DOI:10.3969/J. ISSN. 1672-6693. 2009. 03. 013

一类二阶超定双曲型复方程组的 Riemann-Hilbert 边值问题*

鄢盛勇
(四川教育学院 数学系,成都 610041)

摘要:考察了多双曲复数空间中,一类二阶超定双曲型复方程组 $\left(\frac{\partial^2\omega}{\partial\bar{z}_i\partial\bar{z}_k}\right)=(f_{ik}),i,k=1,2,z\in D$ 在一般柱型域上的 Riemann-Hilbert 边值问题。通过引入新的函数把问题转化为先求两个一阶超定双曲型复方程组,即广义多双曲正则函数在一般柱型域上的 Riemann-Hilbert 边值问题,由已有结果得到它们各自的解,然后再把原问题化为一个一阶超定双曲型复方程组的 Riemann-Hilbert 边值问题,在一般柱型域上通过函数论的方法获得了其可解条件,解的积分表示以及解的唯一性。

关键词:Riemann-Hilbert 边值问题;超定双曲型方程组;多双曲复数
中图分类号:O175.27 **文献标识码**:A **文章编号**:1672-6693(2009)03-0055-05

在文献[1]中通过引入双曲数研究了平面上一阶,二阶双曲型方程,文献[2-3]引入可交换四元数,研究了一些 3 维,4 维的双曲型方程组,文献[4]引入多双曲数研究 4 维空间中一类超定双曲型方程组,文献[5-8]研究了一些高阶高维的椭圆型方程组。本文用函数论的方法考虑一类二阶超定双曲型复方程组

$$\left(\frac{\partial^2\omega}{\partial\bar{z}_i\partial\bar{z}_k}\right)=(f_{ik})\qquad i,k=1,2,z\in D \tag{1}$$

在一般柱形域上的 Riemann-Hilbert 边值问题,即问题 RH。

问题 RH 求方程组(1)式在 D 内的解 $\omega(z)$,使它满足 $\omega\in C_\alpha^1[D]$,适合边界条件

$$\operatorname{Re}[\omega_{\bar{z}_i}(t)]=r_i(t),t\in L,\operatorname{Im}[\omega_{\bar{z}_i}(z_0)]=b_{i0},i=1,2$$
$$\operatorname{Re}[\Lambda(t)\omega(t)]=r(t),t\in L,\operatorname{Im}[\Lambda(z_0)\omega(z_0)]=b_0 \tag{2}$$

这里 b_{10},b_{20},b_0 是实常数,而 $\Lambda(t)=a(t)+\mathrm{j}b(t),r_1(t),r_2(t),r(t),b_{10},b_{20},b_0$ 满足条件

$$C_\alpha[\Lambda(t),L]\leqslant k_0,C_\alpha(r(t),L)\leqslant k_2,C_\alpha^1(r_i(t),L)\leqslant k_2,i=1,2 \tag{3}$$

$$\frac{\partial r_1}{\partial\lambda_2}=\frac{\partial r_2}{\partial\lambda_1},t\in L_{12}\times L_{22};\frac{\partial r_1}{\partial\mu_2}=\frac{\partial r_2}{\partial\mu_1},t\in L_{11}\times L_{21}$$

$$\max_{z\in L_1}\frac{1}{|a(t)-b(t)|}\leqslant k_0,\max_{z\in L_2}\frac{1}{|a(t)+b(t)|}\leqslant k_0;|b_0|,|b_{10}|,|b_{20}|\leqslant k_2 \tag{4}$$

此处 $\alpha(0<\alpha<1),k_0,k_2$ 都是非负常数。

1 预备知识

双曲数是指 $z=x+\mathrm{j}y$,这里 x,y 是两个实数,而 j 是双曲单元,使得 $\mathrm{j}^2=1$。记 $e_1=\frac{1+\mathrm{j}}{2},e_2=\frac{1-\mathrm{j}}{2}$。进而 $\omega=f(z)=u(x,y)+\mathrm{j}v(x,y)$ 称为双曲复变函数,其中 $u(x,y),v(x,y)$ 都是实变量 x,y 的实值函数,分别叫作 $\omega=f(z)$ 的实部与虚部,记作 $\operatorname{Re}\omega=u(z),\operatorname{Im}\omega=v(z)$。显然,$z=x+\mathrm{j}y=\lambda e_1+\mu e_2,\omega=f(z)=u+\mathrm{j}v=\xi e_1+\eta e_2$,其中

$$\lambda=x+y,\mu=x-y,x=\frac{\lambda+\mu}{2},y=\frac{\lambda-\mu}{2};\xi=u+v,\eta=u-v,u=\frac{\xi+\eta}{2},v=\frac{\xi-\eta}{2}$$

