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二阶半线性中立型阻尼微分方程解的振动性*

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摘要:本文研究一类具连续分布滞量的二阶半线性中立型阻尼微分方程的振动性,利用 Yang 不等式、广义 Riccati 变换和 H 函数,给出了此类方程所有解振动新的充分条件为 $\int_T^{+\infty}\left[\frac{C}{r(\xi)}\exp\left(-\int_T^{\xi}\frac{p(s)}{r(s)}\mathrm{d}s\right)\right]^{\frac{1}{\alpha}}\mathrm{d}\xi=+\infty$,且满足 $Q_1(H)>0$,$\left(|H'(t)|+\frac{H(t)\rho'(t)}{(\alpha+1)\rho(t)}-\frac{H(t)p(t)}{(\alpha+1)r(t)}\right)>0$ 和 $\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[H(t,s)k(s)\rho(s)\mu_1(s)-\frac{\rho(s)r(s)\ |h(t,s)|^{\alpha+1}}{(\alpha+1)^{\alpha+1}\ (H(t,s)k(s)g'(s,a))^{\alpha}}\right]\mathrm{d}s=\infty$,所得结果推广和改进了已有文献的结果。

关键词::连续分布滞量;中立型;半线性微分方程;振动性

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近年来,具连续偏差变元的半线性微分方程解的振动性问题引起了学者们广泛关注。文献[1-6]分别讨论了半线性微分方程和半线性阻尼微分方程解的振动性问题,并取得了一些很好的结果。但同时具有连续时滞和阻尼项的二阶半线性中立型微分方程解的振动性目前尚未见报道。本文的目的是利用 Yang 不等式、广义的 Riccati 变换和 H 函数方法,给出方程所有解振动的若干判别准则。所得结果有助于刻画半线性微分方程的动力学行为,为解决力学、物理学、人口动力学、生物学和工程学等学科领域中的许多实际问题(如精密实验室的隔振、防震设计、人口问题中的突发事件导致人口突变、生物种群的生长和桥梁的防震设计等)提供数学理论依据和科学基础。

考虑具有连续分布滞量的二阶半线性中立型阻尼微分方程

$$(r(t)\ |z'(t)|^{\alpha-1}z'(t))'+p(t)\ |z'(t)|^{\alpha-1}z'(t)+\int_a^b q(t,\xi)\ |x(g(t,\xi))|^{\alpha-1}x(g(t,\xi))\mathrm{d}\sigma(\xi)=0,t\geqslant t_0\geqslant 0 \quad (1)$$

其中
$$z(t)=x(t)+c(t)x(h(t)),c(t),p(t)\in C([t_0,\infty),\mathbf{R}_+)$$
$$\mathbf{R}_+=[0,\infty),0\leqslant c(t)\leqslant 1,h(t)\in C^1(\mathbf{R};\mathbf{R})$$
$$h(t)\leqslant t,\lim_{t\to\infty}h(t)=+\infty,r(t)\in C([t_0,\infty];\mathbf{R}_+)$$
且最终不恒为零,$r'(t)\geqslant 0$, $\alpha>1$ 是个常数;$q(t,\xi)\in C([t_0,\infty)\times[a,b],\mathbf{R}_+)$,且 $q(t,\xi)$ 最终不恒为 0;$g(t,\xi)\in C([t_0,\infty)\times[a,b],\mathbf{R})$,$g(t,\xi)\leqslant t$,$g(t,\xi)$ 关于 t,ξ 分别是非减的,$\frac{\partial g(t,a)}{\partial t}$ 是 $g(t,a)$ 关于 t 的偏导数,记 $\frac{\partial g(t,a)}{\partial t}$ 为 $g'(t,a)$,$g'(t,a)$ 最终不恒为零,且有 $\lim_{t\to\infty}\min_{\xi\in[a,b]}\{g(t,\xi)\}=\infty$;$\sigma(\xi)\in C([a,b],\mathbf{R})$,且关于 ξ 非减,且(1)式中的积分是 Stieltjes 积分。

