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不允许卖空限制下跳扩散模型的动态均值-方差资产负债问题*

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摘要:研究了在不允许卖空情况下跳扩散模型的动态均值-方差资产负债问题。本文利用两个黎卡提方程构造出 HJB 方程的一个连续解 $V(t,x)$,然后验证 这个解是方程的粘性解,并利用粘性解和识别定理得到了最优投资策略和有效边界。

关键词:HJB 方程;粘性解;负债;有效边界

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近年来有很多人研究最优投资组合选择问题[1-15],但是一般把股份描述为一个连续的时间过程,即扩散过程,由于受到外界的影响股份在一个有限时间内很可能有突发性跳跃,实证结果也证实了这一点,因此用跳扩散过程描述股票价格的趋势更符合实际情况。对股份跳跃过程的投资问题近年来也有许多研究,Merton[1],Jones[2],Jarrow 与 Rudd[5]研究了跳跃扩散过程的期权定价问题。Guo[3]研究了跳跃扩散过程的关于投资组合选择的均值-方差模式。由于跳跃的因素的存在,关于连续时间的扩散过程的一般最优控制的识别定理不一定成立。Guo[3]证明跳扩散过程的一般最优控制的识别定理,并用动态规划原理通过求解 HJB 议程得到原问题的最优策略,本文研究了关于在负债问题的均值-方差模式,假设股票价格服从跳扩散过程,但是由于模式中含有交易限制,因此不能直接应用移卡提方程方法求解。本文用 HJB 方程来代替黎卡提方程,而且由于控制限制的存在使得 HJB 方程没有平滑解,然后证明了这个连续解是方程的粘性解,并利用粘性解和识别定理得到了最优投资策略和有效边界。

1 市场模型

考虑到一个金融市场,不确定性来自两个驱动因素:一个是概率空间(Ω, F, P, F)上的一维布朗运动 W_t,另一个是概率空间上密度为 λ_t 的一维的 Poisson 过程 N_t。市场中有两个资产,一个是无风险资产,价格过程 S_t^0 满足

$$\begin{cases} \mathrm{d}S_t^0 = r_t S_t^0 \mathrm{d}t, t\in[0,T] \\ S_0^0 = 1 \end{cases} \tag{1}$$

其中 $r_t \in C([0,T];\mathbf{R}^+)$是无风险利率。剩余的是风险资产且它的价格过程满足随机微分方程

$$\begin{cases} \mathrm{d}S_t = S_t\mu_t \mathrm{d}t + S_t\sigma_t \mathrm{d}Wt + S_t\varphi_t \mathrm{d}N_t, t\in[0,T] \\ S_0 = a \in \mathbf{R} \end{cases} \tag{2}$$

假设 μ_t, σ_t 和 φ_t 在$[0, T]$上是可测且一致有界的,投资者的初始财富为 x_0,同时初始负债为 l_0,在 t 时刻的负债记作 l_t,满足

$$\mathrm{d}L_t = u_t \mathrm{d}t + v_t \mathrm{d}B_t,\ L_0 = l \tag{3}$$

其中$\{B_t: t\in[0, T]\}$是在概率空间(Ω, F, P, F)上的一维 Brownian 运动。通常负债不是独立于风险资产价格的,因此 B_t 与 W_t 是相关的,相关系统为 ρ_t,并满足

$$B_t = \rho_t W_t + \sqrt{1-\rho_t^2} W_t^0 \tag{4}$$

其中 W_t^0 独立于 W_t。因此

$$\mathrm{d}L_t = u_t \mathrm{d}t + v_t\rho_t \mathrm{d}W_t + v_t \sqrt{1-\rho_t^2}\, \mathrm{d}W_t^0, L_0 = l \tag{5}$$

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考虑一个投资者在 t 时刻的总财富为 X_t，π_t 表示 t 时刻投资在风险资产上的资金数，由于不允许卖空，所以 $\pi(t)\geqslant 0$。投资者在 t 时刻的财富 X_t 满足

$$\mathrm{d}X_t=(r_tX_t+b_t\pi_t-u_t)\mathrm{d}t+(\pi_t\sigma_t+\delta_t)\mathrm{d}W_t+\delta_t^0\mathrm{d}W_t^0+\pi_t\varphi_t\mathrm{d}N_t,X_0=x \tag{6}$$