---

* 收稿日期:2008-12-08 修回日期:2009-03-25
作者简介:鄢盛勇,男,讲师,硕士,研究方向为复偏微分方程的边值问题。

记$\frac{\partial\omega}{\partial\bar z}=\omega_{\bar z}=\frac{1}{2}(\omega_x+j\omega_y)$,用C表示所有双曲数的集合,即双曲平面。记$\mathbf{C}^2=\{(z_1,z_2):z_i\in\mathbf{C},i=1,2\}$,其中元素称为多双曲数,可见有$\mathbf{C}^2=\mathbf{R}^2+j\mathbf{R}^2$,$\mathbf{R}^2$是2维实空间,记

$$x=(x_1,x_2),y=(y_1,y_2),z_1=x_1+jy_1,z_2=x_2+jy_2,z=(z_1,z_2)=x+jy,\bar z=x-jy$$

定义1[4] 设多双曲复变函数$\omega=f(z_1,z_2)$在区域$D(\in\mathbf{C}^2)$内满足$f_{\bar z_i}=0,i=1,2$,则称$f(z)$是D内的多双曲正则函数。

定义2[4] 多双曲复变函数$\omega=f(z_1,z_2)$在区域$D(\in\mathbf{C}^2)$内满足$f_{\bar z_i}=f_i(z),i=1,2$,则称$f(z)$是$D$内的广义多双曲正则函数。

设一般柱形域$D=\{D_1,D_2\}=D_1\times D_2$,$D_1,D_2$都是双曲复平面C上的一有界单连通区域,其边界分别为$\Gamma_i=L_{i1}\cup L_{i2}\cup L_{i3}\cup L_{i4}$,这里

$$L_{i1}=\{x_i=-y_i,0\leqslant x_i\leqslant R_1\},L_{i2}=\{x_i=y_i+2R_1,R_1\leqslant x_i\leqslant R_2\}$$
$$L_{i3}=\{x_i=-y_i+2R_2-2R_1,R_2-R_1\leqslant x_i\leqslant R_2\},L_{i4}=\{x_i=y_i,0\leqslant x_i\leqslant R_2-R_1\}$$
$$z_{0i}=(1-j)R,i=1,2,R=R_2-R_1$$

记$z_0=(z_{01},z_{02})$,$L_i=L_{i1}\cup L_{i2}$,$L=L_1\times L_2$,还设$R_2\geqslant 2R_1$。

引理1[4] 多双曲正则函数满足边界条件(2)~(4)式的问题RH有解,且可表示为

$$\omega=g_1(\mu_1,\mu_2)e_1+g_2(\lambda_1,\lambda_2)e_2$$

$$g_1(\mu_1,\mu_2)=\frac{2r(0,\mu_1,0,\mu_2)}{a(0,\mu_1,0,\mu_2)+b(0,\mu_1,0,\mu_2)}-\frac{[a(0,\mu_1,0,\mu_2)-b(0,\mu_1,0,\mu_2)][r(z_0)-b_0]}{[a(0,\mu_1,0,\mu_2)+b(0,\mu_1,0,\mu_2)][a(z_0)-b(z_0)]}$$
$$0\leqslant\mu_1\leqslant 2R_1,0\leqslant\mu_2\leqslant 2R_1 \tag{5}$$

$$g_2(\lambda_1,\lambda_2)=\frac{2r(\lambda_1,2R_1,\lambda_2,2R_1)}{a(\lambda_1,2R_1,\lambda_2,2R_1)-b(\lambda_1,2R_1,\lambda_2,2R_1)}-$$
$$\frac{a(\lambda_1,2R_1,\lambda_2,2R_1)+b(\lambda_1,2R_1,\lambda_2,2R_1)}{a(\lambda_1,2R_1,\lambda_2,2R_1)-b(\lambda_1,2R_1,\lambda_2,2R_1)}\times\frac{r(z_0)+b_0}{a(z_0)+b(z_0)},0\leqslant\lambda_1\leqslant 2R,0\leqslant\lambda_2\leqslant 2R \tag{6}$$

