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含有混合时滞的不确定中立系统的稳定性条件*

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摘要:本文研究了一类带有时变时滞的非线性中立系统时滞依赖的稳定性条件,系统模型为:$\dot{x}(t)-C\dot{x}(t-h(t))=Ax(t)+Bx(t-\tau(t))+F(x(t),t)+G(x(t-\tau(t)),t)$。在本文中,时变时滞假定属于一个区间$\tau_m\leqslant\tau(t)\leqslant\tau_M$。目的是对于所容许的不确定性和时滞得到一个新的具有更低保守性的能够使系统达到渐近稳定的充分条件。在本文中有效地利用了时变时滞的变化区间的上下界这一信息,并且利用了牛顿-莱布尼茨公式,构造了新的 Lyapunov 泛函,基于 S-Procedure 引理得到了新的判别带有非线性不确定性中立系统渐近稳定的充分条件:$\sum<0$。最后通过数值算例来验证了此方法的可行性以及更低的保守性。

关键词:中立系统;渐近稳定性;时变时滞;非线性;线性矩阵不等式(LMI)
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在各类工业系统中,不确定性与时滞现象是极其普遍的。如通信系统、传送系统、电力系统等等都是典型的不确定性时滞系统。不确定性与时滞往往是系统不稳定和系统性能变差的根源。因此对不确定时滞系统稳定性问题的研究很重要。在过去几年里,关于中立系统的研究成为控制理论中的一个热点问题,例如文献[1-8,10-12],运用 Lyapunov 泛函的理论、特征方程的方法或者状态解的方法均可以导出中立系统渐进稳定性的条件。文献[1,4,8,10,12]是通过选择适当的 Lyapunov 函数导出稳定性的条件。其中,文献[8,12]是关于多时滞中立系统的时滞依赖的条件。文献[6,8]是通过选择特征方程的方法导出含有时滞中立系统稳定性的条件。但他们状态当中的时滞和状态得以阶导当中的时滞都是相同的并且是定常的。文献[11-12]是关于不确定非线性中立系统的稳定性条件的研究,但是其中立时滞是定常的并且中立项前的系数常阵也是定常的,并且没有考虑下界,不利于降低保守性。文献[13]考虑了时变区间的下界这一有用信息,但是没有考虑系统带有中立项以及非线性不确定性时变时滞。本文在其基础上,考虑了带有非线性不确定性的中立系统,通过构造新的 Lyapunov 泛函,并且利用了牛顿-莱布尼茨公式,得到了新的判别带有非线性不确定性中立系统渐近稳定的充分条件,并且通过数例说明了该方法的有效性以及更低的保守性。

本文采用以下的记号:符号“T”表示一个矩阵的转置,$\mathbf{R}^n$ 和 $\mathbf{R}^{n\times n}$ 分别表示 n 维欧几里德空间和所有的 $n\times n$ 实矩阵集合。对于实对称矩阵 X 和 Y,$X>Y(X\geqslant Y)$ 表示 $X-Y$ 为正定的($X-Y$ 是半正定的);I 和 O 是恰当维的单位矩阵和零矩阵。$\|\cdot\|$ 是欧几里德向量范数;“$*$”表示对称矩阵的对角线以上块矩阵的转置。

1 问题提出

考虑如下形式时滞中立系统

$$\dot{x}(t)-C(t)\dot{x}(t-h(t))=A(t)x(t)+B(t)x(t-\tau(t))+F(t)f(x(t),t)+G(t)g(x(t-\tau(t)),t)\tag{1}$$

$$x(t_0+\gamma)=\Psi(\gamma),\forall\gamma\in[-\max\{\tau_M,h\},0]\tag{2}$$

其中,$x(t)\in\mathbf{R}^n$ 是系统的状态向量,$\Psi(\cdot)$ 是一个具有连续向量值的初值函数,$f(x(t),t)\in\mathbf{R}^n$,$g(x(t-\tau(t)),t)\in\mathbf{R}^n$ 是未知的有界非线性不确定函数,$\tau(t)$,$h(t)$ 是时变时滞并且满足

$$\|f(x(t),t)\|\leqslant\alpha\|x(t)\|,\|g(x(t-\tau(t)),t)\|\leqslant\beta\|x(t-\tau(t))\|,\forall t>0$$

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$$0 \leqslant \tau_m \leqslant \tau(t) \leqslant \tau_M \tag{3}$$

$$0 \leqslant h(t) \leqslant h, \dot{h}(t) \leqslant h_D < 1 \tag{4}$$

不确定矩阵 $A(t), B(t), C(t), F(t), G(t)$ 满足

$$A(t) = A + \Delta A(t), B(t) = B + \Delta B(t), C(t) = C + \Delta C(t), F(t) = F + \Delta F(t), G(t) = G + \Delta G(t)$$

