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Some Equivalent Condition of Generalized Convex Fuzzy Mapping*

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Abstract: In this paper, we obtain that a fuzzy mapping F is (strictly) pseudoconvex if and only if $\widetilde{\nabla}F$ is (strictly) pseudomonotone, and a differenitiable fuzzy mapping F is quasiconvex if and only if $\widetilde{\nabla}F$ is quasimonotone. These results will be useful in checking the generalized convexity of the differenitiable fuzzy mapping and presenting some characterizations of solutions for fuzzy mathematical programming.

Key words: pseudoconvex fuzzy mappings; strictly pseudoconvex fuzzy mappings; quasiconvex fuzzy mappings
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1 Introduction

The occurrence of randomness and fuzziness in the real world is inevitable owing to some unexpected situations. In [1], Zadeh initially introduced the concept of fuzzy number. Since then, theories of fuzzy number and their applications have been extensively and intensively studied by many scholars, one can refer to [2-7]. Mathematical programming under fuzzy environment or which involves fuzziness is called fuzzy mathematical programming. Bellman and Zadeh[8] introduced fuzzy optimization problems and stated that a fuzzy decision can be viewed as the intersection of fuzzy goals and problem constraints.

Nanda and Kar[9] proposed the concept of convex fuzzy mappings and proved that a fuzzy mapping is convex if and only if its epigraph is a convex set. At the same time, some applications to fuzzy mathematical programming problems were studied. The convexity has been playing an important role in fuzzy mathematical programming theory. Some related research work has been carried out, one can refer to [10-19]. But it is obvious that the condition of convex fuzzy mappings is too strict. Recently, different types of generalized convex fuzzy mappings were defined. Some properties and the applications were studied in fuzzy mathematical programming problems. Especially, Panigrahi et al.[20] proposed the concept of quasiconvex fuzzy mappings, which is different from Nanda and Kar[9] as well as Syau[16], and derived the Karush-Kuhn-Tucker optimality conditions for the constrained fuzzy minimization problems. Strict inequality relation between fuzzy numbers is used in [20], which is too much restrictive. Syau[17] introduced the concept of generalized convexity such as pseudoconvexity for fuzzy mappings with several variables and studied some basic differentiability properties of fuzzy mappings from the standpoint of convex analysis.

Motivated by the earlier works of Panigrahi et al.[20], Karamardian[21], Karamardian and Schaible[22], Wang[23] as well as Liu et al.[24], in this paper, we establish some equivalent conditions of (strictly) pseud-

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oconvex and quasiconvex fuzzy mappings.

2 Preliminaries

In this section, we quote some preliminary notations and definitions.

Let **R** be the set of all real numbers. A fuzzy number is a mapping $\mu:\mathbf{R}\to[0,1]$ with the following properties: 1) μ is normal, that is, $[\mu]_1=\{x\in\mathbf{R}:\mu(x)=1\}\neq\emptyset$; 2) μ is upper semicontinuous; 3) μ is convex, that is, $\mu(\lambda x+(1-\lambda)y)\geqslant\min\{\mu(x),\ \mu(y)\}$ for all $x,\ y\in\mathbf{R},\ \lambda\in[0,\ 1]$; 4) the support of μ, $\mathrm{supp}(\mu)=\{x\in\mathbf{R}:\mu(x)>0\}$ and its closure $\mathrm{cl}(\mathrm{supp}(\mu))$ is compact.

Let $\mathfrak{J}$ be the all of fuzzy number on **R**. The α-level set of a fuzzy number $\mu\in\mathfrak{J}$, $0\leqslant\alpha\leqslant1$, denoted by $[\mu]_\alpha$, is defined as $[\mu]_\alpha=\begin{cases}\{x\in\mathbf{R}:\mu(x)\geqslant\alpha\},0<\alpha\leqslant1\\ \mathrm{cl}(\mathrm{supp}(\mu)),\alpha=0\end{cases}$.

It is clear that the α-level set of a fuzzy number is a closed and bounded interval $[\mu_*(\alpha),\ \mu^*(\alpha)]$. $\mu_*(\alpha)$ denotes the left-hand end point of $[\mu]_\alpha$ and $\mu^*(\alpha)$ denotes the right-hand end point of $[\mu]_\alpha$. Also any $m\in\mathbf{R}$ can be regarded as a fuzzy number $\tilde{m}$ defined by $\tilde{m}(t)=\begin{cases}1,t=m\\0,t\neq m\end{cases}$.

