physical meanings from

$$\Pi_C^{(n-\pi^0)}(q)=\frac{G^2}{16\pi^2}\{(\gamma\cdot q)[I_2(q^2)-I_1(q^2)-I_3(q^2)]+im[I_1(q^2)+I_3(q^2)]\}$$

where

$$I_1(q^2)=-[z_1(q^2)K_1(q^2)+z_2(q^2)K_2(q^2)]+(1/\lambda^2)\ln\lambda-(1/\lambda^2)\sqrt{4\lambda^2-1}\times\{\arctan[(2\lambda^2-1)/\sqrt{4\lambda^2-1}]+\arctan(1/\sqrt{4\lambda^2-1})\}$$

$$I_2(q^2)=-1/2[(m^2-\mu^2)/q^2+z_1^2(q^2)K_1(q^2)+z_2^2(q^2)K_2(q^2)]-1/2+1/2\lambda^2+(1/2\lambda^4-1/\lambda^2)\ln\lambda-(1/2\lambda^4)\sqrt{4\lambda^2-1}\{\arctan[(2\lambda^2-1)/\sqrt{4\lambda^2-1}]+\arctan(1/\sqrt{4\lambda^2-1})\}$$

$$I_3(q^2)=-1+2/\lambda^2-(4/\lambda^2-2/\lambda^4)\ln\lambda-[(4/\lambda^2-2/\lambda^4-4)/\sqrt{4\lambda^2-1}]\times\{\arctan[(2\lambda^2-1)/\sqrt{4\lambda^2-1}]+\arctan(1/\sqrt{4\lambda^2-1})\}$$

with

$$z_j(q^2)=\alpha(q^2)+(-1)^{j+1}\begin{cases}-\sqrt{\beta(q^2)}+i0; & q^2>0\\ \sqrt{\beta(q^2)}+i0; & -(m-\mu)^2\leqslant q^2<0\\ 0+i\sqrt{-\beta(q^2)}; & -(m+\mu)^2\leqslant q^2<-(m-\mu)^2\\ \sqrt{\beta(q^2)}+i0; & q^2<-(m+\mu)^2\end{cases}$$

$$K_j(q^2)=\begin{cases}\ln\left|\frac{1-[\alpha(q^2)-(-1)^j\sqrt{\beta(q^2)}]}{\alpha(q^2)-(-1)^j\sqrt{\beta(q^2)}}\right|; & q^2>0\\ \ln\left|\frac{1-[\alpha(q^2)+(-1)^{j+1}\sqrt{\beta(q^2)}]}{\alpha(q^2)+(-1)^{j+1}\sqrt{\beta(q^2)}}\right|; & -(m-\mu)^2\leqslant q^2<0\\ \frac{1}{2}\ln|\lambda^2|+(-1)^{j+1}i\left[\arctan\frac{1-\alpha(q^2)}{\sqrt{-\beta(q^2)}}+\arctan\frac{\alpha(q^2)}{\sqrt{-\beta(q^2)}}\right]; & -(m+\mu)^2\leqslant q^2<-(m-\mu)^2\\ \ln\left|\frac{1-[\alpha(q^2)+(-1)^{j+1}\sqrt{\beta(q^2)}]}{\alpha(q^2)+((-1)^{j+1}\sqrt{\beta(q^2)}}\right|+(-1)^{j+1}i\pi; & q^2<-(m+\mu)^2\end{cases}$$

Here, $j=1,2$, $\lambda=m/\mu$, $\alpha(q^2)=(q^2+m^2-\mu^2)/2q^2$, $\beta(q^2)=\alpha^2(q^2)+\mu^2/q^2$.

To simplify the equation (2), the n-$\bar{n}$ chain propagator can be expressed as the form of renormalized chain propagator as follows

$$S_{F,C}^{(chain)}(q)=\frac{S_F(q)}{1-\Pi_{C-3}(q)\cdot S_F(q)} \tag{3}$$

In order to ensure the correctness of the formula in above, the term $[\Pi_{C-3}(q)\cdot S_F(q)]$ must meet condition $\lim_{n\to\infty}[\Pi_{C-3}(q)\cdot S_F(q)]=0$. From the reference [6], we know that it satisfies the convergent condition in a wide energy range. For shortening the text, we do not put proof here. Employing two corrective factors $R_1(q^2)$ and $R_2(q^2)$, which represent radiative correction factor to the part of non-unit matrix $(\gamma\cdot q)$ and unit matrix in tree propagator $S_F(q)$, respectively, the n-$\bar{n}$ renormalization chain propagator in equation (3) becomes following analytical form