且还有估计式$C_\alpha[\omega,D]\leqslant M_1$($M_1$是只与$\alpha,k_0,k_2,D$有关的非负常数,以下类同)。

引理2 当满足$C_\alpha^1[f_i,D]\leqslant M_0,i=1,2$;$f_{1\bar z_2}=f_{2\bar z_1}$时,广义多双曲正则函数满足边界条件(2)~(4)式的问题RH有解,且可表示为

$$\omega(z)=W+\widetilde W+\Psi,W=g_1(\mu_1,\mu_2)e_1+g_2(\lambda_1,\lambda_2)e_2;\widetilde W=\tilde g_1(\mu_1,\mu_2)e_1+\tilde g_2(\lambda_1,\lambda_2)e_2$$

$$\Psi=\left\{\int_0^{\lambda_1}(\mathrm{Re}f_1+\mathrm{Im}f_1)\Big|_{\lambda_2=0}\mathrm{d}\lambda_1+\int_0^{\lambda_2}(\mathrm{Re}f_2+\mathrm{Im}f_2)\mathrm{d}\lambda_2\right\}e_1+$$
$$\left\{\int_{2R_1}^{\mu_1}(\mathrm{Re}f_1-\mathrm{Im}f_1)\Big|_{\mu_2=2R_1}\mathrm{d}\mu_1+\int_{2R_1}^{\mu_2}(\mathrm{Re}f_2-\mathrm{Im}f_2)\mathrm{d}\mu_2\right\}e_2$$

其中$\tilde g_1(\mu_1,\mu_2)$,$\tilde g_2(\lambda_1,\lambda_2)$类似于(5),(6)式,$\widetilde W(z)$是多双曲正则函数,满足下列边界条件

$$\mathrm{Re}[\Lambda(z)\widetilde W(z)]=-\mathrm{Re}[\Lambda(z)\Psi(z)],z\in L,\mathrm{Im}[\Lambda(z_0)\widetilde W(z_0)]=-\mathrm{Im}[\Lambda(z_0)\Psi(z_0)]=0$$

并且还有估计式$C_\alpha[\omega,D]\leqslant M_2$。

2 问题的转化

假设方程组(1)满足下列条件C。

条件C f_{ik}满足$C_\alpha^1[f_{ik},\overline D]\leqslant M_0$;$f_{ik}=f_{ki}$,$\frac{\partial f_{ik}}{\partial\bar z_l}=\frac{\partial f_{lk}}{\partial\bar z_i}$,$i,k,l=1,2$,令$F_1=\frac{\partial\omega}{\partial\bar z_1}$,$F_2=\frac{\partial\omega}{\partial\bar z_2}$,则它们是广义多双曲正则函数,分别满足下面方程组

$$\begin{cases}\frac{\partial F_1}{\partial\bar z_1}=f_{11}(z)\\ \frac{\partial F_1}{\partial\bar z_2}=f_{21}(z)\end{cases},z\in D;\qquad\begin{cases}\frac{\partial F_2}{\partial\bar z_1}=f_{12}(z)\\ \frac{\partial F_2}{\partial\bar z_2}=f_{22}(z)\end{cases},z\in D \tag{7}$$

由条件 C 和引理 2 知，满足（7）式的 F_1, F_2 可表示成 $F_i = W_i + \widetilde{W}_i + \Psi_i, i = 1,2$　（8）

$$W_i = g_{i1}(\mu_1,\mu_2)e_1 + g_{i2}(\lambda_1,\lambda_2)e_2;\widetilde{W}_i = \widetilde{g}_{i1}(\mu_1,\mu_2)e_1 + \widetilde{g}_{i2}(\lambda_1,\lambda_2)e_2 \tag{9}$$

$$\Psi_i = \left\{\int_0^{\lambda_1}(\mathrm{Re}f_{i1} + \mathrm{Im}f_{i1})\Big|_{\lambda_2=0}\mathrm{d}\lambda_1 + \int_0^{\lambda_2}(\mathrm{Re}f_{i2} + \mathrm{Im}f_{i2})\mathrm{d}\lambda_2\right\}e_1 + \left\{\int_{2R_1}^{\mu_1}(\mathrm{Re}f_{i1} - \mathrm{Im}f_{i1})\Big|_{\mu_2=2R_1}\mathrm{d}\mu_1 + \int_{2R_1}^{\mu_2}(\mathrm{Re}f_{i2} - \mathrm{Im}f_{i2})\mathrm{d}\mu_2\right\}e_2 \tag{10}$$

$$g_{i1}(\mu_1,\mu_2) = 2r_i(0,\mu_1,0,\mu_2) - r_i(z_0) + b_{i0}, 0 \leqslant \mu_1,\mu_2 \leqslant 2R_1 \tag{11}$$

$$g_{i2}(\lambda_1,\lambda_2) = 2r_i(\lambda_1,2R_1,\lambda_2,2R_1) - r_i(z_0) - b_{i0}, 0 \leqslant \lambda_1,\lambda_2 \leqslant 2R \tag{12}$$