其中 $b_t=(\mu_t-r_tl)$，$\delta_t=-v_t\rho_t$，$\delta_t^0=-v_t\sqrt{1-\rho_t^2}$。因此不允许卖空限制下的均值方差资产负债问题为

$$\mathrm{P}\quad \min_{\pi\geqslant 0}(-EX_T,\mathrm{Var}\ X_T) \tag{7}$$

问题(7)是双目标优化问题，由最优控制理论等价于如下最优控制问题

$$\mathrm{P}(\lambda)\quad \min_{\pi\geqslant 0}(-EX_T,\mathrm{Var}\ X_T) \tag{8}$$

其中 $\lambda>0$。

2 最优策略

因为目标函数在动态规划意义下是不可分的，所以直接解问题(8)比较困难。由 Li 和 Zhou[4] 利用嵌套法可以将问题(8)嵌入到一个二次随机优化问题

$$\mathrm{A}(\lambda,\ \omega)\quad \min_{\pi\geqslant 0}E(\lambda X_T^2-\omega X_T) \tag{9}$$

其中 $\lambda>0$，$\omega\in\mathbf{R}$。

问题(9)可以利用动态规划方法求解，其对应的 HJB 方程为

$$\begin{cases}\dfrac{\partial v}{\partial t}(t,x)+\inf\limits_{\pi\geqslant 0}\{\dfrac{\partial v}{\partial x}(t,x)[r_tx_t+b_t\pi-u_t+\dfrac{1}{2}\dfrac{\partial^2 v}{\partial x^2}(t,x)[((\sigma_t\pi_t)+\delta_t)^2+\delta_t^{0^2}]+\\ \lambda_t[V(t,x+\pi_t\varphi_t)-V(t,x)]\}=0\\ V(T,X)=\lambda x^2-\omega x\end{cases} \tag{10}$$

设

$$V=\frac{1}{2}\overline{P}_tx^2+\overline{g}_tx+\overline{c}_t \tag{11}$$

将(11)式代入(10)式可得

$$\left(\frac{1}{2}\dot{\overline{P}}_t+\overline{P}_tr_t\right)x^2+(\dot{\overline{g}}_t-\dot{\overline{P}}_tu_t+\overline{g}_tA_t)x+(\dot{\overline{c}}_t-\overline{g}_tu_t+\frac{1}{2}\overline{P}_t\delta^2+\frac{1}{2}\overline{P}_t\delta_t^{0^2}+$$
$$\frac{1}{2}\overline{P}_t\inf_{\pi\geqslant 0}\{(\sigma_t^2+\lambda_t\varphi_t^2)\pi_t^2+2[\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)x+(b_t+\varphi_t\lambda_t)\eta_t]\pi_t\}=0 \tag{12}$$

其中 $\eta_t=\dfrac{\overline{g}_t}{\overline{P}_t}$。当 $\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)x+(b_t+\varphi_t\lambda_t)\eta_t<0$，有

$$\pi_t^*=-\frac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)x+(b_t+\varphi_t\lambda_t)\eta_t}{\sigma_t^2+\lambda_t\varphi_t^2} \tag{13}$$

将(13)式代入(12)式得 $\overline{P}(t)$，$\overline{g}(t)$ 和 $\overline{c}(t)$ 满足黎卡提方程

$$\begin{cases}\dot{\overline{P}}_t=[-2r_t+\dfrac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}]\overline{P}_t\\ \overline{P}_T=2\lambda\end{cases}$$

$$\begin{cases}\dot{\overline{g}}_t=[-r_t+\dfrac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}]\overline{g}_t+[u_t+\dfrac{\sigma_t\delta_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}]\overline{P}_t\\ \overline{g}_T=-\omega\end{cases} \tag{14}$$

$$\begin{cases}\dot{\overline{c}}_t=[u_t+\dfrac{\sigma_t\delta_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}]\overline{g}_t+\dfrac{1}{2}\overline{P}[\dfrac{\sigma_t^2\varphi_t^2}{\sigma_t^2+\lambda_t\varphi_t^2}]-(\delta_t^2+\delta_t^{0^2})]+\dfrac{1}{2}\dfrac{(B_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}\overline{P}_t^{-1}\overline{g}_t^2\\ \overline{c}_T=0\end{cases}$$

当 $\sigma_t\delta_t(B_t+\varphi_t\lambda_t)x+(B_t+\varphi_t\lambda_t)\overline{\eta}_t>0$，有 $\pi_t^*=0$，则对应的 $\widetilde{P}(t)$，$\widetilde{g}(t)$ 和 $\widetilde{c}(t)$ 满足黎卡提方程

$$\begin{cases}\dot{\widetilde{P}}_t=-2r_t\widetilde{P}_t\\ \widetilde{P}_T=2\lambda\end{cases}$$

$$\begin{cases}\dot{\tilde{g}}_t=u_t\tilde{P}_t-r_t\tilde{g}_t\\ \tilde{g}_T=-\omega\end{cases}$$

$$\begin{cases}\dot{\tilde{c}}_t=u_t\tilde{g}_t-\frac{1}{2}\tilde{P}_t\delta_t^{0^2}-\frac{1}{2}\tilde{P}_t\delta_t^2\\ \tilde{c}_T=0\end{cases} \tag{15}$$

