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Asymptotical Stability of Complex-valued Neural Networks with Time Delayed*

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Abstract: This paper investigated uniqueness and asymptotical stability of equilibrium point for complex-valued neural networks with multiple time delays. Based on the Lyapunov functional method and linear matrix inequalities (LMI), some sufficient conditions for asymptotical stability of the considered neural networks are presented. Finally, a illustrative examples are given to demonstrate the theoretical results.

Key words: complex-valued neural networks; asymptotical stability; time delayed

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In recent years, there have been increasing researches interests in analyzing the dynamical behaviors of neural networks due to their widespread applications in signal processing, pattern recognition, engineering optimization, and associative memory, etc., see[1-7]. Meanwhile, some researchers have investigated impulsive neural networks and its stability. It is well known that the applications of neural networks rely heavily on the dynamical behaviors of the networks. However, time delays, which often occur in the processing of information storage and transmission, may create bad dynamical behaviors of the networks, for example, oscillation, instability and bifurcation[8-10]. Hence, it is necessary to study the dynamical behavior of delayed neural networks, and a great deal of significant results has been reported in the open literatures.

In the neural networks application, complex signals are preferable. Therefore, it is not surprising to see that complex-valued neural networks, which deal with complex-valued date, complex-valued weights, and neuron activation functions, have also been widely studied in recent years[11-13]. Thus, it is important to study the dynamical behaviors of complex-valued recurrent neural networks. As we known, to analyze the stability of neural networks, there are various approaches, such as Lyspunov function method, energy function method and synthesis method. Complex-valued neural networks have received increasing interest due to their promising potential for engineering applications. In [14], the fundamentals of theory and applications of complex-valued neural networks were described. In [15-16], multilayer neural networks based on multivalued neurons were considered. In [17-18], theory and applications on complex-valued learning algorithms were studied. Most of these methods are still applicable to the complex-valued neural networks, see, for example[19-23].

In complex-valued neural networks, their activation function cannot be both bounded and analytic. Therefore, activation functions are main challenge for complex-valued neural networks. There are various types of activation functions in complex domain. For different types of activation functions, we need different approaches to study the relevant neural network, which are quite different from those used in real-valued neural net-

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works. In this paper, we consider a class of activation functions and systematically study the stability problem and provide some useful results.

Motivated by the mentioned above, in this paper, we shall consider the complex-valued neural networks with time delay, which simultaneously consider the delays, complex-valued and. Some new sufficient criteria will be derived for asymptotical stability of its equilibrium point by constructing suitable Lyapunov functional. The stability conditions are viable in the design and analysis of globally stable complex-valued recurrent neural networks, and are of great interest in many applications.

The structure of this paper is outlined as follows: In Section 1, we will interpret the complex-valued neural networks model and some notations. Some globally asymptotically stable conditions are presented in Section 2. A illustrative examples are given to demonstrate the effectiveness of the proposed approach in Section 3. Finally, Section 4 concludes the article.

1 Preliminaries

In this paper, a non-linear delay differential equation of the form is follow complex-valued recurrent neural network

$$\dot{u}(t)=-\boldsymbol{C}u(t)+\boldsymbol{A}f_0(u(t))+\boldsymbol{B}g_0((u(t-\tau))+\boldsymbol{I}, \tag{1}$$

where $u(t)=[u_1(t),u_2(t),\cdots,u_n(t)]^{\mathrm{T}}\in\mathbf{C}^n$ is the neuron state vector, $f_0(u(t))=[f_{01}(u_1(t)),f_{02}(u_2(t)),\cdots,f_{0n}(u_n(t))]^{\mathrm{T}}$ and $g_0(u(t))=[g_{01}(u_1(t)),g_{02}(u_2(t)),\cdots,g_{0n}(u_n(t))]^{\mathrm{T}}$ are the activation functions without and with time delays whose it consist of complex-valued nonlinear functions, $\tau=\tau_{ij}>0$ are time delays parameters, $I=[I_1,I_2,\cdots,I_n]^{\mathrm{T}}\in\mathbf{C}^n$ is the external input vector. $\boldsymbol{C}=\mathrm{diag}(c_1,c_2,\cdots,c_n)\in\mathbf{R}^{n\times n}$ is the self-feedback connection weight matrix, $\boldsymbol{A}=[a_{ij}]_{n\times n}\in\mathbf{C}^{n\times n}$ and $\boldsymbol{B}=[b_{ij}]_{n\times n}\in\mathbf{C}^{n\times n}$ are the connection weight matrix without and with delays, respectively.

The initial condition associated with neural network (1) is given by

$$u_i(t)=\varphi_i(s),-\tau\leqslant s\leqslant 0,i=1,2,\cdots,n,$$

where $\mathrm{Re}(\varphi_i(s))$ and $\mathrm{Im}(\varphi_i(s))$ are continuous on $[-\tau,0]$.