本文总假定方程(1)的非零解是整体存在的。

引理 1[7] 若 X 和 Y 非负,当 $q>1$ 时,有 $X^q+(q-1)Y^q-qXY^{q-1}\geqslant 0$,当且仅当 $X=Y$ 时等号成立。

规定:$D_0=\{(t,s):t>s\geqslant t_0\}$,$D=\{(t,s):t\geqslant s\geqslant t_0\}$。

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定理1　假设对任意 $T\geqslant t_0$，存在 a_0,b_0 且 $T<a_0<b_0$，令 $D(a_0,b_0)=\{u(t)\in C^1[a_0,b_0]:u(t)\neq 0,t\in(a_0,b_0),u(a_0)=u(b_0)=0\}$，如果存在函数 $H(t)\in D(a_0,b_0)$ 和一个正的非减函数 $\rho(t)\in C^1([t_0,\infty),\mathbf{R}_+)$ 满足以下条件：

$$\int_T^{+\infty}\left[\frac{C}{r(\xi)}\exp\left(-\int_T^{\xi}\frac{p(s)}{r(s)}\mathrm{d}s\right)\right]^{\frac{1}{\alpha}}\mathrm{d}\xi=+\infty,C>0\text{ 为一常数}\tag{2}$$

$$Q_1(H)=\int_{a_0}^{b_0}\left\{\rho(t)\left[H^{\alpha+1}(t)\mu_1(t)-\frac{r(t)}{(g'(t,a))^{\alpha}}\left(|H'(t)|+\frac{H(t)\rho'(t)}{(\alpha+1)\rho(t)}-\frac{H(t)p(t)}{(\alpha+1)r(t)}\right)^{\alpha+1}\right]\right\}\mathrm{d}t>0\tag{3}$$

且 $|H'(t)|+\frac{H(t)\rho'(t)}{(\alpha+1)\rho(t)}-\frac{H(t)p(t)}{(\alpha+1)r(t)}>0$，其中 $\mu_1(t)=\int_a^b q(t,\xi)(1-c(g(t,\xi)))^{\alpha}\mathrm{d}\sigma(\xi)$，则方程(1)是振动的。

证明　假设 $x(t)$ 是方程(1)的非振动解，$x(t)\neq 0,t\geqslant t_0$，不失一般性，可设 $x(t)$ 最终为正，即 $x(t)>0,t\geqslant t_0$。$x(t)<0$ 时同理可证。根据 $\lim\limits_{t\to\infty}\min\limits_{\xi\in[a,b]}\{g(t,\xi)\}=\infty,\lim\limits_{t\to\infty}h(t)=\infty$ 可知，存在 $T_0>t_0$，当 $t>T_0$ 时，使得 $x(t)>0$，$x(g(t,\xi))>0,x(h(t))>0$，$\xi\in[a,b]$，即 $z(t)>0$。

下面证明：$z'(t)\geqslant 0,t\geqslant T_0$。若不然，则存在 $T\geqslant T_0\geqslant t_0$，当 $t=T$ 时，$z'(T)<0$。那么根据(1)式，当 $t>T$，有 $(r(t)|z'(t)|^{\alpha-1}z'(t))'+p(t)|z'(t)|^{\alpha-1}z'(t)\leqslant 0$，也就是 $\left(\exp\left(\int_T^t\frac{p(s)}{r(s)}\mathrm{d}s\right)r(s)|z'(t)|^{\alpha-1}z'(t)\right)'\leqslant 0$。即 $|z'(t)|^{\alpha-1}z'(t)$ 是非增的，因此当 $t\geqslant T$，有 $z'(t)<0$。

令 $\mu(t)=-r(t)|z'(t)|^{\alpha-1}z'(t)$，则 $\mu(t)>0$。$\mu'(t)=-(r(t)|z'(t)|^{\alpha-1}z'(t))'=p(t)|z'(t)|^{\alpha-1}z'(t)+\int_a^b q(t,\xi)|x(g(t,\xi))|^{\alpha-1}x(g(t,\xi))\mathrm{d}\sigma(\xi)=-\frac{p(t)}{r(t)}\mu(t)+\int_a^b q(t,\xi)|x(g(t,\xi))|^{\alpha-1}x(g(t,\xi))\mathrm{d}\sigma(\xi)\geqslant-\frac{p(t)}{r(t)}\mu(t)$