而 $\widetilde{W}_i, i = 1,2$ **类似于** $W_i, i = 1,2$，**也是** D **内多双曲正则函数，适合下列边界条件**

$$\mathrm{Re}[\widetilde{W}_i(z)] = -\mathrm{Re}[\Psi_i(z)], z \in L, \mathrm{Im}[\widetilde{W}_i(z_0)] = -\mathrm{Im}[\Psi_i(z_0)], \Psi_i(z_0) = 0$$

于是有

$$\mathrm{Re}[\widetilde{W}_i(z)] = -\frac{1}{2}\left\{\int_{2R_1}^{\mu_1}(\mathrm{Re}f_{i1} - \mathrm{Im}f_{i1})\Big|_{\mu_2=2R_1}\mathrm{d}\mu_1 + \int_{2R_1}^{\mu_2}(\mathrm{Re}f_{i2} - \mathrm{Im}f_{i2})\mathrm{d}\mu_2\right\}, z \in L_{11}\times L_{21}$$

$$\mathrm{Re}[\widetilde{W}_i(z)] = -\frac{1}{2}\left\{\int_{2R_1}^{\mu_1}(\mathrm{Re}f_{i1} - \mathrm{Im}f_{i1})\mathrm{d}\mu_1 + \int_0^{\lambda_2}(\mathrm{Re}f_{i2} + \mathrm{Im}f_{i2})\mathrm{d}\lambda_2\right\}, z \in L_{11}\times L_{22}$$

$$\mathrm{Re}[\widetilde{W}_i(z)] = -\frac{1}{2}\left\{\int_{2R_1}^{\mu_2}(\mathrm{Re}f_{i2} - \mathrm{Im}f_{i2})\mathrm{d}\mu_2 + \int_0^{\lambda_1}(\mathrm{Re}f_{i1} + \mathrm{Im}f_{i1})\mathrm{d}\lambda_1\right\}, z \in L_{12}\times L_{21}$$

$$\mathrm{Re}[\widetilde{W}_i(z)] = -\frac{1}{2}\left\{\int_0^{\lambda_1}(\mathrm{Re}f_{i1} + \mathrm{Im}f_{i1})\Big|_{\lambda_2=0}\mathrm{d}\lambda_1 + \int_0^{\lambda_2}(\mathrm{Re}f_{i2} + \mathrm{Im}f_{i2})\mathrm{d}\lambda_2\right\}, z \in L_{12}\times L_{22}$$

$$\widetilde{g}_{i1}(\mu_1,\mu_2) = -\int_{2R_1}^{\mu_1}(\mathrm{Re}f_{i1} - \mathrm{Im}f_{i1})\Big|_{\substack{\mu_2=2R_1\\ \lambda_1=\lambda_2=0}}\mathrm{d}\mu_1 - \int_{2R_1}^{\mu_2}(\mathrm{Re}f_{i2} - \mathrm{Im}f_{i2})\Big|_{\substack{\lambda_1=0\\ \lambda_2=0}}\mathrm{d}\mu_2, 0 \leqslant \mu_1,\mu_2 \leqslant 2R_1 \tag{13}$$

$$\widetilde{g}_{i2}(\lambda_1,\lambda_2) = -\int_0^{\lambda_1}(\mathrm{Re}f_{i1} + \mathrm{Im}f_{i1})\Big|_{\substack{\mu_1=\mu_2=2R_1\\ \lambda_2=0}}\mathrm{d}\lambda_1 - \int_0^{\mu_1}(\mathrm{Re}f_{i2} + \mathrm{Im}f_{i2})\Big|_{\substack{\mu_1=2R_1\\ \mu_2=2R_1}}\mathrm{d}\lambda_2, 0 \leqslant \lambda_1,\lambda_2 \leqslant 2R \tag{14}$$

并且还有估计式　$C_\alpha^1[F_i, D] \leqslant M_3, i = 1,2$　（15）

于是方程组（1）式的问题 RH 转化为广义多双曲正则函数即方程组（16）式，适合边界条件（5）式的 Riemann-Hilbert **边值问题**

$$\begin{cases}\dfrac{\partial\omega}{\partial\bar{z}_1} = F_1(z)\\[2mm] \dfrac{\partial\omega}{\partial\bar{z}_2} = F_2(z)\end{cases}, z \in D \tag{16}$$