Assumption 1　Let $z=x+\mathrm{i}y$, where i denotes the imaginary unit, that is $\mathrm{i}^2=-1$. $f_j(u)$ can be expressed by separating into its real and imaginary part as

$$f_j(u)=f_{1j}(x)+\mathrm{i}f_{2j}(y),$$

where $f_{1j}(x)\in\mathbf{R}$ and $f_{2j}(y)\in\mathbf{R}$. $f_{1j}(x)$ and $f_{2j}(y)$ satisfies the following condition:

$$0\leqslant\frac{f_{1j}(\alpha)-f_{1j}(\beta)}{\alpha-\beta}\leqslant\xi_j,$$

$$0\leqslant\frac{f_{2j}(\alpha)-f_{2j}(\beta)}{\alpha-\beta}\leqslant\varepsilon_j,\ \forall\,\alpha,\beta\in\mathbf{R},j=1,2,\cdots,n. \tag{3}$$

For notational convenience, we will always shift an intended equilibrium point $u^*=[u_1^*,u_2^*,\cdots,u_n^*]^{\mathrm{T}}\in\mathbf{C}^n$ of system (1) to the origin by letting $z(t)=u(t)-u_1^*$, which yields the following system:

$$\dot{z}(t)=-\boldsymbol{C}z(t)+\boldsymbol{A}f(z(t))+\boldsymbol{B}g((z(t-\tau(t))), \tag{5}$$

where $z(t)=[z_1(t),z_2(t),\cdots,z_n(t)]^{\mathrm{T}}\in\mathbf{C}^n$ is the neuron state vector, $f(z(t))=[f_1(z_1(t)),f_2(z_2(t)),\cdots,f_n(z_n(t))]^{\mathrm{T}}$ and $g(z(t))=[g_1(z_1(t)),g_2(z_2(t)),\cdots,g_n(z_n(t))]^{\mathrm{T}}$ denotes the activation function vector with $f_i(z_i)=f_{0i}(z_i+u_i^*)-f_{0i}(u_i^*)$ and $g_i(z_i)=g_{0i}(z_i+u_i^*)-g_{0i}(u_i^*)$, $i=1,2,\cdots,n$. Note that function $f_i(z(t))$ and $g_i(z_i(t))$ here satisfy condition (3) and (4).

Lemma 1　Let $X,Y\in\mathbf{R}^n$, matrix $\boldsymbol{P}\in\mathbf{R}^{n\times n}$ and $\boldsymbol{Q}$ a positive definite matrix with appropriate dimensions, then we have following inequality hold:

$$X^{\mathrm{T}}Y+Y^{\mathrm{T}}X\leqslant X^{\mathrm{T}}\boldsymbol{Q}X+Y\boldsymbol{Q}^{-1}Y.$$

Lemma 2　(Schur Complement) Let X be a symmetric matrix given by

$$\boldsymbol{X}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{B}^{\mathrm{T}} & \boldsymbol{C}\end{bmatrix},$$

let S be the Schur complement of $\boldsymbol{A}$ in X, that is $S=\boldsymbol{A}-\boldsymbol{B}\boldsymbol{C}^{-1}\boldsymbol{B}^{\mathrm{T}}$.

Let $x(t)=\mathrm{Re}(z(t))$, $y(t)=\mathrm{Im}(z(t))$, $A_1=\mathrm{Re}(\boldsymbol{A})$, $A_2=\mathrm{Im}(\boldsymbol{A})$, $B_2=\mathrm{Re}(\boldsymbol{B})$, $B_2=\mathrm{Im}(\boldsymbol{B})$, $\varphi_1(t)=\mathrm{Re}(\varphi_i(s))$, $\varphi_2(t)=\mathrm{Im}(\varphi_i(s))$; then neural network (5) can be rewritten as

$$\begin{aligned}\dot{x}(t)&=-\boldsymbol{C}x(t)+A_1f_1(x(t))-A_2f_2(y(t))+B_1g_1(x(t))-B_2g_2(y(t)),\\ \dot{y}(t)&=-\boldsymbol{C}y(t)+A_2f_1(x(t))+A_1f_2(y(t))+B_2g_1(x(t))+B_1g_2(y(t)).\end{aligned}\tag{6}$$

The initial condition of neural network (6) is of the form

$$\begin{cases}x_i(s)=\varphi_{1i}(s)\\ y_i(s)=\varphi_{2i}(s)\end{cases},-\tau\leqslant s\leqslant 0,i=1,2,\cdots,n.$$

In this paper, we also use the following notations. Let $\boldsymbol{A}$ be a complex-valued matrix, $\boldsymbol{A}^*$ denotes the complex conjugate transpose of $\boldsymbol{A}$. Let z be a complex number, $\hat{z}$ denotes the complex conjugate of z.

