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一类四阶非线性微分方程的渐近稳定性*

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摘要:假设存在常数 $h>0,k>0,m>0,\varepsilon>0$,使得当 $|y|\leqslant h,|z|\leqslant k,|y|\leqslant m|z|$ 时,函数 $G(y)$ 具有连续的二阶导数,四阶非线性微分方程 $x^{(4)}+ax^{(3)}+G'(x')x^{(2)}+cx'+f(x)=0,f(0)=0$,在满足:$acG'(y)-c^2-a^2\geqslant\varepsilon>0,|G'(y)|\leqslant\frac{\varepsilon}{(am^2+c)k},|f'(x)|\leqslant\frac{2a}{2a+1},2a^2+a>c,(f(x)+cy)\operatorname{sgn} z\geqslant 0,(az+u)\operatorname{sgn} y\geqslant 0$ 的条件下,利用 Lyapunov 函数构造法,给出了其零解的全局渐近稳定性的充分性准则,所得结果包含并改进了相关文献的结果。

关键词:微分方程;Lyapunov 函数;渐近稳定性
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1 引言及引理

对于二阶、三阶非线性系统,可以用类比的方法和 Lyapunov 第二方法,其稳定性的研究,已有很多很好的结果[1-21],但对于四阶及以上的系统,应用这一方法,也有一些结果[22-26],但还有很多问题没有得到解决。

1995 年,梁在中[24]研究了

$$x^{(4)}+ax^{(3)}+bx^{(2)}+cx'+f(x)=0,f(0)=0,\tag{1}$$

$$x^{(4)}+ax^{(3)}+G'(x')x^{(2)}+cx'+dx=0\tag{2}$$

的平凡解的渐近稳定性。

2004 年,Sadek A I[27]研究了如下四阶时滞微分方程

$$x^{(4)}+\alpha_1x^{(3)}+\alpha_2x^{(2)}+\alpha_3x'+f(x(t-\gamma))=0,\tag{3}$$

$$x^{(4)}+\alpha_1x^{(3)}+\alpha_2x^{(2)}+G(x'(t-\gamma))+f(x)=0\tag{4}$$

的平凡解的渐近稳定性。

笔者将讨论如下四阶非线性方程零解的渐近稳定性问题:

$$x^{(4)}+ax^{(3)}+G'(x')x^{(2)}+cx'+f(x)=0,f(0)=0,\tag{5}$$

其中 $a>0,c>0$,将方程化为等价系统

$$\begin{cases}x'=y,\\ y'=z,\\ z'=u,\\ u'=-f(x)-cy-G'(y)z-au。\end{cases}\tag{6}$$

2 主要结论

定理 1　假设存在常数 $h>0,k>0,m>0,\varepsilon>0$,使得当 $|y|\leqslant h,|z|\leqslant k,|y|\leqslant m|z|$ 时,函数 $G(y)$ 具有连续的二阶导数,且满足下列条件:

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1) $acG'(y)-c^2-a^2\geqslant\varepsilon>0$；

2) $|G'(y)|\leqslant\frac{\varepsilon}{(am^2+c)k}$；

3) $|f'(x)|\leqslant\frac{2a}{2a+1}$；

4) $2a^2+a>c$；

5) $(f(x)+cy)\mathrm{sgn}z\geqslant0$；

6) $(az+u)\mathrm{sgn}y\geqslant0$。

则系统(6)的平凡解是渐近稳定的。

证明　作 Lyapunov 函数

$$V=af^2(x)+[aG'(y)-c+ac^2]y^2+[a^2c+cG'(y)]z^2+cu^2+2acyf(x)+2czf(x)+$$
$$2(c^2+a^2)yz+2ayu+2aczu+2a\int_0^x f(\xi)[1-f'(\xi)]\mathrm{d}\xi=$$
$$[aG'(y)-c]y^2+[cG'(y)-a]z^2+a[f(x)+cy]^2+c(az+u)^2+2c(f(x)+cy)z+$$
$$2a(az+u)y+2a\int_0^x f(\xi)[1-f'(\xi)]\mathrm{d}\xi+az^2\ 。$$

由于① $\frac{\mathrm{d}}{\mathrm{d}t}\{[aG'(y)-c]y^2\}=aG''(y)zy^2+2ayzG'(y)-2cyz$；

② $\frac{\mathrm{d}}{\mathrm{d}t}\{[cG'(y)-a]z^2\}=cG''(y)z^3+2czuG'(y)-2azu$；

③ $\frac{\mathrm{d}}{\mathrm{d}t}\{a[f(x)+cy]^2\}=2af(x)f'(x)y+2acy^2f'(x)2acf(x)+2ac^2yz$；

④ $\frac{\mathrm{d}}{\mathrm{d}t}[c(az+u)^2]=-2aczf(x)-2cuf(x)-2ac^2yz-2c^2yu-2caG'(y)z^2-2czuG'(y)$；

⑤ $\frac{\mathrm{d}}{\mathrm{d}t}[2c(f(x)+cy)z]=2cyzf'(x)+2c^2z^2+2cuf(x)+2c^2yu$；