In particular, the fuzzy number $\tilde{0}$ is defined as $\tilde{0}(t)=1$ if $t=0$ and otherwise $\tilde{0}(t)=0$. Thus, fuzzy number μ can be identified by a parameterized triples $\{(\mu_*(\alpha),\mu^*(\alpha),\alpha):\alpha\in[0,1]\}$.

For fuzzy number μ and ν parameterized by $\{(\mu_*(\alpha),\mu^*(\alpha),\alpha):\alpha\in[0,1]\}$ and $\{(\nu_*(\alpha),\nu^*(\alpha),\alpha):\alpha\in[0,1]\}$, respectively, and each nonnegative real number k, we define the addition $\mu\tilde{+}\nu$ and nonnegative scalar multiplication $k\mu$ as follows

$$\mu\tilde{+}\nu=\{(\mu_*(\alpha)+\nu_*(\alpha),\mu^*(\alpha)+\nu^*(\alpha),\alpha):\alpha\in[0,1]\},k\mu=\{(k\mu_*(\alpha),k\mu^*(\alpha),\alpha):\alpha\in[0,1]\}.$$

Obviously, for each real number r, $\mu\tilde{+}r=\{(\mu_*(\alpha)+r,\mu^*(\alpha)+r,\alpha):\alpha\in[0,1]\}$.

Moreover, define the opposite of a fuzzy number μ to be the fuzzy number $-\mu$ satisfying $(-\mu)(x)=\mu(-x)$. In other words, if μ is represented by the parametric form $\{(\mu_*(\alpha),\mu^*(\alpha),\alpha):\alpha\in[0,1]\}$, then $-\mu$ is represented by the corresponding parametric form $\{(-\mu_*(\alpha),-\mu^*(\alpha),\alpha):\alpha\in[0,1]\}$. We represent a fuzzy number μ as $[\mu_*(\alpha),\mu^*(\alpha)]$.

A fuzzy number $\mu=[\mu_*(\alpha),\mu^*(\alpha)]$ is said to be a triangular fuzzy number if $\mu_*(1)=\mu^*(1)$. Moreover, if $\mu_*(\alpha)$ and $\mu^*(\alpha)$ are liner, then we say μ a liner triangular fuzzy number. We denote by $\langle\mu_*(0);\mu^*(1);\mu^*(0)\rangle$.

Definition 1[20] For $u,v\in\mathfrak{J}$, we say that $u\preceq v$ if for each $\alpha\in[0,1]$, $\mu_*(\alpha)\leqslant\nu_*(\alpha),\mu^*(\alpha)\leqslant\nu^*(\alpha)$. If $u\preceq v,\nu\preceq\mu$, then $u=\nu$. We say that $u\prec\nu$, if $u\preceq v$ and there exists $\alpha_0\in[0,1]$ such that $\mu_*(\alpha_0)<\nu_*(\alpha_0)$ or $\mu^*(\alpha_0)<\nu^*(\alpha_0)$. For $u,v\in\mathfrak{J}$, If either $u\preceq v$ or $\nu\preceq\mu$, then u and ν are comparable, otherwise non-comparable.

A mapping $F:K\subseteq\mathbf{R}^n\to\mathfrak{J}$ is said to be a fuzzy mapping. For any $\alpha\in[0,1]$ and for any $x\in K$, we denote $F(x)=[F_*(x,\alpha),F^*(x,\alpha)]$.

Definition 2[20] Let $F:K\subseteq\mathbf{R}^n\to\mathfrak{J}$ is fuzzy mapping. Then, F is said to be comparable if for each pair $x\neq y\in K$, $F(x)$ and $F(y)$ are comparable. Otherwise, F is said to be non-comparable.

Definition 3[20] Let $K\subseteq\mathbf{R}^n$ be an open set and assume that $F:K\to\mathfrak{J}$ be a fuzzy mapping. Let $x=(x_1,x_2,\cdots,x_n)\in K$ and $D_{x_i},i=1,2,\cdots,n$ stand for the partial dierentiation with respect to the ith variable x_i. Assume that for all $\alpha\in[0,1]$, $F_*(x,\alpha)$ and $F^*(x,\alpha)$ have continuous partial derivatives. Define $D_{x_i}F(x)[\alpha]=[D_{x_i}F_*(x,\alpha),D_{x_i}F^*(x,\alpha)]$, for $i=1,2,\cdots,n,\alpha\in[0,1]$.