2 Stability analysis

In the section, two criteria are obtained for the stability of neural network with time-vary delays via Lyapinov stability theorem for functional differential equations and linear matrix inequality (LMI) technique.

Theorem 1 The network activation function satisfy Assumption 1. Then, neural network (1) is Asymptotical stability if there exist positive definite matrix $\boldsymbol{P}$ and positive definite Hermitian matrices $Q=Q_1+\mathrm{i}Q_2$, $S=S_1+\mathrm{i}S_2$, $M=M_1+\mathrm{i}M_2$, and $N=N_1+\mathrm{i}N_2$ such that the following LMIs hold:

$$\Lambda=\begin{pmatrix}\Omega_1 & 0\\ 0 & \Omega_2\end{pmatrix}>0,\tag{7}$$

Where

$$\Omega_1=\begin{pmatrix}\Xi_1 & \boldsymbol{P}A_1\Sigma_1 & \boldsymbol{P}A_2\Sigma_1 & \boldsymbol{P}B_1 & \boldsymbol{P}B_2\\ * & Q_1 & 0 & 0 & 0\\ * & * & S_2 & 0 & 0\\ * & * & * & M_1 & 0\\ * & * & * & * & N_1\end{pmatrix},\Omega_2=\begin{pmatrix}\Xi_2 & \boldsymbol{P}A_1\Sigma_2 & \boldsymbol{P}A_2\Sigma_2 & \boldsymbol{P}B_1 & \boldsymbol{P}B_2\\ * & Q_2 & 0 & 0 & 0\\ * & * & S_1 & 0 & 0\\ * & * & * & M_2 & 0\\ * & * & * & * & N_2\end{pmatrix}\tag{8}$$

$\Xi_1=\boldsymbol{PC}+\boldsymbol{PC}-\Sigma_1(Q_1+S_1)\Sigma_1-\Sigma_3(M_1+N_1)\Sigma_3$, $\Xi_1=\boldsymbol{PC}+\boldsymbol{PC}-\Sigma_2(Q_2+S_2)\Sigma_2-\Sigma_3(M_2+N_2)\Sigma_3$.

Proof To prove the theorem 1, we will divide to two steps,

Step 1: We assume that the solution $\hat{z}\neq(0,0,\cdots,0)^{\mathrm{T}}$ is also its equilibrium, i. e.,

$$-\boldsymbol{C}\hat{z}+\boldsymbol{A}f(\hat{z})+\boldsymbol{B}g(\hat{z})=0,$$

Multiplying both sides of above equation by $2\hat{z}^*\boldsymbol{P}$, we obtain

$$-2\hat{z}^*\boldsymbol{PC}\hat{z}+2\hat{z}^*\boldsymbol{PA}f(\hat{z})+2\hat{z}^*\boldsymbol{PB}g(\hat{z})=0,\tag{9}$$

Noting that

$$\begin{aligned}&2\hat{z}^*\boldsymbol{PA}f(\hat{z})\leqslant 2\mathrm{Re}(\hat{z}^*\boldsymbol{PA}f(\hat{z}))=\hat{x}^{\mathrm{T}}\boldsymbol{P}A_1f_1(\hat{x})+(f_1(\hat{x}))^{\mathrm{T}}A_1^{\mathrm{T}}\boldsymbol{P}\hat{x}-\\ &\hat{x}^{\mathrm{T}}\boldsymbol{P}A_2f_2(\hat{y})-(f_2(\hat{y}))^{\mathrm{T}}A_2^{\mathrm{T}}\boldsymbol{P}\hat{x}+\hat{y}^{\mathrm{T}}\boldsymbol{P}A_1f_2\,\hat{y}+(f_2(\hat{y}))^{\mathrm{T}}A_1^{\mathrm{T}}\boldsymbol{P}y+\\ &\hat{y}^{\mathrm{T}}\boldsymbol{P}A_2f_1(\hat{x})+(f_1(\hat{x}))^{\mathrm{T}}A_2^{\mathrm{T}}\boldsymbol{P}\hat{y}\leqslant(f_1(\hat{x}))^{\mathrm{T}}\boldsymbol{Q}_1f_1(\hat{x})+\\ &\hat{x}^{\mathrm{T}}\boldsymbol{P}A_1\boldsymbol{Q}_1^{-1}A_1^{\mathrm{T}}\boldsymbol{P}\hat{x}+(f_2(\hat{y}))^{\mathrm{T}}S_2f_2(\hat{y})+\hat{x}^{\mathrm{T}}\boldsymbol{P}A_2S_2^{-1}A_2^{\mathrm{T}}\boldsymbol{P}\hat{x}+\\ &(f_2(\hat{y}))^{\mathrm{T}}\boldsymbol{Q}_2f_2(\hat{y})+\hat{y}^{\mathrm{T}}\boldsymbol{P}A_1\boldsymbol{Q}_2^{-1}A_1^{\mathrm{T}}\boldsymbol{P}\hat{y}+(f_1(\hat{x}))^{\mathrm{T}}S_1f_1(\hat{x})+\\ &\hat{y}^{\mathrm{T}}\boldsymbol{P}A_2S_1^{-1}A_2^{\mathrm{T}}\boldsymbol{P}\hat{y}=\hat{x}(\boldsymbol{P}A_1Q_1^{-1}A_1^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}A_2S_2^{-1}A_2^{\mathrm{T}}\boldsymbol{P})\hat{x}+\\ &(f_1(\hat{x}))^{\mathrm{T}}(Q_1+S_1)f_1(\hat{x})+\hat{y}^{\mathrm{T}}(\boldsymbol{P}A_1Q_2^{-1}A_1^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}A_2S_1^{-1}A_2^{\mathrm{T}}\boldsymbol{P})\hat{y}+\\ &(f_2(\hat{y}))^{\mathrm{T}}(Q_2+S_2)f_2\,\hat{y}\leqslant\hat{x}^{\mathrm{T}}(\boldsymbol{P}A_1Q_1^{-1}A_1^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}A_2S_2^{-1}A_2^{\mathrm{T}}\boldsymbol{P}+\Sigma_1(Q_1+S_1)\Sigma_1)\hat{x}+\\ &\hat{y}^{\mathrm{T}}(\boldsymbol{P}A_1Q_2^{-1}A_1^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}A_2S_1^{-1}A_2^{\mathrm{T}}\boldsymbol{P}+\Sigma_2(Q_2+S_2)\Sigma_2)\hat{y},\end{aligned}\tag{10}$$