其中 :$A \in \mathbf{R}^{n\times n}, B \in \mathbf{R}^{n\times n}, C \in \mathbf{R}^{n\times n}, F \in \mathbf{R}^{n\times n}, G \in \mathbf{R}^{n\times n}$,是已知的实矩阵,$\Delta A(t), \Delta B(t), \Delta C(t), \Delta F(t), \Delta G(t)$ 由下列形式给出

$$\Delta A(t) = D_1\Sigma_1(t)E_1, \Delta B(t) = D_2\Sigma_2(t)E_2, \Delta C(t) = D_3\Sigma_3(t)E_3, \Delta F(t) = D_4\Sigma_4(t)E_4, \Delta G(t) = D_5\Sigma_5(t)E_5 \tag{5}$$

其中 $D_i, E_i, i = 1,2,\cdots,5$ 是已知相应维数的实矩阵,且 $\Sigma_i^{\mathrm{T}}(t)\sum_i(t) \leqslant I$。

引理 1[1]　(S-Procedure 引理) 对 $k = 0,1,2,\cdots,N$,设 $\sigma_k: V \to \mathbf{R}$ 是定义在一个线性向量空间 V(例如 $V = \mathbf{R}^n$)上的实值泛函,考虑以下的两个条件

1) S_1 对使得 $\sigma_k(y) \geqslant 0, k = 1,2,\cdots,N$ 的所有 $y \in V$,有 $\sigma_0(y) \geqslant 0$;

2) S_2 存在标量 $\tau_k \geqslant 0, k = 1,2,\cdots,N$,使得对任意的 $y \in V$,$\sigma_0(y) - \sum_{k=1}^{N}\tau(k)\sigma_k(y) \geqslant 0$

那么条件 S_2 可以推出条件 S_1。

引理 2[2]　对给定的正定矩阵 $M \in \mathbf{R}^{n\times n}$,$0 \leqslant h_m \leqslant h(t) \leqslant h_M, t \geqslant 0$,以及任意的可微向量函数 $x(t) \in \mathbf{R}^n$,下面两个命题成立

1) $\left(\int_{t-h_m}^{t}\dot{x}(s)\mathrm{d}s\right)^{\mathrm{T}}M\left(\int_{t-h_m}^{t}\dot{x}(s)\mathrm{d}s\right) \leqslant h_m\int_{t-h_m}^{t}\dot{x}^{\mathrm{T}}(s)M\dot{x}(s)\mathrm{d}s, t \geqslant 0$

2) $\left(\int_{t-h(t)}^{t-h_m}\dot{x}(s)\mathrm{d}s\right)^{\mathrm{T}}M\left(\int_{t-h(t)}^{t-h_m}\dot{x}(s)\mathrm{d}s\right) \leqslant (h(t) - h_m)\int_{t-h(t)}^{t-h_m}\dot{x}^{\mathrm{T}}(s)M\dot{x}(s)\mathrm{d}s \leqslant (h_M - h_m)\int_{t-h_M}^{t-h_m}\dot{x}^{\mathrm{T}}(s)M\dot{x}(s)\mathrm{d}s$, $t \geqslant 0$

假定 1　矩阵 $C(t)$ 的所有特征值都在单位圆内。

2 主要结论

首先考查当没有不确定性时,即系统为名义系统时

$$\Delta A(t) = 0, \Delta B(t) = 0, \Delta C(t) = 0, \Delta F(t) = 0, \Delta G(t) = 0$$

$$\dot{x}(t) - C\dot{x}(t - h(t)) = Ax(t) + Bx(t - \tau(t)) + Ff(x(t),t) + Gg(x(t - \tau(t)),t) \tag{6}$$

定理 1　设假定 1 成立,在初始条件(2) 下,系统(6) 是渐近稳定的,如果存在恰当维的矩阵 $\begin{pmatrix} Q_1 & Q_2 \\ Q_2^{\mathrm{T}} & Q_3 \end{pmatrix} > 0$,$\begin{pmatrix} Q_4 & Q_5 \\ Q_5^{\mathrm{T}} & Q6 \end{pmatrix} > 0$,$P > 0, R_1 > 0, R_2 > 0, R_3 > 0, M > 0, S > 0, \varepsilon_1 \geqslant 0, \varepsilon_2 \geqslant 0$,使得 $\Sigma < 0$ 成立,其中

$$\Sigma = \begin{pmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & 0 & \Sigma_{15} & 0 & R_3 & \Sigma_{18} & \Sigma_{19} & PG + A^{\mathrm{T}}\Theta G \\
* & \Sigma_{22} & 0 & S & 0 & S & 0 & \Sigma_{28} & \Sigma_{29} & B^{\mathrm{T}}\Theta G \\
* & * & \Sigma_{33} & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Sigma_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Sigma_{55} & -Q_5 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Sigma_{66} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -R_3 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Sigma_{88} & \Sigma_{89} & C^{\mathrm{T}}\Theta G \\
* & * & * & * & * & * & * & * & \Sigma_{99} & F^{\mathrm{T}}\Theta G \\
* & * & * & * & * & * & * & * & * & G^{\mathrm{T}}\Theta G - \varepsilon_2 I
\end{pmatrix}$$