If for each $i=1,2,\cdots,n$, $D_{x_i}F(x)[\alpha]$ defines the α-cut of a fuzzy mapping number at x, and we denote by $\tilde{\nabla}F(x)=(D_{x_1}F(x),D_{x_2}F(x),\cdots,D_{x_n}F(x))$.

We call $\tilde{\nabla}F(x)$, the gradient of the fuzzy mapping F at x. A fuzzy number F is said to be differentiable at x if $\tilde{\nabla}F(x)$ exists and both $F_*(x,\alpha),F^*(x,\alpha)$ for each $\alpha\in[0,1]$ are differentiable at x.

Definition 4[20] Let $K\subseteq\mathbf{R}^n$ be a nonempty open convex set and $F:K\rightarrow\mathfrak{J}$ be a differentiable fuzzy mapping. F is said to be pseudconvex if for each $x,y\in K$, $\tilde{0}\preceq\tilde{\nabla}F(x)^{\mathrm{T}}(y-x)$ implies that $F(x)\preceq F(y)$.

Definition 5[20] Let $K\subseteq\mathbf{R}^n$ be a nonempty convex set and $F:K\rightarrow\mathfrak{J}$ be a fuzzy mapping. F is said to be quasiconvex if for each $x,y\in K$ and for $\lambda\in(0,1)$, The following implication hold $F(\lambda x+(1-\lambda)y)\preceq\max\{F(x),F(y)\}$, where $F(x)$ and $F(y)$ are comparable.

Definition 6[23] Let $K\subseteq\mathbf{R}^n$ be a nonempty convex set and $F:K\rightarrow\mathfrak{J}$ be a differentiable fuzzy mapping. F is said to be strictly pseudconvex if for each $x,y\in K$, $\tilde{0}\preceq\tilde{\nabla}F(x)^{\mathrm{T}}(y-x)$ implies that $F(x)\prec F(y)$.

In the following sections, we always assume that $K\subseteq\mathbf{R}^n$ be a nonempty convex set, $F:K\subseteq\mathbf{R}^n\rightarrow\mathfrak{J}$ be a differentiable fuzzy mapping and F be comparable.

3 Pseudoconvexity of fuzzy mappings

In this section, we establish the equivalent conditions of pseudoconvexity and strictly pseudoconvexity fuzzy mappings. We first give some lemmas which will be used in the sequel.

Lemma 1[23] Assume that F be a pseudoconvexity fuzzy mappings. Then F is a quasiconvex fuzzy mapping.

Lemma 2[20] F is a quasiconvex fuzzy mapping if and only if for each $x,y\in K$, $F(x)\preceq F(y)$ implies that $\tilde{\nabla}F(y)^{\mathrm{T}}(x-y)\preceq\tilde{0}$.

Theorem 1 F is a pseudoconvexity fuzzy mappings if and only if for each $x,y\in K$, $\tilde{0}\preceq\tilde{\nabla}F(x)^{\mathrm{T}}(y-x)$ implies that $\tilde{0}\preceq\tilde{\nabla}F(y)^{\mathrm{T}}(y-x)$.

Proof Suppose that F is a pseudoconvexity fuzzy mappings. Let $x,y\in K$ be such that

$$\tilde{0}\preceq\tilde{\nabla}F(x)^{\mathrm{T}}(y-x). \tag{1}$$

We need to show that $\tilde{0}\preceq\tilde{\nabla}F(y)^{\mathrm{T}}(y-x)$. Assume to the contrary that

$$\tilde{\nabla}F(y)^{\mathrm{T}}(y-x)\prec\tilde{0}. \tag{2}$$

By the pseudoconvexity of F and (1), we have

$$F(x)\preceq F(y). \tag{3}$$

By Lemma 1 and Lemma 2, it follows from (3) that $\tilde{0}\preceq\tilde{\nabla}F(y)^{\mathrm{T}}(y-x)$, which contradicts to (2).