Similarly, we have that

$$2\hat{z}^*\boldsymbol{PB}g(\hat{z})\leqslant 2\mathrm{Re}(\hat{z}^*\boldsymbol{PB}g(\hat{z}))\leqslant$$
$$\hat{x}^{\mathrm{T}}(\boldsymbol{P}B_1M_1^{-1}B_1^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}B_2N_2^{-1}B_2^{\mathrm{T}}\boldsymbol{P}+\Sigma_3(M_1+N_1)\Sigma_3)\hat{x}+$$
$$\hat{y}^{\mathrm{T}}(\boldsymbol{P}B_1M_2^{-1}M_1^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}B_2N_1^{-1}B_2^{\mathrm{T}}\boldsymbol{P}+\Sigma_4(M_2+N_2)\Sigma_4)\hat{y}. \tag{11}$$

Introducing (10) and (11) to (9), we can derive

$$-\hat{x}^{\mathrm{T}}(\boldsymbol{PC}+\boldsymbol{CP}-\boldsymbol{P}A_1Q_1^{-1}A_1^{\mathrm{T}}\boldsymbol{P}-\boldsymbol{P}A_2S_2^{-1}A_2^{\mathrm{T}}\boldsymbol{P}-\Sigma_1(Q_1+S_1)\Sigma_1)\hat{x}-$$
$$\hat{y}^{\mathrm{T}}(\boldsymbol{PC}+\boldsymbol{CP}-\boldsymbol{P}A_1\boldsymbol{Q}_2^{-1}A_1^{\mathrm{T}}\boldsymbol{P}-\boldsymbol{P}A_2S_1^{-1}A_2^{\mathrm{T}}\boldsymbol{P}-\Sigma_2(\boldsymbol{Q}_2+S_2)\Sigma_2)\hat{y}-$$
$$\hat{x}^{\mathrm{T}}(\boldsymbol{P}B_1M_1^{-1}B_1^{\mathrm{T}}\boldsymbol{P}-\boldsymbol{P}B_2N_2^{-1}B_2^{\mathrm{T}}\boldsymbol{P}-\Sigma_3(M_1+N_1)\Sigma_3)\hat{x}-$$
$$\hat{y}^{\mathrm{T}}(\boldsymbol{P}B_1M_2^{-1}M_1^{\mathrm{T}}\boldsymbol{P}-\boldsymbol{P}B_2N_1^{-1}B_2^{\mathrm{T}}\boldsymbol{P}-\Sigma_4(M_2+N_2)\Sigma_4)\hat{y}\leqslant$$
$$-(\hat{x}^{\mathrm{T}}\Theta_1\hat{x}+\hat{y}^{\mathrm{T}}\Theta_2\hat{y})\leqslant-\Xi^{\mathrm{T}}\Lambda\Xi\geqslant 0,$$