$$\Sigma_{11}=PA+A^{\mathrm T}P-R_1-R_2-R_3+Q_1+Q_4+A^{\mathrm T}\Theta A+\varepsilon_1\alpha^2 I,\Sigma_{12}=PB+A^{\mathrm T}\Theta B,\Sigma_{13}=R_1+Q_2,$$
$$\Sigma_{15}=R_2+Q_5,\Sigma_{18}=PC+A^{\mathrm T}\Theta C,\Sigma_{19}=PF+A^{\mathrm T}\Theta F,\Sigma_{22}=-2S+B^{\mathrm T}\Theta B+\varepsilon_2\beta^2 I,\Sigma_{28}=B^{\mathrm T}\Theta C,$$
$$\Sigma_{29}=B^{\mathrm T}\Theta F,\Sigma_{33}=-R_1+Q_3-Q_1,\Sigma_{44}=-S-Q_3,\Sigma_{55}=-R_2+Q_6-Q_4,\Sigma_{66}=-S-Q_6,$$
$$\Sigma_{88}=-M+C^{\mathrm T}\Theta C,\Sigma_{89}=C^{\mathrm T}\Theta F,\Sigma_{99}=F^{\mathrm T}\Theta F-\varepsilon_1 I,\delta=\tau_M-\tau_m,$$
$$\Theta=\frac{1}{4}\tau_m^2R_1+\frac{1}{4}\tau_M^2R_2+\delta^2S+(1-h_D)^{-1}M+h^2R_3$$

证明 构造 Lyapunov 函数

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)+V_5(t)+V_6(t)+V_7(t)+V_8(t)$$

其中

$$V_1(t)=x^{\mathrm T}(t)Px(t),V_2(t)=\frac{\tau_m}{2}\int_{-\frac{\tau_m}{2}}^{0}\mathrm ds\int_{t+s}^{t}\dot x^{\mathrm T}(\theta)R_1\dot x(\theta)\mathrm d\theta$$

$$V_3(t)=\int_{t-\frac{\tau_m}{2}}^{t}\left(x^{\mathrm T}(s),x^{\mathrm T}\left(s-\frac{\tau_m}{2}\right)\right)\begin{pmatrix}Q_1&Q_2\\Q_2^{\mathrm T}&Q_3\end{pmatrix}\begin{pmatrix}x(s)\\x\left(s-\frac{\tau_m}{2}\right)\end{pmatrix}\mathrm ds$$

$$V_4(t)=\int_{t-\frac{\tau_M}{2}}^{t}\left(x^{\mathrm T}(s),x^{\mathrm T}\left(s-\frac{\tau_M}{2}\right)\right)\begin{pmatrix}Q_4&Q_5\\Q_5^{\mathrm T}&Q_6\end{pmatrix}\begin{pmatrix}x(s)\\x\left(s-\frac{\tau_M}{2}\right)\end{pmatrix}\mathrm ds$$

$$V_5(t)=\frac{\tau_M}{2}\int_{-\frac{\tau_M}{2}}^{0}\mathrm ds\int_{t+s}^{t}\dot x^{\mathrm T}(\theta)R_2\dot x(\theta)\mathrm d\theta,V_6(t)=\delta\int_{-\tau_M}^{-\tau_m}\mathrm ds\int_{t+s}^{t}\dot x^{\mathrm T}(\theta)S\dot x(\theta)\mathrm d\theta$$

$$V_7(t)=(1-h_D)^{-1}\int_{t+s}^{t}\dot x^{\mathrm T}(s)M\dot x(s)\mathrm ds,V_8(t)=h\int_{-h}^{0}\mathrm ds\int_{t+s}^{t}\dot x^{\mathrm T}(\theta)R_3\dot x(\theta)\mathrm d\theta$$

沿系统(6)的轨迹对 $V_i(t),i=1,2,\cdots,8$。关于时间 t 求导为

$$\dot V_1(t)=2x^{\mathrm T}(t)P\dot x(t)=$$
$$2x^{\mathrm T}(t)P[Ax(t)+Bx(t-\tau(t))+C\dot x(t-h(t))+Ff(x(t),t)+Gg(x(t-\tau(t)),t)]=$$
$$x^{\mathrm T}(t)PAx(t)+x^{\mathrm T}(t)A^{\mathrm T}Px(t)+2x^{\mathrm T}(t)PBx(t-\tau(t))+2x^{\mathrm T}(t)PC\dot x(t-h(t))+$$
$$2x^{\mathrm T}(t)PFf(x(t),t)+2x^{\mathrm T}(t)PGg(x(t-\tau(t)),t)$$

$$\dot V_2(t)=\frac{\tau_m^2}{4}\dot x^{\mathrm T}(t)R_1\dot x(t)-\frac{\tau_m}{2}\int_{t-\frac{\tau_m}{2}}^{t}\dot x^{\mathrm T}(\theta)R_1\dot x(\theta)\mathrm d\theta$$