从而有

$$\frac{\mu'(t)}{\mu(t)}\geqslant-\frac{p(t)}{r(t)}\tag{4}$$

对(4)式在 $[T,t]$ 上积分，可得 $\int_T^t\frac{\mu'(s)}{\mu(s)}\mathrm{d}s\geqslant-\int_T^t\frac{p(s)}{r(s)}\mathrm{d}s$，也就是 $\mu(t)\geqslant C\exp\left(\int_T^t-\frac{p(s)}{r(s)}\mathrm{d}s\right)$，其中 $C>0$ 为常数。故有

$$z'(t)\leqslant-\left(\frac{C}{r(t)}\exp\left(\int_T^t\left(-\frac{p(s)}{r(s)}\mathrm{d}s\right)\right)\right)^{\frac{1}{\alpha}}\tag{5}$$

对(5)式在 $[T,t]$ 上积分，得 $z(t)\leqslant z(T)-\int_T^t\left[\frac{C}{r(\xi)}\exp\left(\int_T^{\xi}\left(-\frac{p(s)}{r(s)}\mathrm{d}s\right)\right)\right]^{\frac{1}{\alpha}}\mathrm{d}\xi$。令 $t\to\infty$，并结合(2)式有 $\lim\limits_{t\to\infty}z(t)\leqslant z(T)-\lim\limits_{t\to\infty}\int_T^t\frac{C}{r(\xi)}\exp\left(\int_T^{\xi}\left(-\frac{p(s)}{r(s)}\mathrm{d}s\right)\right)^{\frac{1}{\alpha}}\mathrm{d}\xi\to-\infty$。这与 $z(t)>0$ 矛盾。所以 $z'(t)\geqslant 0$，因此有 $z''(t)\leqslant 0$。

又由 $g(t,\xi)$ 关于 ξ 是非减的，因此 $g(t,\xi)\geqslant g(t,a),\xi\in[a,b]$，从而有 $z(g(t,\xi))\geqslant z(g(t,a)),z'(g(t,a))\geqslant z'(t)$，又因为 $z(t)=x(t)+c(t)x(h(t))$，所以 $z(t)\geqslant x(t)$。从而有 $x(t)\geqslant(1-c(t))z(t)$，即有 $x(g(t,\xi))\geqslant(1-c(g(t,\xi)))z(g(t,\xi))\geqslant(1-c(g(t,\xi)))z(g(t,a))$。所以有

$$(r(t)|z'(t)|^{\alpha-1}z'(t))'+p(t)|z'(t)|^{\alpha-1}z'(t)+(z(g(t,a)))^{\alpha}\int_a^b q(t,\xi)(1-c(g(t,\xi)))^{\alpha}\mathrm{d}\sigma(\xi)\leqslant 0\tag{6}$$

于是(6)式可写为

$$(r(t)|z'(t)|^{\alpha-1}z'(t))'+p(t)|z'(t)|^{\alpha-1}z'(t)+(z(g(t,a)))^{\alpha}\mu_1(t)\leqslant 0\tag{6'}$$

令

$$w(t)=\frac{r(t)|z'(t)|^{\alpha-1}z'(t)}{(z(g(t,a)))^{\alpha}}\tag{7}$$

则 $w(t)>0$。

$$w'(t)=\frac{(r(t)|z'(t)|^{\alpha-1}z'(t))'(z(g(t,a)))^{\alpha}}{(z(g(t,a)))^{2\alpha}}-\frac{\alpha(z(g(t,a)))^{\alpha-1}z'(g(t,a))g'(t,a)r(t)|z'(t)|^{\alpha-1}z'(t)}{(z(g(t,a)))^{2\alpha}}\leqslant$$