where $\Xi=(\hat{x}^{\mathrm{T}},\hat{y}^{\mathrm{T}})$, $\Theta_1=\boldsymbol{PC}+\boldsymbol{CP}-\boldsymbol{P}A_1Q_1^{-1}A_1^{\mathrm{T}}\boldsymbol{P}-\boldsymbol{P}A_2S_2^{-1}A_2^{\mathrm{T}}\boldsymbol{P}-\Sigma_1(Q_1+S_1)\Sigma_1+\boldsymbol{P}B_1M_1^{-1}B_1^{\mathrm{T}}\boldsymbol{P}-\boldsymbol{P}B_2N_2^{-1}B_2^{\mathrm{T}}\boldsymbol{P}-\Sigma_3(M_1+N_1)\Sigma_3$, $\Theta_2=\boldsymbol{PC}+\boldsymbol{CP}-\boldsymbol{P}A_1\boldsymbol{Q}_2^{-1}A_1^{\mathrm{T}}\boldsymbol{P}-\boldsymbol{P}A_2S_1^{-1}A_2^{\mathrm{T}}\boldsymbol{P}-\Sigma_2(\boldsymbol{Q}_2+S_2)\Sigma_2+\boldsymbol{P}B_1M_2^{-1}M_1^{\mathrm{T}}\boldsymbol{P}-\boldsymbol{P}B_2N_1^{-1}B_2^{\mathrm{T}}\boldsymbol{P}-\Sigma_4(M_2+N_2)\Sigma_4$.

Which implies $\Lambda\leqslant 0$. It leads to a contradiction with Eq. (7). Therefore, there exists unique equilibrium of system (1).

Step 2: To prove the asymptotical stability of the origin of system (1), we construct the following Lyapunov functional:

$$V(z(t))=z^*(t)Pz(t)+\int_{t-\tau}^{t}(g_1(x(s)))^{\mathrm{T}}(M_1+N_1)g_1(x(s))\mathrm{d}s+$$
$$\int_{t-\tau}^{t}(g_2(x(s)))^{\mathrm{T}}(M_2+N_2)g_2(x(s))\mathrm{d}s$$

Deriving the derivative of $V(z(t))$, we can obtain that

$$D^+V(z(t))=\dot{z}^*(t)(t)\boldsymbol{P}z(t)+z^*(t)\boldsymbol{P}\dot{z}(t)+(g_1(x(t)))^{\mathrm{T}}(M_1+N_1)g_1(x(t))+$$
$$(g_2(x(t)))^{\mathrm{T}}(M_2+N_2)g_2(x(t))-(g_1(x(t-\tau)))^{\mathrm{T}}(M_1+N_1)g_1(x(t-\tau))-$$
$$(g_2(x(t-\tau)))^{\mathrm{T}}(M_2+N_2)g_2(x(t-\tau))=-z^*(t)(\boldsymbol{CP}+\boldsymbol{PC})z(t)+$$
$$z^*(t)\boldsymbol{PA}f(z(t))+(f(z(t)))^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}Pz(t)+z^*(t)\boldsymbol{PB}g(z(t-\tau))+$$
$$(g(z(t-\tau)))^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}z(t)+(g_1(x(t)))^{\mathrm{T}}(M_1+N_1)g_1(x(t))+$$
$$(g_2(x(t)))^{\mathrm{T}}(M_2+N_2)g_2(x(t))-(g_1(x(t-\tau)))^{\mathrm{T}}(M_1+N_1)g_1(x(t-\tau))-$$
$$(g_2(x(t-\tau)))^{\mathrm{T}}(M_2+N_2)g_2(x(t-\tau)).$$

By (10) and (11), we have

$$D^+V(z(t))\leqslant -z^*(t)(\boldsymbol{CP}+\boldsymbol{PC})z(t)+2\mathrm{Re}(z^*(t)\boldsymbol{PA}f(z(t)))+$$
$$2\mathrm{Re}(z^*(t)\boldsymbol{PB}g(z(t-\tau(t))))+(g_1(x(t)))^{\mathrm{T}}(M_1+N_1)g_1(x(t))+$$
$$(g_2(x(t)))^{\mathrm{T}}(M_2+N_2)g_2(x(t))-(g_1(x(t-\tau)))^{\mathrm{T}}(M_1+N_1)g_1(x(t-\tau))-$$
$$(g_2(x(t-\tau)))^{\mathrm{T}}(M_2+N_2)g_2(x(t-\tau))\leqslant$$
$$-x^{\mathrm{T}}(\boldsymbol{PC}+\boldsymbol{CP}-\boldsymbol{P}A_1Q_1^{-1}A_1^{\mathrm{T}}\boldsymbol{P}-\boldsymbol{P}A_2S_2^{-1}A_2^{\mathrm{T}}\boldsymbol{P}-\Sigma_1(Q_1+S_1)\Sigma_1)x-$$
$$y^{\mathrm{T}}(\boldsymbol{PC}+\boldsymbol{CP}-\boldsymbol{P}A_1\boldsymbol{Q}_2^{-1}A_1^{\mathrm{T}}\boldsymbol{P}-\boldsymbol{P}A_2S_1^{-1}A_2^{\mathrm{T}}\boldsymbol{P}-\Sigma_2(\boldsymbol{Q}_2+S_2)\Sigma_2)y+$$
$$x^{\mathrm{T}}(\boldsymbol{P}B_1M_1^{-1}B_1^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}B_2N_2^{-1}B_2^{\mathrm{T}}\boldsymbol{P}+\Sigma_3(M_1+N_1)\Sigma_3)x+$$
$$y^{\mathrm{T}}(\boldsymbol{P}B_1M_2^{-1}B_1^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}B_2N_1^{-1}B_2^{\mathrm{T}}\boldsymbol{P}+\Sigma_4(M_2+N_2)\Sigma_4)y\leqslant-(x^{\mathrm{T}}\Theta_1x+y^{\mathrm{T}}\Theta_2y)\leqslant-\Xi'^{\mathrm{T}}\Lambda\Xi',$$