定理 1 问题(9)的最优策略

$$\pi_t^*=\begin{cases}-\frac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)x+(b_t+\varphi_t\lambda_t)\eta_t}{\sigma_t^2+\lambda_t\varphi_t^2},\text{当 } x-\int_t^T\left[u_t+\frac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]\times\\ e^{\int_z^T r_s ds}dz-\gamma e^{-\int_z^T r_s ds}+\frac{\sigma_t\delta_t}{b_t+\varphi_t\lambda_t}\leqslant 0\text{ 时}\\ 0,\text{当 } x-\int_t^T\left[u_t+\frac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]e^{\int_z^T r_s ds}dz-\gamma e^{-\int_z^T r_s ds}+\frac{\sigma_t\delta_t}{b_t+\varphi_t\lambda_t}>0\text{ 时}\end{cases} \tag{16}$$

且

$$V(t,x)=\begin{cases}\frac{1}{2}\bar{P}_t x^2+g_t x+\bar{c}_t,\text{当 } x-\int_t^T\left[u_t+\frac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]\times\\ e^{\int_z^T r_s ds}dz-\gamma e^{-\int_z^T r_s ds}+\frac{\sigma_t\delta_t}{b_t+\varphi_t\lambda_t}\leqslant 0\text{ 时}\\ \frac{1}{2}\tilde{P}_t x^2+\tilde{g}_t x+\tilde{c}_t,\text{当 } x-\int_t^T\left[u_t+\frac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]\times\\ e^{\int_z^T r_s ds}dz-\gamma e^{-\int_z^T r_s ds}+\frac{\sigma_t\delta_t}{b_t+\varphi_t\lambda_t}>0\text{ 时}\end{cases} \tag{17}$$

是 HJB 方程(10)的一个粘性解,其中

$$\begin{cases}\bar{P}_t=e^{\int_t^T\left(2r_s-\frac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}\right)ds}\\ \bar{g}_t=e^{\int_t^T\left(r_s-\frac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}\right)ds}\int_t^T\left[u_t+\frac{\sigma_t\delta_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]e^{\int_z^T r_s ds}dz\\ \dot{\bar{c}}_t=\left[u_t+\frac{\sigma_t\delta_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]e^{\int_t^T(r_s-\frac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}ds}\int_t^T\left[u_t+\frac{\sigma_t\delta_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]e^{\int_z^T r_s ds}dz+\frac{1}{2}e^{\int_t^T\left(2r_s-\frac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}\right)ds}\times\\ \left[\frac{\sigma_t^2}{\sigma_t^2+\lambda_t\varphi_t^2}-(\sigma_t^2+\delta_t^{0^2})\right]+\frac{1}{2}\frac{(B_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}e^{\int_t^T-\frac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}ds}\left\{\left[\int_t^T u_t+\frac{\sigma_t\delta_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]e^{\int_z^T r_s ds}dz\right\}^2\\ \tilde{P}_t=e\int_t^T 2r_s ds\\ \tilde{g}_t=e^{\int_t^T A_s ds}\int_t^T-u_z e^{\int_z^T r_s ds}dz\\ \tilde{c}_t=e^{\int_t^T A_s ds}\int_t^T-u_z e^{\int_z^T r_s ds}dz u_t+\frac{1}{2}e^{\int_t^T 2r_s ds}(\sigma_t^{0^2}+\delta_t^2)dv\end{cases}$$

3 有效前沿

策略选择的一个重要部分就是确定有效前沿,其描述了期望和方差在最优策略下的关系在一般投资组合问题下,有效前沿是最优策略下终端财富期望和终端财富方差之间的关系,而这里要在负债下讨论端财富期望和终端财富方差之间的关系。