$$\frac{-\left[p(t)\,|z'(t)|^{\alpha-1}z'(t)+\mu_1(t)(z(g(t,a)))^{\alpha}\right]}{(z(g(t,a)))^{\alpha}}-\frac{\alpha\,(z(g(t,a)))^{\alpha-1}z'(g(t,a))g'(t,a)r(t)\,|z'(t)|^{\alpha-1}z'(t)}{(z(g(t,a)))^{2\alpha}}=$$

$$-\frac{p(t)}{r(t)}w(t)-\mu_1(t)-\frac{\alpha r(t)\,|z'(t)|^{\alpha-1}z'(t)z'(g(t,a))g'(t,a)}{(z(g(t,a)))^{\alpha+1}}\leqslant-\frac{p(t)}{r(t)}w(t)-\mu_1(t)-\alpha\frac{w^{1+\frac{1}{\alpha}}(t)g'(t,a)}{(r(t))^{\frac{1}{\alpha}}}$$

即

$$w'(t)\leqslant-\frac{p(t)}{r(t)}w(t)-\mu_1(t)-\alpha\frac{w^{1+\frac{1}{\alpha}}(t)g'(t,a)}{(r(t))^{\frac{1}{\alpha}}}\tag{8}$$

根据假设,可选择 $a_0,b_0\geqslant T_0$ 且 $a_0<b_0$,$H(t)\in D(a_0,b_0)$ 是由假设条件中给出的,(8)式的两端同时乘以 $H^{\alpha+1}(t)\rho(s)$,并在 (a_0,b_0) 上积分,利用 $H(a_0)=H(b_0)=0$,可得

$$\int_{a_0}^{b_0}H^{\alpha+1}(t)\rho(t)\mu_1(t)\mathrm{d}t\leqslant-H^{\alpha+1}(t)\rho(t)w(t)\Big|_{a_0}^{b_0}+\int_{a_0}^{b_0}w(t)\left((\alpha+1)H^{\alpha}(t)H'(t)\rho(t)+H^{\alpha+1}(t)\rho'(t)\right)\mathrm{d}t-$$

$$\int_{a_0}^{b_0}H^{\alpha+1}(t)\rho(t)\frac{p(t)}{r(t)}w(t)\mathrm{d}t-\int_{a_0}^{b_0}\alpha H^{\alpha+1}(t)\rho(t)\frac{w^{1+\frac{1}{\alpha}}(t)g'(t,a)}{(r(t))^{\frac{1}{\alpha}}}\mathrm{d}t=$$

$$\int_{a_0}^{b_0}(\alpha+1)H^{\alpha}(t)\rho(t)\left(H'(t)+\frac{H(t)\rho'(t)}{(\alpha+1)\rho(t)}-\frac{H(t)p(t)}{(\alpha+1)r(t)}\right)w(t)\mathrm{d}t-$$

$$\alpha\int_{a_0}^{b_0}H^{\alpha+1}(t)\rho(t)\frac{w^{1+\frac{1}{\alpha}}(t)g'(t,a)}{(r(t))^{\frac{1}{\alpha}}}\mathrm{d}t\tag{9}$$

令 $q=1+\dfrac{1}{\alpha}$,$X=(\alpha\rho(t)g'(t,a))^{\frac{\alpha}{\alpha+1}}\dfrac{w(t)H^{\alpha}(t)}{(r(t))^{\frac{1}{\alpha+1}}}$,$Y=\left[\dfrac{\alpha\rho(t)r(t)}{(g'(t,a))^{\alpha}}\right]^{\frac{\alpha}{\alpha+1}}\cdot\left[|H'(t)|+\dfrac{H(t)\rho'(t)}{(\alpha+1)\rho(t)}-\dfrac{H(t)p(t)}{(\alpha+1)r(t)}\right]^{\alpha}$