where $\Xi'=(x^{\mathrm{T}},y^{\mathrm{T}})$.

Hence, $D^+V(z(t))<0$ when $\Lambda>0$. Then, by lemma 2, $\Lambda>0$ if and only if LMI (7) holds, which completes the proof of the theorem 1.

If neural network (1) is not complex-valued, thus it becomes common networks. We will get following corollary.

Corollary 1　The network activation function satisfy Assumption 1. Then, neural network (1) is Asymptotical stability if there exist positive definite matrix $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{S}$, such that the following LMIs hold:

$$\Lambda'=\begin{pmatrix}\Psi & \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} & \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\\ * & -\boldsymbol{Q} & 0\\ * & * & -\boldsymbol{S}\end{pmatrix}<0, \quad (12)$$

where $\Psi=-\boldsymbol{PC}-\boldsymbol{PC}+\Sigma_1\boldsymbol{Q}\Sigma_1+\Sigma_2\boldsymbol{S}\Sigma_2$.

Proof To prove the Corollary 1, we construct the following Lyapunov functional:

$$V(z(t))=z^{\mathrm{T}}(t)\boldsymbol{P}z(t)+\int_{t-\tau}^{t}(g(x(s)))^{\mathrm{T}}\boldsymbol{S}g(x(s))\mathrm{d}s$$

Deriving the derivative of $V(z(t))$, we can obtain that

$$D^{+}V(z(t))\leqslant -z^{\mathrm{T}}(t)(\boldsymbol{CP}+\boldsymbol{PC})z(t)+2z^{\mathrm{T}}(t)\boldsymbol{PA}f(z(t))+2z^{\mathrm{T}}(t)\boldsymbol{PB}g(z(t-\tau(t)))\times$$
$$(g(z(t)))^{\mathrm{T}}\boldsymbol{S}g_1(x(t))-(g_1(x(t-\tau)))^{\mathrm{T}}\boldsymbol{S}g_1(x(t-\tau))\leqslant$$
$$z^{\mathrm{T}}(t)(-\boldsymbol{PC}-\boldsymbol{CP}+\boldsymbol{PA}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{AP}+\boldsymbol{PB}^{\mathrm{T}}\boldsymbol{S}^{-1}\boldsymbol{BP}+\Sigma_1\boldsymbol{Q}\Sigma_1+\Sigma_2\boldsymbol{S}\Sigma_2)z(t)<0,$$

Hence, $D^{+}V(z(t))<0$ when $\Lambda'<0$. Then, by lemma 2, $\Lambda'<0$ if and only if LMI (12) holds, which completes the proof of the Corollary 1.

3 Numerical examples

In this section, we will present a examples to illustrate the effectives of our results. Consider a two-neuron complex-valued recurrent neural network described as follows:

$$\dot{z}_1(t)=c_1z_1(t)+\sum_{j=1}^{2}a_{1j}f_j(z_j(t))+\sum_{j=1}^{2}b_{1j}g_j(z_j(t-\tau_{ij}))+I_1,$$
$$\dot{z}_2(t)=c_2z_2(t)+\sum_{j=1}^{2}a_{2j}f_j(z_j(t))+\sum_{j=1}^{2}b_{2j}g_j(z_j(t-\tau_{ij}))+I_2, \quad (13)$$

Assume that the network parameters of neural system (13) are given as follows:

$$A=\begin{pmatrix}-1.5+\mathrm{i} & -1-1.3\mathrm{i}\\ -1-\mathrm{i} & -2+\mathrm{i}\end{pmatrix},\ B=\begin{pmatrix}1+1.8\mathrm{i} & 1-\mathrm{i}\\ -1+2.4\mathrm{i} & 1+2\mathrm{i}\end{pmatrix},\ C=\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix},$$

$I_1=-0.1+0.1\mathrm{i}$, $I_2=0.1-0.1\mathrm{i}$, $I_2=0.1-0.1\mathrm{i}$, $\tau_{11}=0.5$, $\tau_{11}=0.4$, $\tau_{11}=0.3$, $\tau_{11}=0.6$.