⑥ $\frac{\mathrm{d}}{\mathrm{d}t}[2a(az+u)y]=2a^2yu-2ayf(x)-2acy^2-2aG'(y)yz-2a^2yu$；

⑦ $\frac{\mathrm{d}}{\mathrm{d}t}\left\{2a\int_0^x f(\xi)[1-f'(\xi)]\mathrm{d}\xi\right\}=2ayf(x)-2ayf(x)f'(x)$；

⑧ $\frac{\mathrm{d}}{\mathrm{d}t}(az^2)=2azu$。

则
$$\frac{\mathrm{d}V}{\mathrm{d}t}=aG''(y)zy^2+cG''(y)z^3+2acy^2f'(x)^2-2acG'(y)z^2+2cyzf'(x)+2c^2z^2-2acy\leqslant$$
$$|G''(y)|\left(a\left|\frac{y}{z}\right|^2+c\right)|z|z^2+(2c^2-2acG'(y))z^2+(2ac|f'(x)|-2ac)y^2+c|f'(x)|(y^2+z^2)=$$
$$|G''(y)|\left(a\left|\frac{y}{z}\right|^2+c\right)|z|z^2+[2c^2-2acG'(y)+c|f'(x)|]z^2+[2ac|f'(x)|-2ac+c|f'(x)|]y^2\leqslant$$
$$\varepsilon z^2+\left(-2\varepsilon-2a^2+\frac{2ac}{2a+1}\right)z^2+\left(2ac\frac{2a}{2a+1}-2ac+c\frac{2a}{2a+1}\right)y^2\leqslant-\varepsilon z^2。$$

且只有平凡解才能使$\frac{\mathrm{d}V}{\mathrm{d}t}\equiv0$，则由关于渐近稳定性的李雅普诺夫稳定性[13]可知，系统(5)的平凡解是渐近稳定的。　　证毕

3 推论

推论 1　若 $G'(x')=b$ 时，即为文献[24]中的定理 1。

推论 2　若 $f(x)=d\cdot x$ 时，即为文献[24]中的定理 2。

参考文献:

[1] 姚洪兴,孟伟业.一类三阶双滞量时滞微分方程的全局渐近稳定性[J].应用数学,2008,21(3):576-580.
Yao H X,Meng W Y. Global asymptotic stability of a class of three order differential equations with two delays[J]. Applied Mathematics,2008,21(3):576-580.

[2] Tune C. New results about stability and boundedness of solutions of certain non-linear third-order delay differentinal equations[J]. The Arabian Journal for Science and Engineering,2006,31(2A):186-196.

[3] 康慧燕,张丽娟.一类三阶非线性时滞系统的全局稳定性[J].江苏工业学院学报,2005,17(4):48-50.
Kang H Y,Zhang L J. Global stability of a class of three order nonlinear delay system[J]. Journal of Jiangsu Polytechnic University,2005,17(4):48-50.

[4] 张丽娟,斯力更.一类三阶非线性系统的全局渐近稳定性[J].应用数学学报,2007,30(1):99-102.
Zhang L J,Si L G. Global asymptotic stability of a class of three order nonlinear system[J]. Journal of Applied Mathematics,2007,30(1):99-102.

[5] 耿翊翔.一类三阶非线性系统的全局渐近稳定性[J].曲靖师范学院学报,2003,22(3):32-33.
Geng Y X. Global asymptotic stability of a class of three order nonlinear system[J]. Journal of Qujing Normal University,2003,22(3):32-33.

[6] 张丽娟,吴雁.一类三阶非线性系统的全局渐近稳定性[J].大学数学,2007,23(1):70-74.
Zhang L J,Wu Y. Global asymptotic stability of a class of three order nonlinear system[J]. University Mathematics,2007,23(1):70-74.

[7] 刘俊.一类三阶非线性非自治系统的全局稳定性[J].数学理论与应用,2004,24(1):28-31.
Liu J. The global stability of a class of three order nonlinear non autonomous system[J]. Mathematical Theory and Application,2004,24(1):28-31.

[8] 张同斌.一类三阶非线性微分方程解的全局渐近稳定性[J].河南大学学报,2000,30(2):45-48.
Zhang T B. Global asymptotic stability of a class of three order nonlinear differential equation[J]. Journal of Henan University,2000,30(2):45-48.

[9] 蒋自国.两类三阶非线性系统的全局渐近稳定性[J].四川师范大学学报:自然科学版,2011,34(2):217-220.
Jiang Z G. Global asymptotic stability of two class of three order nonlinear system[J]. Journal of Sichuan Normal University:Natural Science Edition,2011,34(2):217-220.

[10] 蒋自国.一类三阶非线性系统的全局渐近稳定性[J].四川师范大学学报:自然科学版,2008,31(6):667-669.
Jiang Z G. Global asymptotic stability of a class of three order nonlinear system[J]. Journal of Sichuan Normal University:Natural Science Edition,2008,31(6):667-669.