由引理2得

$$-\frac{\tau_m}{2}\int_{t-\frac{\tau_m}{2}}^{t}\dot x^{\mathrm T}(\theta)R_1\dot x(\theta)\mathrm d\theta\leqslant-\left(\int_{t-\frac{\tau_m}{2}}^{t}\dot x(\theta)\mathrm d\theta\right)^{\mathrm T}R_1\left(\int_{t-\frac{\tau_m}{2}}^{t}\dot x(\theta)\mathrm d\theta\right)$$

根据牛顿-莱布尼茨公式 $\int_{t-\frac{\tau_m}{2}}^{t}\dot x(\theta)\mathrm d\theta=x(t)-x\left(t-\frac{\tau_m}{2}\right)$，所以

$$-\frac{\tau_m}{2}\int_{t-\frac{\tau_m}{2}}^{t}\dot x^{\mathrm T}(\theta)R_1\dot x(\theta)\mathrm d\theta\leqslant\left(x^{\mathrm T}(t)x^{\mathrm T}\left(t-\frac{\tau_m}{2}\right)\right)\begin{pmatrix}-R_1&R_1\\R_1&-R_1\end{pmatrix}\begin{pmatrix}x(t)\\x\left(t-\frac{\tau_m}{2}\right)\end{pmatrix}$$

$$\dot V_2(t)\leqslant\frac{\tau_m^2}{4}\dot x^{\mathrm T}(t)R_1\dot x(t)+\left(x^{\mathrm T}(t)x^{\mathrm T}\left(t-\frac{\tau_m}{2}\right)\right)\begin{pmatrix}-R_1&R_1\\R_1&-R_1\end{pmatrix}\begin{pmatrix}x(t)\\x\left(t-\frac{\tau_m}{2}\right)\end{pmatrix}$$

$$\dot V_3(t)=\left(x^{\mathrm T}(t)x^{\mathrm T}\left(t-\frac{\tau_m}{2}\right)\right)\begin{pmatrix}Q_1&Q_2\\Q_2^{\mathrm T}&Q_3\end{pmatrix}\begin{pmatrix}x(t)\\x\left(t-\frac{\tau_m}{2}\right)\end{pmatrix}-\left(x^{\mathrm T}\left(t-\frac{\tau_m}{2}\right)x^{\mathrm T}(t-\tau_m)\right)\begin{pmatrix}Q_1&Q_2\\Q_2^{\mathrm T}&Q_3\end{pmatrix}\begin{pmatrix}x\left(t-\frac{\tau_m}{2}\right)\\x(t-\tau_m)\end{pmatrix}$$

$$\dot V_4(t)=\left(x^{\mathrm T}(t)x^{\mathrm T}\left(t-\frac{\tau_M}{2}\right)\right)\begin{pmatrix}Q_4&Q_5\\Q_5^{\mathrm T}&Q_6\end{pmatrix}\begin{pmatrix}x(t)\\x\left(t-\frac{\tau_M}{2}\right)\end{pmatrix}-\left(x^{\mathrm T}\left(t-\frac{\tau_M}{2}\right)x^{\mathrm T}(t-\tau_M)\right)\begin{pmatrix}Q_4&Q_5\\Q_5^{\mathrm T}&Q_6\end{pmatrix}\begin{pmatrix}x\left(t-\frac{\tau_M}{2}\right)\\x(t-\tau_M)\end{pmatrix}$$

$$\dot V_5(t)=\frac{\tau_M^2}{4}\dot x^{\mathrm T}(t)R_2\dot x(t)-\frac{\tau_M}{2}\int_{t-\frac{\tau_M}{2}}^{t}\dot x^{\mathrm T}(\theta)R_2\dot x(\theta)\mathrm d\theta$$

$$\dot V_6(t)=\delta^2\dot x^{\mathrm T}(t)S\dot x(t)-\delta\int_{t-\tau_M}^{t-\tau_m}\dot x^{\mathrm T}(\theta)S\dot x(\theta)\mathrm d\theta$$

$$\dot{V}_8(t) = h^2\dot{x}^{\mathrm{T}}(t)R_3\dot{x}(t) - h\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(\theta)R_3\dot{x}(\theta)\,\mathrm{d}\theta$$