We select the activation function is following

$$f_j(z_j(t))=\frac{1-\mathrm{e}^{-x_j}}{1+\mathrm{e}^{-x_j}}+\mathrm{i}\frac{1}{1+\mathrm{e}^{-y_j}},\ g_j(z_j(t))=\frac{1-\mathrm{e}^{-y_j}}{1+\mathrm{e}^{-y_j}}+\mathrm{i}\frac{1}{1+\mathrm{e}^{-x_j}}(j=1,2).$$

We use the Matlab LMI Control Toolbox to solve the LMIs in (7), and obtain the following feasible solution

$$P=\mathrm{e}^{-11}\begin{bmatrix}0.9113 & -0.3645\\ -0.3645 & 0.7496\end{bmatrix},Q_1=\mathrm{e}^{-11}\begin{bmatrix}0.0970 & 0.0045\\ 0.0045 & 0.1137\end{bmatrix},$$
$$Q_2=\mathrm{e}^{-11}\begin{bmatrix}0 & 0.9113\\ -0.9113 & 0\end{bmatrix},S_1=\mathrm{e}^{-10}\begin{bmatrix}0.1092 & -0.0702\\ -0.0702 & 0.1355\end{bmatrix},$$
$$S_2=\mathrm{e}^{-11}\begin{bmatrix}0 & 0.9113\\ -0.9113 & 0\end{bmatrix},M_1=\mathrm{e}^{-11}\begin{bmatrix}0.7955 & 0.0208\\ 0.0208 & 0.8271\end{bmatrix},$$
$$M_2=\mathrm{e}^{-11}\begin{bmatrix}0 & 0.9113\\ -0.9113 & 0\end{bmatrix},N_1=\mathrm{e}^{-10}\begin{bmatrix}0.2157 & 0.0694\\ -0.0694 & 0.2333\end{bmatrix},$$
$$N_2=\mathrm{e}^{-11}\begin{bmatrix}0 & 0.9113\\ -0.9113 & 0\end{bmatrix}.$$

Therefore, the delayed neural network is asymptotical stable..

Fig. 1 depicts the time responses of the variables of the neural networks (13) with input $I=(0.1+0.1\mathrm{i}, 0.1-0.1\mathrm{i})$.

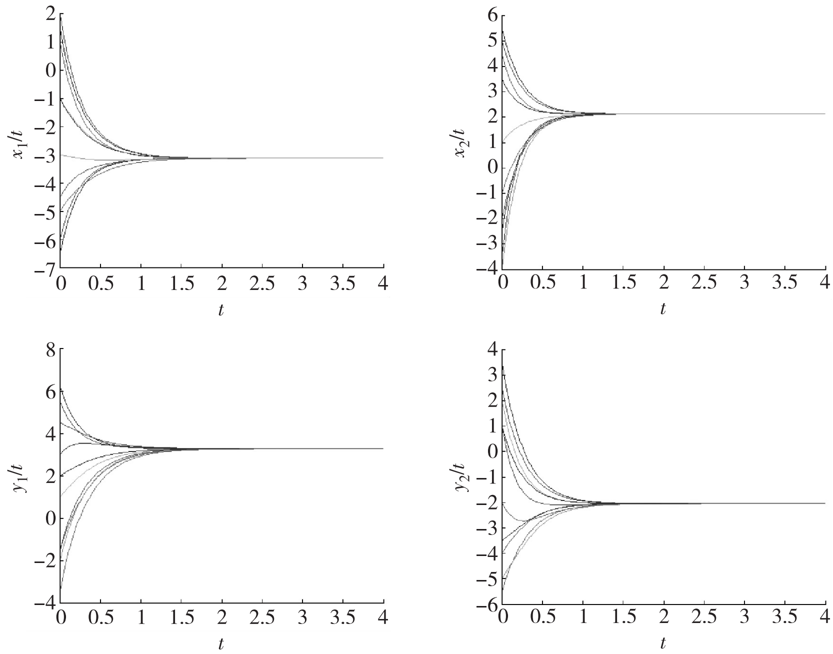

Fig. 1　Transient stats of the neural network in example

4 Conclusions

Recently, numerous works have been published on the stability analysis of various real-valued neural networks, little attention has been paid to the investigation on the stability of complex-valued neural networks. This paper has focused on uniqueness and asymptotical stability of equilibrium point for complex-valued neural networks with multiple time delays with respect to the Assumption 1 activation functions. Some new criterions for robust of complex-valued neural networks with time-delays has been presented that established a new time-independent relationship between the networks of the neural system. A illustrate examples are given to demonstrate the results. Some research methods used in other complex-valued NNs could be practicable.

References:

[1] Yu W. Multiple recurrent neural networks for stable adaptive control[J]. Neurocomputing, 2006, 70(1): 430-444.

[2] Rubio, J, Yu, W. Nonlinear system identification with recurrent neural networks and dead-zone Kalman filter algorithm[J]. Neurocomputing, 2007, 70(13), 2460-2466.