则由引理1,可以得到 $(\alpha+1)H^{\alpha}(t)\rho(t)\left(H'(t)+\dfrac{H(t)\rho'(t)}{(\alpha+1)\rho(t)}-\dfrac{H(t)p(t)}{(\alpha+1)r(t)}\right)w(t)-\alpha H^{\alpha+1}(t)\rho(t)\dfrac{w^{1+\frac{1}{\alpha}}(t)g'(t,a)}{(r(s))^{\frac{1}{\alpha}}}\leqslant$ $\dfrac{\rho(t)r(t)}{(g'(t,a))^{\alpha}}\left[|H'(t)|+\dfrac{H(t)\rho'(t)}{(\alpha+1)\rho(t)}-\dfrac{H(t)p(t)}{(\alpha+1)r(t)}\right]^{\alpha+1}$。这样,得到

$$\int_{a_0}^{b_0}H^{\alpha+1}(t)\rho(t)\mu_1(t)\mathrm{d}t\leqslant\int_{a_0}^{b_0}\frac{\rho(t)r(t)}{(g'(t,a))^{\alpha}}\cdot\left[|H'(t)|+\frac{H(t)\rho'(t)}{(\alpha+1)\rho(t)}-\frac{H(t)p(t)}{(\alpha+1)r(t)}\right]^{\alpha+1}\mathrm{d}t\text{ 。}$$

此式与(3)式矛盾,定理得证。

推论1 如果定理1中 $\rho(t)=1$,且(3)式被替换为

$$\int_{a_0}^{b_0}\left\{H^{\alpha+1}(t)\mu_1(t)-\frac{r(t)}{(g'(t,a))^{\alpha}}\left[|H'(t)|-\frac{H(t)p(t)}{(\alpha+1)r(t)}\right]^{\alpha+1}\right\}\mathrm{d}t>0\tag{3'}$$

其中 $\mu_1(t)=\int_a^b q(t,\xi)\,(1-c(g(t,\xi)))^{\alpha}\mathrm{d}\sigma(\xi)$,则方程(1)是振动的。

定理2 设 $H\in C(D;\mathbf{R})$,$h\in C(D_0;\mathbf{R})$,$\kappa,\rho\in C^1([t_0,\infty);(0,\infty))$ 满足以下条件:

(i) $H(t,s)>0,(t,s)\in D_0$;$H(t,t)=0,t\geqslant t_0$;

(ii) $H(t,s)$ 在 D_0 上对第二个变量有连续非正的偏导数;

(iii) $-\dfrac{\partial}{\partial s}(H(t,s)k(s))+H(t,s)k(s)\left[\dfrac{p(s)}{r(s)}-\dfrac{\rho'(s)}{\rho(s)}\right]=h(t,s),(t,s)\in D_0$;

如果

$$\int_T^{+\infty}\left[\frac{C}{r(\xi)}\exp\left(-\int_T^{\xi}\frac{p(s)}{r(s)}\mathrm{d}s\right)\right]^{\frac{1}{\alpha}}\mathrm{d}\xi=+\infty;C>0\text{ 为常数}\tag{10}$$

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[H(t,s)k(s)\rho(s)\mu_1(s)-\frac{\rho(s)r(s)\,|h(t,s)|^{\alpha+1}}{(\alpha+1)^{\alpha+1}\,(H(t,s)k(s)g'(s,a))^{\alpha}}\right]\mathrm{d}s=\infty\tag{11}$$

成立,其中 $\mu_1(s)=\int_a^b q(s,\xi)\,(1-c(g(s,\xi)))^{\alpha}\mathrm{d}\sigma(\xi)$,则方程(1)是振动的。

证明　根据定理1的证明，同样可以得到(6′)式 $(r(t)|z'(t)|^{\alpha-1}z'(t))'+p(t)|z'(t)|^{\alpha-1}z'(t)+(z(g(t,a)))^{\alpha}\mu_1(t)\leqslant 0$。令

$$w(t)=\rho(t)\frac{r(t)|z'(t)|^{\alpha-1}z'(t)}{(z(g(t,a)))^{\alpha}} \tag{12}$$