由引理 2 及牛顿-莱布尼茨公式

$$\dot{V}_5(t) \leqslant \frac{\tau_M^2}{4}\dot{x}^{\mathrm{T}}(t)R_2\dot{x}(t) + (x^{\mathrm{T}}(t)\;x^{\mathrm{T}}(t-\frac{\tau_M}{2}))\begin{pmatrix}-R_2 & R_2\\ R_2 & -R_2\end{pmatrix}\begin{pmatrix}x(t)\\ x(t-\frac{\tau_M}{2})\end{pmatrix}$$

$$\dot{V}_6(t) \leqslant \delta^2\dot{x}^{\mathrm{T}}(t)S\dot{x}(t) + (x^{\mathrm{T}}(t-\tau(t))\;x^{\mathrm{T}}(t-\tau_M))\begin{pmatrix}-S & S\\ S & -S\end{pmatrix}\begin{pmatrix}x(t-\tau(t))\\ x(t-\tau_M)\end{pmatrix} +$$
$$(x^{\mathrm{T}}(t-\tau(t))\;x^{\mathrm{T}}(t-\tau_m))\begin{pmatrix}-S & S\\ S & -S\end{pmatrix}\begin{pmatrix}x(t-\tau(t))\\ x(t-\tau_m)\end{pmatrix}$$

$$\dot{V}_6(t) \leqslant \delta^2\dot{x}^{\mathrm{T}}(t)S\dot{x}(t) + (x^{\mathrm{T}}(t-\tau(t))\;x^{\mathrm{T}}(t-\tau_M))\begin{pmatrix}-S & S\\ S & -S\end{pmatrix}\begin{pmatrix}x(t-\tau(t))\\ x(t-\tau_M)\end{pmatrix}$$

$$\dot{V}_8(t) \leqslant h^2\dot{x}^{\mathrm{T}}(t)R_3\dot{x}(t) + (x^{\mathrm{T}}(t)\;x^{\mathrm{T}}(t-h))\begin{pmatrix}-R_3 & R_3\\ R_3 & -R_3\end{pmatrix}\begin{pmatrix}x(t)\\ x(t-h)\end{pmatrix}$$

$$\dot{V}_7(t) = (1-h_D)^{-1}[\dot{x}^{\mathrm{T}}(t)M\dot{x}(t) - (1-\dot{h}(t))\dot{x}^{\mathrm{T}}(t-h(t))M\dot{x}(t-h(t))] \leqslant$$
$$(1-h_D)^{-1}\dot{x}^{\mathrm{T}}(t)M\dot{x}(t) - \dot{x}^{\mathrm{T}}(t-h(t))M\dot{x}(t-h(t))$$

综上

$$\dot{V}(t) \leqslant \eta^{\mathrm{T}}(t)\Sigma_0\eta(t)$$

其中

$$\Sigma_0 = \begin{pmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & 0 & \Sigma_{15} & 0 & R_3 & \Sigma_{18} & \Sigma_{19} & PG + A^{\mathrm{T}}\Theta G\\
* & \Sigma_{22} & 0 & S & 0 & S & 0 & \Sigma_{28} & \Sigma_{29} & B^{\mathrm{T}}\Theta G\\
* & * & \Sigma_{33} & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & \Sigma_{44} & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & \Sigma_{55} & -Q_5 & 0 & 0 & 0 & 0\\
* & * & * & * & * & \Sigma_{66} & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & -R_3 & 0 & 0 & 0\\
* & * & * & * & * & * & * & \Sigma_{88} & \Sigma_{89} & C^{\mathrm{T}}\Theta G\\
* & * & * & * & * & * & * & * & \Sigma_{99} & F^{\mathrm{T}}\Theta G\\
* & * & * & * & * & * & * & * & * & G^{\mathrm{T}}\Theta G
\end{pmatrix}$$

$$\Sigma_{11} = PA + A^{\mathrm{T}}P - R_1 - R_2 - R_3 + Q_1 + Q_4 + A^{\mathrm{T}}\Theta A,\ \Sigma_{12} = PB + A^{\mathrm{T}}\Theta B,\ \Sigma_{13} = R_1 + Q_2$$
$$\Sigma_{15} = R_2 + Q_5,\ \Sigma_{18} = PC + A^{\mathrm{T}}\Theta C,\ \Sigma_{19} = PF + A^{\mathrm{T}}\Theta F,\ \Sigma_{22} = -2S + B^{\mathrm{T}}\Theta B,\ \Sigma_{28} = B^{\mathrm{T}}\Theta C$$
$$\Sigma_{29} = B^{\mathrm{T}}\Theta F,\ \Sigma_{33} = -R_1 + Q_3 - Q_1,\ \Sigma_{44} = -S - Q_3,\ \Sigma_{55} = -R_2 + Q_6 - Q_4,\ \Sigma_{66} = -S - Q_6$$
$$\Sigma_{88} = -M + C^{\mathrm{T}}\Theta C,\ \Sigma_{89} = C^{\mathrm{T}}\Theta F,\ \Sigma_{99} = F^{\mathrm{T}}\Theta F,\ \delta = \tau_M - \tau_m$$
$$\Theta = \frac{1}{4}\tau_m^2R_1 + \frac{1}{4}\tau_M^2R_2 + \delta^2S + (1-h_D)^{-1}M + h^2R_3$$