当 $x-\int_t^T\left[u_t+\frac{\sigma_t\delta_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]e^{\int_z^T r_s ds}dz-\gamma e^{-\int_z^T r_s ds}+\frac{\sigma_t\delta_t}{b_t+\varphi_t\lambda_t}\leqslant 0$ 时,把(16)式代入(6)式得

$$\begin{aligned}\mathrm{d}X_t=&\left[\left(r_t-\frac{b_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right)X_t+\frac{b_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}(\gamma-\eta_t)-\left(\frac{b_t\delta_t\sigma_t)}{\sigma_t^2+\lambda_t\varphi_t^2}+u_t\right)\right]\mathrm{d}t+\\&\left[-\frac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)x+(b_t+\varphi_t\lambda_t)(\eta_t-\gamma)}{\sigma_t^2+\lambda_t\varphi_t^2}\times\sigma_t+\delta_t\right]\mathrm{d}W_t+\\&\delta_t^0\mathrm{d}W_t^0+\left[-\frac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)x+(b_t+\varphi_t\lambda_t)(\eta_t-\gamma)}{\sigma_t^2+\lambda_t\varphi_t^2}\varphi_t\mathrm{d}N_t\text{。}\right.\end{aligned}\tag{18}$$

等号两边取期望

$$\begin{aligned}\mathrm{d}EX_t=&\left[\left(r_t-\frac{b_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right)EX_t+\frac{b_t(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}(\gamma-\eta_t)-\left(\frac{b_t\delta_t\sigma_t}{\sigma_t^2+\lambda_t\varphi_t^2}+u_t\right)\right]\mathrm{d}t-\\&\frac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)EX_t+(b_t+\varphi_t\lambda_t)(\eta_t-\gamma)}{\sigma_t^2+\lambda_t\varphi_t^2}\varphi_t\lambda_t\mathrm{d}t=\\&\left[\left(r_t-\frac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}\right)EX_t+\frac{(b_t+\varphi_t\lambda_t)^2(\eta_t-\gamma)}{\sigma_t^2+\lambda_t\varphi_t^2}+\frac{\delta_t\sigma_t(b_t-\lambda_t\varphi_t)}{\sigma_t^2+\lambda_t\varphi_t^2}+u_t\right]\mathrm{d}t\end{aligned}\tag{19}$$

解(19)式得

$$EX_t=x\mathrm{e}^{\int_0^t\left(r_t-\frac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}\mathrm{d}z\right)}+\int_0^T\left[\frac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}\gamma-\frac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}\eta_t-\mathcal{L}_t\right]\times\mathrm{e}^{\int_s^t(r_t-\frac{(b_t+\varphi_t\lambda_t)^2}{\sigma_t^2+\lambda_t\varphi_t^2}\mathrm{d}z}\mathrm{d}s\tag{20}$$

其中$\mathcal{L}_t=\dfrac{\delta_t\sigma_t(b_t-\lambda_t\varphi_t)}{\sigma_t^2+\lambda_t\varphi_t^2}+u_t$,因此

$$EX_T=\alpha+\beta\gamma\tag{21}$$

其中 $\alpha=-\int_0^T\mathcal{L}_t\mathrm{e}^{\int_t^T r_z\mathrm{d}z-\int_0^T\frac{(b_z+\varphi_z\lambda_z)^2}{\sigma_z^2+\lambda_z\varphi_z^2}\mathrm{d}z}\mathrm{d}t+x\mathrm{e}^{\int_0^T\left(r_z-\frac{(b_z+\varphi_z\lambda_z)^2}{\sigma_z^2+\lambda_z\varphi_z^2}\mathrm{d}z\right)}$,$\beta=1-\mathrm{e}^{-\int_0^T\frac{(b_z+\varphi_z\lambda_z)^2}{\sigma_z^2+\lambda_z\varphi_z^2}\mathrm{d}z}$

类似于上述步骤可得

$$EX_T^2=\eta+\beta\gamma^2\tag{22}$$

其中

$$\begin{aligned}\eta=&-2x\mathrm{e}^{\int_0^T\left(r_z-\frac{(b_z+\varphi_z\lambda_z)^2}{\sigma_z^2+\lambda_z\varphi_z^2}\right)\mathrm{d}z}\int_0^T(-\mathcal{L}_t)\mathrm{e}^{\int_t^T r_z\mathrm{d}z}\mathrm{d}t+\mathrm{e}^{\int_0^T\frac{(b_z+\varphi_z\lambda_z)^2}{\sigma_z^2+\lambda_z\varphi_z^2}\mathrm{d}z}\left(\int_0^T(-\mathcal{L}_t)\mathrm{e}^{\int_t^T r_z\mathrm{d}z-\int_0^T\frac{(b_z+\varphi_z\lambda_z)^2}{\sigma_z^2+\lambda_z\varphi_z^2}\mathrm{d}z}\mathrm{d}t\right)^2+\\&x^2\mathrm{e}^{\int_0^T\left(2r_z-\frac{(b_t+\varphi_t\lambda_t)[2\varphi_t(\sigma_t^2+\lambda_t\varphi_t^2)-\varphi_t^3\lambda_t^2+b_t\sigma_t^2]}{(\sigma_t^2+\lambda_t\varphi_t^2)^2}\right)\mathrm{d}z}+\int_0^T\left(\delta_t^{0^2}+\frac{\lambda_t\varphi_t^2\sigma_t^2\delta_t^2}{(\sigma_t^2+\lambda_t\varphi_t^2)}\right)\mathrm{e}^{\int_t^T\left(2r_z-\frac{(b_t+\varphi_t\lambda_t)[2\varphi_t(\sigma_t^2+\lambda_t\varphi_t^2)-\varphi_t^3\lambda_t^2+b_t\sigma_t^2]}{(\sigma_t^2+\lambda_t\varphi_t^2)^2}\right)\mathrm{d}z}\mathrm{d}t\end{aligned}$$