Conversely, let $x,y\in K$ be such that

$$\tilde{0}\preceq\tilde{\nabla}F(x)^{\mathrm{T}}(y-x). \tag{4}$$

We need to show that $F(x)\preceq F(y)$. Assume that contrary, that is, $F(y)\prec F(x)$. Hence, For each $\alpha\in[0,1]$, $F_*(y,\alpha)\leqslant F_*(x,\alpha)$, $F^*(y,\alpha)\leqslant F^*(x,\alpha)$, and there exists an $\alpha_0\in[0,1]$, such that $F_*(y,\alpha_0)<F_*(x,\alpha_0)$ or $F^*(y,\alpha_0)<F^*(x,\alpha_0)$. Without loss of generality, we assume that

$$F_*(y,\alpha_0)<F_*(x,\alpha_0). \tag{5}$$

From the mean-value theorem, we have

$$F_*(y,\alpha_0)-F_*(x,\alpha_0)=\nabla F_*(\bar{x},\alpha_0)^{\mathrm{T}}(y-x). \tag{6}$$

where $\bar{x}=x+\lambda(y-x)$ for some $\lambda\in(0,1)$. From (5) and (6), we have

$$0<\nabla F_*(\bar{x},\alpha_0)^{\mathrm{T}}(x-\bar{x}). \tag{7}$$

On the other hand, from (4), it follows that $\tilde{0}\preceq\tilde{\nabla}F(x)^{\mathrm{T}}(\bar{x}-x)$. From the comparability assumption of F, this implies that $\tilde{0}\preceq\tilde{\nabla}F(\bar{x})^{\mathrm{T}}(\bar{x}-x)$. Then, for each $\alpha\in[0,1]$, we have $0\leqslant\nabla F_*(\bar{x},\alpha)^{\mathrm{T}}(\bar{x}-x)$, which contradicts to (7).

The proof of Theorem 1 is completed.

Remark 1 Theorem 1 generalizes Karamardian's result (Theorem 3.1 in [21]) to fuzzy mapping case.

Theorem 2 F is a strictly pseudoconvex fuzzy mapping if and only if for each $x,y\in K$, $x\neq y$, $\tilde{0}\preceq\tilde{\nabla}F(x)^{\mathrm{T}}(y-x)$ implies that $\tilde{0}\prec\tilde{\nabla}F(y)^{\mathrm{T}}(y-x)$.

Proof Suppose that F is a strictly pseudoconvex fuzzy mapping. Let $x,y\in K$, $x\neq y$, such that

$$\tilde{0}\preceq\tilde{\nabla}F(x)^{\mathrm{T}}(y-x). \tag{8}$$

We need to show that $\tilde{0}\prec\tilde{\nabla}F(y)^{\mathrm{T}}(y-x)$. Assume to the contrary that $$\tilde{\nabla}F(y)^{\mathrm{T}}(y-x)\preceq\tilde{0}. \tag{9}$$
Combine with (8) and from strict pseudoconvexity of F, we have $$F(x)\prec F(y). \tag{10}$$

On the other hand, (9) can be written as $\tilde{0}\preceq\tilde{\nabla}F(y)^{\mathrm{T}}(x-y)$. From strict pseudoconvexity of F, it follows that $F(y)\prec F(x)$, which contradicts to (10).

Conversely, let $x,y\in K, x\neq y$ be such that $$\tilde{0}\preceq\tilde{\nabla}F(x)^{\mathrm{T}}(y-x). \tag{11}$$
We need to show $F(x)\prec F(y)$. Assume to the contrary that $F(y)\preceq F(x)$. Hence, for each $\alpha\in[0,1]$,
$$F_*(y,\alpha)\leqslant F_*(x,\alpha), F^*(y,\alpha)\leqslant F^*(x,\alpha). \tag{12}$$

From the mean-value theorem, we have
$$F_*(y,\alpha)-F_*(x,\alpha)=\nabla F_*(\bar{x},\alpha)^{\mathrm{T}}(y-x). \tag{13}$$
where $\bar{x}=x+\lambda(y-x)$ for some $\lambda\in(0,1)$. From (12) and (13), we have
$$0\leqslant\nabla F_*(\bar{x},\alpha)^{\mathrm{T}}(x-\bar{x}). \tag{14}$$

On the other hand, from (11), it follows that $\tilde{0}\preceq\tilde{\nabla}F(x)^{\mathrm{T}}(\bar{x}-x)$, which implies that $\tilde{0}\prec\tilde{\nabla}F(\bar{x})^{\mathrm{T}}(\bar{x}-x)$. Then, for each $\alpha\in[0,1]$, $0<F_*(\bar{x},\alpha)^{\mathrm{T}}(\bar{x}-x)$, which contradicts to (14).

The proof of Theorem 2 is completed.