要使系统(6)是渐进稳定的,必须 $\eta^{\mathrm{T}}(t)\Sigma_0\eta(t) < 0$,对所有的 $\eta(t) \neq 0$ 成立。基于 S-Procedure,假如存在 $\varepsilon_1 \geqslant 0$,$\varepsilon_2 \geqslant 0$,使得

$$\eta^{\mathrm{T}}(t)\Sigma\eta(t) = \eta^{\mathrm{T}}(t)\Sigma_0\eta(t) + \varepsilon_1[\alpha^2x^{\mathrm{T}}(t)x(t) - f^{\mathrm{T}}(x(t),t)f(x(t),t)] +$$
$$\varepsilon_2[\beta^2x^{\mathrm{T}}(t-\tau(t))x(t-\tau(t)) - g^{\mathrm{T}}(x(t-\tau(t)),t)g(x(t-\tau(t)),t)] < 0,\ \forall\eta(t) \neq 0$$

就等价于 $\Sigma < 0$,而不等式 $\Sigma < 0$ 表明:$\dot{V}(t) \leqslant -v\|x(t)\|^2$,其中 v 是充分小的正数,而假定 1 保证了系统 $x(t) - C(t)x(t-h(t)) = 0$ 的稳定性,因此系统(6)在初始条件(2)下是渐进稳定的。

注 1　对于系统(6)在由 $0 \leqslant \tau(t) \leqslant \tau_M$ 所描述的通常的时滞情况,只需令定理 1 证明中构造的 Lyapunov 泛函中的 $\tau_m = 0$,就得到 $0 \leqslant \tau(t) \leqslant \tau_M$ 情况下相应的 Lyapunov 泛函。

在定理 1 的基础上,下面给出含有不确定性(3)和(5)的系统(1)的渐进稳定性的充分条件。

定理2 设假定1成立,在初始条件(2)下,系统(1)是渐进稳定的,如果存在恰当维的矩阵$P>0$,$R_1>0$,$R_2>0$,$R_3>0$,$M>0$,$S>0$,$\varepsilon_1\geqslant 0$,$\varepsilon_2\geqslant 0$,$\varepsilon_3\geqslant 0$,$\varepsilon_4\geqslant 0$,$\varepsilon_5\geqslant 0$,$\varepsilon_6\geqslant 0$,$\varepsilon_7\geqslant 0$,$\begin{pmatrix} Q_1 & Q_2 \\ Q_2^{\mathrm{T}} & Q_3 \end{pmatrix}>0$,$\begin{pmatrix} Q_4 & Q_5 \\ Q_5^{\mathrm{T}} & Q_6 \end{pmatrix}>0$,使得$\Sigma<0$成立。$\Sigma$为下面的矩阵

$$\begin{pmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & 0 & \Sigma_{15} & 0 & R_3 & \Sigma_{18} & \Sigma_{19} & \Sigma_{110} & \Sigma_{111} & \Sigma_{112} & \Sigma_{113} & \Sigma_{114} & \Sigma_{115} \\
* & \Sigma_{22} & 0 & S & 0 & S & 0 & \Sigma_{28} & \Sigma_{29} & \Sigma_{210} & \Sigma_{211} & \Sigma_{212} & \Sigma_{213} & \Sigma_{214} & \Sigma_{215} \\
* & * & \Sigma_{33} & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Sigma_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Sigma_{55} & -Q_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Sigma_{66} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -R_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Sigma_{88} & \Sigma_{89} & \Sigma_{810} & \Sigma_{811} & \Sigma_{812} & \Sigma_{813} & \Sigma_{814} & \Sigma_{815} \\
* & * & * & * & * & * & * & * & \Sigma_{99} & \Sigma_{910} & \Sigma_{911} & \Sigma_{912} & \Sigma_{913} & \Sigma_{914} & \Sigma_{915} \\
* & * & * & * & * & * & * & * & * & \Sigma_{1010} & \Sigma_{1011} & \Sigma_{1012} & \Sigma_{1013} & \Sigma_{1014} & \Sigma_{1015} \\
* & * & * & * & * & * & * & * & * & * & \Sigma_{1111} & \Sigma_{1112} & \Sigma_{1113} & \Sigma_{1114} & \Sigma_{1115} \\
* & * & * & * & * & * & * & * & * & * & * & \Sigma_{1212} & \Sigma_{1213} & \Sigma_{1214} & \Sigma_{1215} \\
* & * & * & * & * & * & * & * & * & * & * & * & \Sigma_{1313} & \Sigma_{1314} & \Sigma_{1315} \\
* & * & * & * & * & * & * & * & * & * & * & * & * & \Sigma_{1414} & \Sigma_{1415} \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & \Sigma_{1515}
\end{pmatrix}$$