则 $w(t)>0$。 $w'(t)=\frac{\rho'(t)r(t)|z'(t)|^{\alpha-1}z'(t)}{(z(g(t,a)))^{\alpha}}+\rho(t)\frac{(r(t)|z'(t)|^{\alpha-1}z'(t))'(z(g(t,a)))^{\alpha}}{(z(g(t,a)))^{2\alpha}}-$

$$\frac{\alpha(z(g(t,a)))^{\alpha-1}z'(g(t,a))g'(t,a)r(t)|z'(t)|^{\alpha-1}z'(t)}{(z(g(t,a)))^{2\alpha}}\leqslant\frac{\rho'(t)}{\rho(t)}w(t)+$$

$$\rho(t)\frac{-[p(t)|z'(t)|^{\alpha-1}z'(t)+\mu_1(t)(z(g(t,a)))^{\alpha}]}{(z(g(t,a)))^{\alpha}}-$$

$$\rho(t)\frac{\alpha(z(g(t,a)))^{\alpha-1}z'(g(t,a))g'(t,a)r(t)|z'(t)|^{\alpha-1}z'(t)}{(z(g(t,a)))^{2\alpha}}=$$

$$\left[\frac{\rho'(t)}{\rho(t)}-\frac{p(t)}{r(t)}\right]w(t)-\rho(t)\mu_1(t)-\frac{\alpha\rho(t)r(t)|z'(t)|^{\alpha-1}z'(t)z'(g(t,a))g'(t,a)}{(z(g(t,a)))^{\alpha+1}}\leqslant$$

$$\left[\frac{\rho'(t)}{\rho(t)}-\frac{p(t)}{r(t)}\right]w(t)-\rho(t)\mu_1(t)-\alpha\frac{w^{1+\frac{1}{\alpha}}(t)g'(t,a)}{(\rho(t)r(t))^{\frac{1}{\alpha}}}$$

即

$$w'(t)\leqslant\left[\frac{\rho'(t)}{\rho(t)}-\frac{p(t)}{r(t)}\right]w(t)-\rho(t)\mu_1(t)-\alpha\frac{w^{1+\frac{1}{\alpha}}(t)g'(t,a)}{(\rho(t)r(t))^{\frac{1}{\alpha}}} \tag{13}$$

对(13)式两边同时用乘以 $H(t,s)k(s)$，在 $[T,t]$ 上积分，并结合条件(iii)有

$$\int_T^t H(t,s)k(s)\rho(s)\mu_1(s)\mathrm{d}s\leqslant H(t,T)k(T)w(T)-\int_T^t\left(-\frac{\partial}{\partial s}(H(t,s)k(s))\right)w(s)\mathrm{d}s-$$

$$\int_T^t H(t,s)k(s)\left(\frac{p(s)}{r(s)}-\frac{\rho'(s)}{\rho(s)}\right)w(s)\mathrm{d}s-\int_T^t\alpha H(t,s)k(s)\frac{w^{1+\frac{1}{\alpha}}(t)g'(s,a)}{(\rho(s)r(s))^{\frac{1}{\alpha}}}\mathrm{d}s=$$

$$H(t,T)k(T)w(T)-\int_T^t h(t,s)w(s)\mathrm{d}s-\int_T^t\alpha H(t,s)k(s)\frac{w^{1+\frac{1}{\alpha}}(t)g'(s,a)}{(\rho(s)r(s))^{\frac{1}{\alpha}}}\mathrm{d}s \tag{14}$$

令 $q=1+\frac{1}{\alpha}$，$X=(\alpha H(t,s)k(s)g'(s,a))^{\frac{\alpha}{\alpha+1}}\frac{w(s)}{(\rho(s)r(s))^{\frac{1}{\alpha+1}}}$，$Y=\left[\frac{\alpha}{\alpha+1}\right]^{\alpha}\left[\frac{\rho(s)r(s)}{(\alpha H(t,s)k(s)g'(t,a))^{\alpha}}\right]^{\frac{\alpha}{\alpha+1}}|h(t,s)|^{\alpha}$