由(21)式可得

$$\gamma=\frac{EX_T-\alpha}{\beta}$$

$$\mathrm{Var}\ X_T=\beta(1-\beta)\gamma^2-2\alpha\beta\gamma+\eta-\alpha^2=\frac{1-\beta}{\beta}\left[EX_T-\frac{\alpha}{1-\beta}-\frac{\alpha^2}{1-\beta}\right]^2+\eta\tag{23}$$

当 $x-\int_t^T\left[u_t+\dfrac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]\mathrm{e}^{\int_z^T r_s\mathrm{d}s}\mathrm{d}z-\gamma\mathrm{e}^{-\int_z^T r_s\mathrm{d}s}+\dfrac{\sigma_t\delta_t}{b_t+\varphi_t\lambda_t}>0$ 时,有

$$\mathrm{Var}\ X_T=-\int_0^T(\delta_t^{0\,2}+\delta_t^2)\mathrm{e}^{\int_t^T 2r_z\mathrm{d}z}=v$$

定理 2　问题(7)的有效边界为

$$\mathrm{Var}\ X_T=\begin{cases}\dfrac{\mathrm{e}^{-\int_0^T\frac{(b_z+\varphi_z\lambda_z)^2}{\sigma_z^2+\lambda_z\varphi_z^2}\mathrm{d}z}}{1-\mathrm{e}^{-\int_0^T\frac{(b_z+\varphi_z\lambda_z)^2}{\sigma_z^2+\lambda_z\varphi_z^2}\mathrm{d}z}}\times[EX_T-(x\mathrm{e}^{\int_0^T r_z\mathrm{d}z}-\int_0^T\mathcal{L}_t\mathrm{e}^{\int_0^T r_z\mathrm{d}z}\mathrm{d}t)]^2+\\\int_0^T\left(\delta_t^{0^2}+\dfrac{\lambda_t\varphi_t^2\sigma_t^2\delta_t^2}{(\sigma_t^2+\lambda_t\varphi_t^2)}\right)\times\mathrm{e}^{\int_t^T(2r_z-\frac{(b_t+\varphi_t\lambda_t)[2\varphi_t(\sigma_t^2+\lambda_t\varphi_t^2)-\varphi_t^3\lambda_t^2+b_t\sigma_t^2]}{(\sigma_t^2+\lambda_t\varphi_t^2)}\mathrm{d}z]}\mathrm{d}t,\text{当}\\x-\int_t^T\left[u_t+\dfrac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]\mathrm{e}^{\int_z^T r_s\mathrm{d}s}\mathrm{d}z-\gamma\mathrm{e}^{-\int_z^T r_s\mathrm{d}s}+\dfrac{\sigma_t\delta_t}{b_t+\varphi_t\lambda_t}\leqslant 0\text{ 时}\\v,\text{当 }x-\int_t^T\left[u_t+\dfrac{\sigma_t\delta_t+(b_t+\varphi_t\lambda_t)}{\sigma_t^2+\lambda_t\varphi_t^2}\right]\mathrm{e}^{\int_z^T r_s\mathrm{d}s}\mathrm{d}z-\gamma\mathrm{e}^{-\int_z^T r_s\mathrm{d}s}+\dfrac{\sigma_t\delta_t}{b_t+\varphi_t\lambda_t}>0\text{ 时}\end{cases}$$

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Dynamic Mean-variance Asset-liability Problem for Jump-diffusion Model with No-shorting Constraints

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Abstract: This paper deals with a mean-variance asset-liability problem for jump-diffusion model under no-shorting constraints. A continuous solution of the HJB equation is constructed through two Ricttati equations, and show that this function is a viscosity solution of the HJB equation. Using the viscosity solution and verification theorem, explicitly the optimal strategy and the mean-variance efficient frontier in closed forms are obtained.

Key words: HJB equation; viscosity solution; liability; effcient frontier.

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