$$\Sigma_{11}=PA+A^{\mathrm{T}}P-R_1-R_2-R_3+Q_1+Q_4+A^{\mathrm{T}}\Theta A+\varepsilon_1\alpha^2 I+\varepsilon_3E_1^{\mathrm{T}}E_1$$

$$\Sigma_{12}=PB+A^{\mathrm{T}}\Theta B,\Sigma_{13}=R_1+Q_2,\Sigma_{15}=R_2+Q_5,\Sigma_{18}=PC+A^{\mathrm{T}}\Theta C$$

$$\Sigma_{19}=PF+A^{\mathrm{T}}\Theta F,\Sigma_{110}=PF+A^{\mathrm{T}}\Theta G,\Sigma_{111}=PD_1+A^{\mathrm{T}}\Theta D_1,\Sigma_{112}=PD_2+A^{\mathrm{T}}\Theta D_2$$

$$\Sigma_{113}=PD_3+A^{\mathrm{T}}\Theta D_3,\Sigma_{114}=PD_4+A^{\mathrm{T}}\Theta D_4,\Sigma_{115}=PD_5+A^{\mathrm{T}}\Theta D_5$$

$$\Sigma_{22}=-2S+B^{\mathrm{T}}\Theta B+\varepsilon_2\beta^2I+\varepsilon_4E_2^{\mathrm{T}}E_2,\Sigma_{28}=B^{\mathrm{T}}\Theta C,\Sigma_{29}=B^{\mathrm{T}}\Theta F,\Sigma_{210}=B^{\mathrm{T}}\Theta G,\Sigma_{211}=B^{\mathrm{T}}\Theta D_1$$

$$\Sigma_{212}=B^{\mathrm{T}}\Theta D_2,\Sigma_{213}=B^{\mathrm{T}}\Theta D_3,\Sigma_{214}=B^{\mathrm{T}}\Theta D_4,\Sigma_{215}=B^{\mathrm{T}}\Theta D_5,\Sigma_{33}=-R_1+Q_3-Q_1,\Sigma_{44}=-S-Q_3$$

$$\Sigma_{55}=-R_2+Q_6-Q_4,\Sigma_{66}=-S-Q_6,\Sigma_{88}=-M+C^{\mathrm{T}}\Theta C+\varepsilon_5E_3^{\mathrm{T}}E_3,\Sigma_{89}=C^{\mathrm{T}}\Theta F,\Sigma_{810}=C^{\mathrm{T}}\Theta G$$

$$\Sigma_{811}=C^{\mathrm{T}}\Theta D_1,\Sigma_{812}=C^{\mathrm{T}}\Theta D_2,\Sigma_{813}=C^{\mathrm{T}}\Theta D_3,\Sigma_{814}=C^{\mathrm{T}}\Theta D_4,\Sigma_{815}=C^{\mathrm{T}}\Theta D_5$$

$$\Sigma_{99}=-\varepsilon_1I+F^{\mathrm{T}}\Theta F+\varepsilon_6E_4^{\mathrm{T}}E_4,\Sigma_{910}=F^{\mathrm{T}}\Theta G,\Sigma_{911}=F^{\mathrm{T}}\Theta D_1,\Sigma_{912}=F^{\mathrm{T}}\Theta D_2,\Sigma_{913}=F^{\mathrm{T}}\Theta D_3$$

$$\Sigma_{914}=F^{\mathrm{T}}\Theta D_4,\Sigma_{915}=F^{\mathrm{T}}\Theta D_5,\Sigma_{1010}=-\varepsilon_2I+G^{\mathrm{T}}\Theta G+\varepsilon_7E_5^{\mathrm{T}}E_5,\Sigma_{1011}=G^{\mathrm{T}}\Theta D_1,\Sigma_{1012}=G^{\mathrm{T}}\Theta D_2$$

$$\Sigma_{1013}=G^{\mathrm{T}}\Theta D_3,\Sigma_{1014}=G^{\mathrm{T}}\Theta D_4,\Sigma_{1015}=G^{\mathrm{T}}\Theta D_5,\Sigma_{1111}=-\varepsilon_3I+D_1^{\mathrm{T}}\Theta D_1,\Sigma_{1112}=D_1^{\mathrm{T}}\Theta D_2$$

$$\Sigma_{1113}=D_1^{\mathrm{T}}\Theta D_3,\Sigma_{1114}=D_1^{\mathrm{T}}\Theta D_4,\Sigma_{1115}=D_1^{\mathrm{T}}\Theta D_5,\Sigma_{1212}=-\varepsilon_4I+D_2^{\mathrm{T}}\Theta D_2,\Sigma_{1213}=D_2^{\mathrm{T}}\Theta D_3$$