$$S_{F,C}^{(chain)}(q)=-\frac{(\gamma\cdot q)}{q^2+m^2}R_1(q^2)-\frac{im}{q^2+m^2}R_2(q^2)$$

where

$$R_1(q^2)=\frac{1-5g\rho(q^2)+5g\frac{m^2}{q^2+m^2}I_2(q^2)}{[1-5g\rho(q^2)]^2+\left[5g\frac{mq}{q^2+m^2}I_2(q^2)\right]^2};$$

$$R_2(q^2)=\frac{1-5g\rho(q^2)-5g\frac{q^2}{q^2+m^2}I_2(q^2)}{[1-3g\rho(q^2)]^2+\left[5g\frac{mq}{q^2+m^2}I_2(q^2)\right]^2}.$$

with $\rho(q^2)=I_1(q^2)+I_3(q^2)-q^2/(q^2+m^2)I_2(q^2)$, $g=G^2/16\pi^2$.

2.2 Differential cross section of $p\bar{p}\to\pi^+\pi^-$ with n-$\bar{n}$ renormalized chain propagator

In particle physics, the study of the reaction cross section has a very important significance for experiment. According to scattering theory in the Quantum Field Theory, the differential cross section of $p\bar{p}\to\pi^+\pi^-$ in the center of mass frame can be formulated by[3]:

$$\frac{d\sigma}{d\Omega}=\frac{G^2}{(2\pi)^2}\frac{m^2}{16p_0^2}\frac{\sqrt{p_0^2-\mu^2}}{\sqrt{p_0^2-m^2}}\frac{1}{4}\sum_{\sigma=1}^{2}\sum_{\sigma'=1}^{2}|M_{fi}|^2 \tag{4}$$

Here, m and μ stand for the masses of proton or antiproton and charged meson $\pi^\pm$, respectively. p_0 is the incident energy in the center of mass frame, and M_{fi} is the matrix element of transition. $\frac{1}{4}\sum_{\sigma=1}^{2}\sum_{\sigma'=1}^{2}$ represents initial particles spin average and final particles quantum number summation.

From equation (4), the main procedure for the calculation of $d\sigma/d\Omega$ is the calculation of $|M_{fi}|^2$. For the scattering process $p\bar{p}\to\pi^+\pi^-$, we only need to calculate one the transition probability $|M_{fi}|^2$. There is not an exchange diagram. According to Feynman diagram in Fig. 1 and Feynman rules, the transition amplitude is written easily. To apply a standard trace technology[3] and to calculate complex, $|M_{fi}|^2$ can be written in the following simple and direct form

$$|\overline{M}_{fi}|^2=\frac{1}{4}\sum_{\sigma=1}^{2}\sum_{\sigma'=1}^{2}|M_{fi}|^2=\frac{G^4}{4}\frac{1}{4m^2}\frac{1}{q^2+m^2}T(q^2)$$

with

$$T(q^2)=8m^4(R_2-R_1)(R_1^*-R_2^*)-8\mu^2p_0^2R_1R_1^*-m^2p_0^2(R_1-R_2)(R_1^*-R_2^*)+8p_0^4R_1R_1^*+4(m^2-p_0^2)\cos\theta[m^2(2R_1R_1^*-R_1^*R_2-R_1R_2^*)-2p_0^2R_1R_1^*]-8\sqrt{p_0^2-m^2}\sqrt{p_0^2-\mu^2}\cos\theta\times\{m^2(2R_1R_1^*-R_1^*R_2-R_1R_2^*)+[2p_0^2+2(p_0^2-m^2)\cos\theta]R_1R_1^*\}$$

Here, we have used the condition of center of mass as follows

$\vec{p}+\vec{p}'=0$; $\vec{k}+\vec{k}'=0$; $p_0=p_0'=k_0=k_0'$, $\vec{p}\cdot\vec{k}=|\vec{p}||\vec{k}|\cos\theta$, $|\vec{p}|=\sqrt{p_0^2-m^2}$, $|\vec{k}|=\sqrt{k_0^2-\mu^2}$

$$q^2=(p-k)^2=2p_0^2-m^2-\mu^2-2\sqrt{p_0^2-m^2}\times\sqrt{p_0^2-\mu^2}\cos\theta$$