[3] Li T, Fei S M, Zhang K J. Synchronization control of recurrent neural networks with distributed delays[J]. Physica A: Statistical Mechanics and its Applications, 2008, 387 (4), 982-996.

[4] Zeng Z, Wang J. Global exponential stability of recurrent neural networks with time-varying delays in the presence of strong external stimuli[J]. Neural Networks, 2006, 19 (10): 1528-1537.

[5] Ahn C K. Passive learning and input-to-state stability of switched Hopfield neural networks with time-delay[J]. Information Sciences, 2010, 180(23): 4582-4594.

[6] Li C, Li C, Huang T. Exponential stability of impulsive high-order Hopfield-type neural networks with delays and reaction-diffusion[J]. International Journal of Computer Mathematics, 2011, 88(15): 3150-3162.

[7] Zhang W, Li C, Huang T, et al. Global exponential synchronization for coupled switched delayed recurrent neural networks with stochastic perturbation and impulsive effects [J]. Neural Computing and Applications, 2014, 25 (6): 1275-1283.

[8] He X, Li C, Huang T, et al. Bogdanov-Takens singularity in tri-neuron network with time delay[J]. Neural Networks and Learning Systems, IEEE Transactions on, 2013, 24(6): 1001-1007.

[9] He X, Li C, Huang T, et al. Codimension two bifurcation in a delayed neural network with unidirectional coupling[J]. Nonlinear Analysis: Real World Applications, 2013, 14(2): 1191-1202.

[10] Ma S Q, Lu Q S, Wang Q Y, et al. Effects of time delay on two neurons interaction Morris-Lecar model[J]. International Journal of Biomathematics, 2008, 1(2): 161-170.

[11] Bohner M, Rao V S H, Sanyal S. Global stability of complex-valued neural networks on time scales[J]. Differential Equations and Dynamical Systems, 2011, 19(1-2): 3-11.

[12] Hirose A. Complex-valued neural networks[M]. NewYork: Springer Science & Business Media, 2006.

[13] Nitta, T. Complex-valued neural networks: utilizing high-dimensional parameters[M]. NewYork: IGI Global, 2009.

[14] Aizenberg I, Aizenberg N N, Vandewalle J P L. Multi-valued and universal binary neurons: theory, learning and applications[M]. NewYork: Springer Science & Business Media, 2000.

[15] Aizenberg I, Paliy D V, Zurada J M, et al. Blur identification by multilayer neural network based on multivalued neurons[J]. Neural Networks, IEEE Transactions on, 2008, 19(5): 883-898.

[16] Aizenberg I, Moraga C. Multilayer feedforward neural network based on multi-valued neurons (MLMVN) and a backpropagation learning algorithm[J]. Soft Computing, 2007, 11(2): 169-183.

[17] Kuroe Y, Hashimoto N, Mori T. On energy function for complex-valued neural networks and its applications[C]//Neural information processing, 2002. ICONIP′02. Proceedings of the 9th international conference on IEEE. DC: IEEE Computer Society Washington, 2002, 3: 1079-1083.

[18] Zhou W, Zurada J M. Discrete-time recurrent neural networks with complex-valued linear threshold neurons[J]. IEEE Transactions on Circuits and Systems Ⅱ, Express Briefs, 2009, 56(8): 669-678.

[19] Zhang W, Li C, Huang T. Global robust stability of complex-valued recurrent neural networks with time-delays and uncertainties[J]. International Journal of Biomathematics, 2014, 7(2): 1450016-1-24.

[20] Sree Hari Rao V, Murthy G R. Global dynamics of a class of complex valued neural networks[J]. International Journal of Neural Systems, 2008, 18(2): 165-171.

[21] Zhou W, Zurada J M. Discrete-time recurrent neural networks with complex-valued linear threshold neurons[J]. IEEE Transactions on Circuits and Systems Ⅱ, Express Briefs, 2009, 56(8): 669-677.

[22] Kuroe Y, Yoshid M, Mori T. On activation functions for complex-valued neural networks—existence of energy functions[M]//Artificial neural networks and neural information processing—ICANN/ICONIP 2003. Heidelberg: Springer Berlin, 2003: 985-992.

[23] Hu J, Wang J. Global stability of complex-valued recurrent neural networks with time-delays[J]. Neural Networks and Learning Systems, IEEE Transactions on, 2012, 23(6): 853-865.

复域时滞神经网络的渐近稳定

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摘要：对复域时滞神经网络的唯一性和在平衡点处的渐近稳定进行了研究，并基于 Lyapinov 函数方法和线性矩阵不等式(LMI)，复域时滞神经网络的一些渐近稳定的有效条件得到推导。最后通过一个有效的例子来验证了结论的有效性和正确性。

关键词：复域神经网络；渐进稳定；时滞

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