$$\Sigma_{1214}=D_2^{\mathrm{T}}\Theta D_4,\Sigma_{1215}=D_2^{\mathrm{T}}\Theta D_5,\Sigma_{1313}=-\varepsilon_5I+D_3^{\mathrm{T}}\Theta D_3,\Sigma_{1314}=D_3^{\mathrm{T}}\Theta D_4,\Sigma_{1315}=D_3^{\mathrm{T}}\Theta D_5$$

$$\Sigma_{1414}=-\varepsilon_6I+D_4^{\mathrm{T}}\Theta D_4,\Sigma_{1415}=D_4^{\mathrm{T}}\Theta D_5,\Sigma_{1515}=-\varepsilon_7I+D_5^{\mathrm{T}}\Theta D_5,\delta=\tau_M-\tau_m$$

$$\Theta=\frac{1}{4}\tau_m^2R_1+\frac{1}{4}\tau_M^2R_2+\delta^2S+(1-h_D)^{-1}M+h^2R_3$$

3 数值算例

例1 考虑名义系统(6),其中$A=\begin{pmatrix} -2 & 0 \\ 0 & -0.9 \end{pmatrix}$,$B=\begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$,$C=0$,$F=0$,$G=0$,$\alpha=0$,$\beta=0$,$\tau_m=0$,$h_D=0$,$h=0$,则$\tau_M=1.3453$,$\varepsilon_1=172.1500$,$\varepsilon_2=172.1500$。且具体的可行解如下。

$$P=\begin{pmatrix}83.5963 & 0.0005\\ 0.0005 & 0.0001\end{pmatrix},Q_1=\begin{pmatrix}428.0087 & 6.0615\\ 6.0615 & 157.5079\end{pmatrix},Q_2=\begin{pmatrix}-398.3429 & -6.0642\\ -6.1991 & -157.5077\end{pmatrix},$$

$$Q_3=\begin{pmatrix}509.9569 & 6.1935\\ 6.1935 & 157.5089\end{pmatrix},Q_4=\begin{pmatrix}171.0848 & 0.0029\\ 0.0029 & 0.0002\end{pmatrix},Q_5=\begin{pmatrix}-0.1751 & 0.0000\\ 0.0000 & -0.0000\end{pmatrix},$$

$$Q_6=\begin{pmatrix}161.0190 & 0.0032\\ 0.0032 & 0.0002\end{pmatrix},R_1=\begin{pmatrix}893.9192 & 15.4303\\ 15.4303 & 195.2079\end{pmatrix},R_2=\begin{pmatrix}0.3160 & -0.0000\\ -0.0000 & 0.0000\end{pmatrix},$$

$$R_3=\begin{pmatrix}114.7665 & -0.0225\\ -0.0225 & 65.3461\end{pmatrix},M=\begin{pmatrix}0.2525 & -0.0000\\ -0.0000 & 0.0000\end{pmatrix},S=\begin{pmatrix}19.0038 & 0.0001\\ 0.0001 & 0.0001\end{pmatrix}$$

例 2　利用定理 1 考虑如下的系统

$$\dot{x}(t)=\begin{pmatrix}-2 & 0\\ 0 & -0.9\end{pmatrix}x(t)+\begin{pmatrix}-1 & 0\\ -1 & -1\end{pmatrix}x(t-\tau(t))$$

由定理 1 的结论可以求得当τ_m 取不同的数值时,得到的区间上界具有更低的保守性,见表 1。

表 1　当τ_m 取不同的数值时τ_M 的值

	τ_m	1	2	3	4
文献[14]	τ_M	1.64	2.39	3.20	4.06
文献[15]	τ_M	1.742 4	2.432 8	3.223 4	4.064 4
本文	τ_M	1.804 3	2.504 9	3.259 1	4.188 0

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Stability Criteria in Uncertain Neutral Systems with Mixed Time-Varying Delays

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Abstract: This paper investigates the stability condition in a class of uncertain neutral systems with interval time-varying delays and nonlinear uncertainties. In this paper, the time-varying delays are assumed to belong to an interval: $\tau_m \leqslant \tau(t) \leqslant \tau_M$. The purpose is to derive a new delay-dependent stability condition with much less conservative in the neutral system, irrespective of the uncertainties and the time delays. Based on both the lower and upper bounds of time-varying delay interval, in the choice of the approriate Lyapunov function, used the transform of Leibnitz-Newton formula, a new delay-dependent stability condition of nonlinear neutral systems with time-varying delays is derived which is based on S-prosedure: $\Sigma < 0$. Finally, numerical examples show the effectiveness of our results and much less conservative.

Key words: neutral system; asymptotically; time-varying delays; nonlinear; linear matrix inequality(LMI)

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