[11] 刘昌东.一类三阶非线性微分方程的全局渐近稳定性[J].湛江师范学院学报:自然科学版,1999,20(2):16-18.
Liu C D. Global asymptotic stability of a class of three order nonlinear differential equation[J]. Journal of Zhanjiang Normal University:Natural Science Edition,1999,20(2):16-18.

[12] 康慧燕.关于三阶非线性系统的全局稳定性两个定理的推广[J].应用数学,2001,14(增):7-9.
Kang H Y. Two theorems on global stability of three order nonlinear systems[J]. Applied Mathematics,2001,14(Suppl):7-9.

[13] 王联,王慕秋.非线性常微分方程定性分析[M].哈尔滨:哈尔滨工业大学出版社,1987.
Wang L,Wang M Q. Qualitative analysis of nonlinear ordinary differential equations[M]. Harbin:Harbin Institute of Technology Press,1987.

[14] Qian C. On global stability of third-order nonlinear differential equations[J]. Non-linear Analysis,2000,42:651-661.

[15] Further O O. results on global stability of third-order nonlinear differential equations[J]. Nonlinear Analysis,2007,67:3394-3400.

[16] 康慧燕.一类三阶非线性系统的全局稳定性[J],江苏工业学院学报,2004,19(3):41-42.
Kang H Y. The global stability of a class of three order nonlinear system[J]. Journal of Jiangsu Polytechnic University,2004,19(3):41-42.

[17] Gaiko V A. Limit cycles of quadratic systems[J]. Nonlinear Anal,2008,69:2150-2157.

[18] Samli R,Arik S. New results for global stability of a class of neutral-type neural systems with time delays[J]. Applied Mathematics and Computation,2009,210(2):564-570.

[19] Liu Z J,Fan M,Chen L S. Globally asymptotic stability in two periodic delayed competitive systems[J]. Applied Mathematics and Computation,2008,197(1):271-287.

[20] Ge Z H,He Y N. Diffusion effect and stability analysis of a predator-prey system described by a delayed reaction-diffusion equations[J]. J Math Anal Appl,2008,339(2):1432-1450.

[21] 董彪.变时滞细胞神经网络模型的全局指数稳定性[J].四川师范大学学报:自然科学版,2007,30(4):454-457.
Dong B. Global exponential stability for delayed cellular neural network model[J]. Journal of Sichuan Normal University:Natural Science Edition,2007,30(4):454-457.

[22] 胡爱莲.一类四阶非线性微分方程解的有界性及稳定性[J].四川师范大学学报:自然科学版,2006,29(6):692-695.
Hu A L. A class of four order nonlinear differential equation, boundedness and stability of solutions[J]. Journal of Sichuan Normal University: Natural Science Edition, 2006,29(6):692-695.

[23] 徐静,李玉洁.一类四阶非线性系统的稳定性[J].工科数学,2001,17(1):47-49.
Xu J, Li Y J. Stability of a class of four order nonlinear system[J]. Engineering Mathematics, 2001,17(1):47-49.

[24] 梁在中.关于一类四阶非线性系统李雅普诺夫函数构造的研究[J].应用数学和力学,1995,16(2):181-188.
Liang Z Z. Study on a class of four order nonlinear Lyapunov function[J]. Applied Mathematics and Mechanics, 1995,16(2):181-188.

[25] 姚洪兴,王经天.一类四阶微分方程的渐近稳定性[J].江苏大学学报:自然科学版,2010,31(5):616-620.
Yao H X, Wang J T. Asymptotic stability of a class of four order differential equations[J]. Journal of Jiangsu University: Natural Science Edition, 2010,31(5):616-620.

[26] 西密尔·通兹.某些四阶时滞微分方程解的稳定性[J].应用数学和力学,2006,27(8):994-1000.
Tong C. The stability of certain four order delay differential equations[J]. Applied Mathematics and Mechanics, 2006,27(8):994-1000.

[27] Sadek A I. On the stability of solutions of some nonautonomous delay differential equations of the third order[J]. Asymptotic Anal, 2005,43(1/2):1-7.

Asymptotic Stability of a Certain Fourth-order Differential Equation

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Abstract: Under the assumption that there is constant $h>0$, $k>0$, $m>0$ and $\varepsilon>0$, which $|y|\leqslant h$, $|z|\leqslant k$ and $|y|\leqslant m|z|$. The function $G(y)$ with continuous two order derivatives. Four nonlinear differential equations in $x^{(4)}+ax^{(3)}+G'(x')x^{(2)}+cx'+f(x)=0$, $f(0)=0$ (5), to meet the conditions of $acG'(y)-c^2-a^2\geqslant\varepsilon>0$, $|g''(y)|\leqslant\frac{\varepsilon}{(am^2+c)k}$, $|f'(x)|\leqslant\frac{2a}{2a+1}$, $2a^2+a>c$, $(f(x)+cy)\mathrm{sgn}z\geqslant 0$, and $(az+u)\mathrm{sgn}y\geqslant 0$. By constructing Lyapunov functional, given equation (5) in the adequacy criterion for the global asymptotic stability. The results include and improve the related results in the literatures.

Key words: differential equation; Lyapunov function; asymptotic stability

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