Finally, through the phase-space integral, we can deduce the expression of differential cross-section of the $p\bar{p}\to\pi^+\pi^-$ with the n-$\bar{n}$ renormalization chain propagator

$$\frac{d\sigma_{(chain)}}{d\Omega}=\frac{G^4}{64}\frac{1}{(2\pi)^2}\frac{1}{p_0^2}\frac{\sqrt{p_0^2-\mu^2}}{\sqrt{p_0^2-m^2}}\times\frac{1}{(2p_0^2-\mu^2-2\sqrt{p_0^2-m^2}\sqrt{p_0^2-\mu^2}\cos\theta)^2}T(q^2)$$

2.3 Differential cross section of $p\bar{p}\to\pi^+\pi^-$ with one-loop and tree propagator

Similarly, the propagator of the n-$\bar{n}$ renormalized one-loop is written from the Fig. 1 as follows

$$S_{F,C}^{(loop)}(q)=S_F(q)\cdot[1+\Pi_{C-3}(q)\cdot S_F(q)]=-\frac{(\gamma\cdot q)}{q^2+m^2}\xi_1(q^2)-\frac{im}{q^2+m^2}\xi_2(q^2)$$

with

$$\xi_1(q^2)=1+5g[I_1(q^2)+I_3(q^2)-\frac{q^2-m^2}{q^2+m^2}I_2(q^2)],\ \xi_2(q^2)=1+5g[I_1(q^2)+I_3(q^2)-\frac{2q^2}{q^2+m^2}I_2(q^2)]$$

According to results of $S_{F,C}^{(chain)}(q)$ above, we can deduce simple expression of $S_{F,C}^{(loop)}(q)$. Here, we only make some replacement to the two functions $\xi_1(q^2)$ and $\xi_2(q^2)$ on q^2 as follows

$$R_1(q^2)\to\xi_1(q^2),R_2(q^2)\to\xi_2(q^2)\quad(5)$$

After to replace, and to deal with the same calculation and simplification as chain, we could get the accurate theoretical computed results of the differential cross-section of $p\bar{p}\to\pi^+\pi^-$ with the n-$\bar{n}$ renormalized one-loop propagator contribution

$$\frac{d\sigma_{(loop)}}{d\Omega}=\frac{G^4}{64}\frac{1}{(2\pi)^2}\frac{1}{p_0^2}\frac{\sqrt{p_0^2-\mu^2}}{\sqrt{p_0^2-m^2}}\times\frac{1}{(2p_0^2-\mu^2-2\sqrt{p_0^2-m^2}\sqrt{p_0^2-\mu^2}\cos\theta)^2}T(q^2)$$

with

$$T(q^2)=8m^4(\xi_2-\xi_1)(\xi_1^*-\xi_2^*)-8\mu^2p_0^2\xi_1\xi_1^*-m^2p_0^2(\xi_1-\xi_2)(\xi_1^*-\xi_2^*)+8p_0^4\xi_1\xi_1^*+4(m^2-p_0^2)\cos\theta[m^2(2\xi_1\xi_1^*-\xi_1^*\xi_2-\xi_1\xi_2^*)-2p_0^2\xi_1\xi_1^*]-8\sqrt{p_0^2-m^2}\sqrt{p_0^2-\mu^2}\cos\theta\times\{m^2(2\xi_1\xi_1^*-\xi_1^*\xi_2-\xi_1\xi_2^*)+[2p_0^2+2(p_0^2-m^2)\cos\theta]\xi_1\xi_1^*\}$$

To make a similar replacement asequation (5) for$R_1(q^2)\to1,R_2(q^2)\to1$, we can easily obtain the differential cross section in the tree diagram as follows

$$\frac{d\sigma_{(tree)}}{d\Omega}=\frac{G^4}{64}\frac{1}{(2\pi)^2}\frac{1}{p_0^2}\frac{\sqrt{p_0^2-\mu^2}}{\sqrt{p_0^2-m^2}}\times\frac{1}{(2p_0^2-\mu^2-2\sqrt{p_0^2-m^2}\sqrt{p_0^2-\mu^2}\cos\theta)^2}T(q^2)$$

with

$$T(q^2)=8p_0^4-8\mu^2p_0^2-8(m^2-p_0^2)p_0^2\cos\theta-8\sqrt{p_0^2-m^2}\sqrt{p_0^2-\mu^2}\cos\theta[2p_0^2+2(p_0^2-m^2)\cos\theta]$$

3 Numericresults

In order to show the chain and loop effective correction, we have not only present the numerical results of differential cross section in Tab. 1, but also give the correction of one-loop and chain diagram to tree in the different collision energy in the Fig. 2, where the mass of proton or anti-proton and charged mesons $\pi^\pm$ are taken as $m=0.938$