Remark 2 Theorem 2 generalizes Karamardian and Schaible's result (Proposition 4.1 in [22]) to fuzzy mapping case.

4 Quasiconvexity of fuzzy mappings

In this section, we establish an equivalent condition of a differentiable quasiconvex fuzzy mapping.

Theorem 3 F is a quasiconvex fuzzy mapping if and only if for each $x,y\in K$, $\tilde{0}\prec\tilde{\nabla}F(x)^{\mathrm{T}}(y-x)$ implies that $\tilde{0}\preceq\tilde{\nabla}F(y)^{\mathrm{T}}(y-x)$.

Proof Suppose that F is a quasiconvex fuzzy mapping. Let $x,y\in K$ be such that
$$\tilde{0}\prec\tilde{\nabla}F(x)^{\mathrm{T}}(y-x). \tag{15}$$
The relation $F(y)\preceq F(x)$ is not possible. Otherwise, it will imply that $\tilde{\nabla}F(x)^{T}(y-x)\preceq\tilde{0}$, according to Lemma 2, which contradicts to (15). From the compatibility,
$$F(x)\prec F(y). \tag{16}$$
From Lemma 2 and (16), it follows that $\tilde{\nabla}F(y)^{\mathrm{T}}(x-y)\preceq\tilde{0}$, i.e., $\tilde{0}\preceq\tilde{\nabla}F(y)^{\mathrm{T}}(y-x)$.

Conversely, assume that F is not a quasiconvex fuzzy mapping. Then, there exists $\alpha\in[0,1]$, such that $F_*(y,\alpha)\leqslant F_*(x,\alpha)<F_*(\bar{x},\alpha)$, or $F^*(y,\alpha)\leqslant F^*(x,\alpha)<F^*(\bar{x},\alpha)$. Without loss of generality, we assume that $$F_*(y,\alpha)\leqslant F_*(x,\alpha)<F_*(\bar{x},\alpha). \tag{17}$$

By the mean-value theorem, then there exist z_1, z_2 such that
$$F_*(\bar{x},\alpha)-F_*(x,\alpha)=\nabla F_*(z_1,\alpha)^{\mathrm{T}}(\bar{x}-x), \tag{18}$$
$$F_*(\bar{x},\alpha)-F_*(y,\alpha)=\nabla F_*(z_2,\alpha)^{\mathrm{T}}(\bar{x}-y), \tag{19}$$
where $$z_1=x+\lambda_1(y-x), z_2=x+\lambda_2(y-x), 0<\lambda_1<\lambda<\lambda_2<1. \tag{20}$$
From (17), (18) and (19), it follow that $0<\nabla F_*(z_1,\alpha)^{\mathrm{T}}(\bar{x}-x)$, $0<\nabla F_*(z_2,\alpha)^{\mathrm{T}}(\bar{x}-y)$. Thus, (20) yields
$$0<\nabla F_*(z_1,\alpha)^{\mathrm{T}}(z_2-z_1), \tag{21}$$ $$0<\nabla F_*(z_2,\alpha)^{\mathrm{T}}(z_1-z_2), \tag{22}$$

On the other hand, from (20) and the hypothesis: for each $x,y\in K$, $\tilde{0}\prec\tilde{\nabla}F(x)^{\mathrm{T}}(y-x)$ implies that $\tilde{0}\preceq\tilde{\nabla}F(y)^{\mathrm{T}}(y-x)$. We obtain that $\tilde{0}\preceq\tilde{\nabla}F(z_2)^{\mathrm{T}}(z_2-z_1)$. Hence $0\leqslant\nabla F_*(z_2,\alpha)^{\mathrm{T}}(z_2-z_1)$, which contradicts to (22).

The proof of Theorem 3 is completed.

Remark 3 Theorem 3 generalizes Karamardian and Schaible's result (Proposition 5.2 in [22] to fuzzy mapping case.

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运筹学与控制论

广义凸模糊映射的若干等价条件

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摘要：在本文中，证明了 F 是(严格)伪凸模糊映射当且仅当 $\tilde{\nabla}F$ 是(严格)伪单调的，F 是拟凸模糊映射当且仅当 $\tilde{\nabla}F$ 是拟单调的等几个广义凸模糊映射的等价条件。在模糊数学规划中，这些结果在检验模糊映射的广义凸性以及刻画其解集时，将会产生非常重要的作用。

关键词：伪凸模糊映射；严格伪凸模糊映射；拟凸模糊映射

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