则由引理1可以得到

$$|h(t,s)w(s)|-\alpha H(t,s)k(s)\frac{w^{1+\frac{1}{\alpha}}(t)g'(s,a)}{(\rho(s)r(s))^{\frac{1}{\alpha}}}\leqslant\frac{\rho(s)r(s)}{(\alpha+1)^{\alpha+1}(H(t,s)k(s)g'(s,a))^{\alpha}}|h(t,s)|^{\alpha+1} \tag{15}$$

又由(13)、(14)式可以得到

$$\int_T^t\left[H(t,s)k(s)\rho(s)\mu_1(s)-\frac{\rho(s)r(s)}{(\alpha+1)^{\alpha+1}(H(t,s)k(s)g'(s,a))^{\alpha}}|h(t,s)|^{\alpha+1}\right]\mathrm{d}s\leqslant$$

$$H(t,T)k(T)|w(T)|,t\geqslant T\geqslant T_0\geqslant t_0 \tag{16}$$

进而对每个 $t\geqslant T\geqslant T_0\geqslant t_0$，有

$$\int_{t_0}^t[H(t,s)k(s)\rho(s)\mu_1(s)-\frac{\rho(s)r(s)}{(\alpha+1)^{\alpha+1}(H(t,s)k(s)g'(s,a))^{\alpha}}|h(t,s)|^{\alpha+1}]\mathrm{d}s=$$

$$-\left\{\int_{t_0}^{T_0}+\int_{T_0}^{t}\right\}\left[H(t,s)k(s)\rho(s)\mu_1(s)-\frac{\rho(s)r(s)}{(\alpha+1)^{\alpha+1}(H(t,s)k(s)g'(s,a))^{\alpha}}|h(t,s)|^{\alpha+1}\mathrm{d}s\right]\leqslant$$

$$H(t,t_0)\left[\int_{t_0}^{T_0}|k(s)\rho(s)\mu_1(s)|\mathrm{d}s+k(T_0)|w(T_0)|\right]$$

两边同时除以 $H(t,t_0)$，并令 $t\to\infty$ 有

$$\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^t\left[H(t,s)k(s)\rho(s)\mu_1(s)-\frac{\rho(s)r(s)|h(t,s)|^{\alpha+1}}{(\alpha+1)^{\alpha+1}(H(t,s)k(s)g'(s,a))^{\alpha}}\right]\mathrm{d}s\leqslant$$

$$\int_{t_0}^{T_0} |k(s)\rho(s)\mu_1(s)|\mathrm{d}s + k(T_0)|w(T_0)| < \infty$$

这与(11)式矛盾,故假设不成立,亦即方程(1)的解是振动的。 证毕

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Oscillation of Second Order Half-linear Neutral Damped Differential Equation

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Abstract: In this paper, Oscillation of a class of half-linear neutral damped differential equation with distributed deviating arguments is studied. By means of Yang inequality, the generalized Riccati transformation and function H, We obtain several new sufficient conditions for oscillation of all solutions of the equation $\int_T^{+\infty}\left[\frac{C}{r(\xi)}\exp\left(-\int_T^{\xi}\frac{p(s)}{r(s)}\mathrm{d}s\right)\right]^{\frac{1}{\alpha}}\mathrm{d}\xi=+\infty$, and such that $Q_1(H)>0$, $\left(|H'(t)|+\frac{H(t)\rho'(t)}{(\alpha+1)\rho(t)}-\frac{H(t)p(t)}{(\alpha+1)r(t)}\right)>0$ and $\limsup_{t\to\infty}\frac{1}{H(t,t_0)}\int_{t_0}^{t}\left[H(t,s)k(s)\rho(s)\mu_1(s)-\frac{\rho(s)r(s)|h(t,s)|^{\alpha+1}}{(\alpha+1)^{\alpha+1}(H(t,s)k(s)g'(s,a))^{\alpha}}\right]\mathrm{d}s=\infty$. Which generalize and improve some knownresults.

Key words: distributed deviating arguments; neutral; half-linear differential equation; oscillation

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