3 问题的解

定理 1　**当条件 C 满足时，方程组（1）式的问题 RH 有解，且可表示为（17）~（22）式，并且还有估计式** $C_\alpha^1[\omega, D] \leqslant M_4$。

证明　**方程组（1）式的解** $\omega(z)$ **满足（16）式，其中** F_1, F_2 **见（8）式，因** $W_1, W_2, \widetilde{W}_1, \widetilde{W}_2$ **都是多双曲正则函数，由条件 C 得** $\dfrac{\partial F_1}{\partial\bar{z}_2} = f_{12} = \dfrac{\partial F_2}{\partial\bar{z}_1}$，**结合（15）式，由引理 2 可得方程组（16）式的解为**

$$\omega(z) = W + \widetilde{W} + \Psi, W = g_1(\mu_1,\mu_2)e_1 + g_2(\lambda_1,\lambda_2)e_2;\widetilde{W} = \widetilde{g}_1(\mu_1,\mu_2)e_1 + \widetilde{g}_2(\lambda_1,\lambda_2)e_2 \tag{17}$$

$$g_1(\mu_1,\mu_2) = \frac{2r(0,\mu_1,0,\mu_2)}{a(0,\mu_1,0,\mu_2) + b(0,\mu_1,0,\mu_2)} - \frac{[a(0,\mu_1,0,\mu_2) - b(0,\mu_1,0,\mu_2)][r(z_0) - b_0]}{[a(0,\mu_1,0,\mu_2) + b(0,\mu_1,0,\mu_2)][a(z_0) - b(z_0)]}, 0 \leqslant \mu_1 \leqslant 2R_1, 0 \leqslant \mu_2 \leqslant 2R_1 \tag{18}$$

$$g_2(\lambda_1,\lambda_2) = \frac{2r(\lambda_1,2R_1,\lambda_2,2R_1)}{a(\lambda_1,2R_1,\lambda_2,2R_1) - b(\lambda_1,2R_1,\lambda_2,2R_1)} - \frac{a(\lambda_1,2R_1,\lambda_2,2R_1) + b(\lambda_1,2R_1,\lambda_2,2R_1)}{a(\lambda_1,2R_1,\lambda_2,2R_1) - b(\lambda_1,2R_1,\lambda_2,2R_1)} \times \frac{r(z_0) + b_0}{a(z_0) + b(z_0)}, 0 \leqslant \lambda_1 \leqslant 2R, 0 \leqslant \lambda_2 \leqslant 2R \tag{19}$$

$$\Psi = \{\int_0^{\lambda_1}(\mathrm{Re}[W_1+\widetilde{W}_1+\Psi_1]+\mathrm{Im}[W_1+\widetilde{W}_1+\Psi_1])\Big|_{\lambda_2=0}\mathrm{d}\lambda_1 + \int_0^{\lambda_2}(\mathrm{Re}[W_2+\widetilde{W}_2+\Psi_2]+\mathrm{Im}[W_2+\widetilde{W}_2+\Psi_2])\mathrm{d}\lambda_2\}e_1 + \{\int_{2R_1}^{\mu_1}(\mathrm{Re}[W_1+\widetilde{W}_1+\Psi_1]-\mathrm{Im}[W_1+\widetilde{W}_1+\Psi_1])\Big|_{\mu_2=2R_1}\mathrm{d}\mu_1 + \int_{2R_1}^{\mu_2}(\mathrm{Re}[W_2+\widetilde{W}_2+\Psi_2]-\mathrm{Im}[W_2+\widetilde{W}_2+\Psi_2])\mathrm{d}\mu_2\}e_2$$

利用条件C和(9)~(14)式化简$\Psi(z)$即得

$$\Psi = \{\sum_{i=1}^{2}\lambda_i[g_{i1}-\int_{2R_1}^{\mu_2}(\mathrm{Re}f_{i2}-\mathrm{Im}f_{i2})\Big|_{\substack{\lambda_1=0\\ \lambda_2=0}}\mathrm{d}\mu_1-\int_{2R_1}^{\mu_1}(\mathrm{Re}f_{i1}-\mathrm{Im}f_{i1})\Big|_{\substack{\lambda_1=\lambda_2=0\\ \mu_2=2R_1}}\mathrm{d}\mu_1] + \int_0^{\lambda_1}\mathrm{d}\lambda_1\int_0^{\lambda_1}(\mathrm{Re}f_{11}+\mathrm{Im}f_{11})\Big|_{\lambda_2=0}\mathrm{d}\lambda_1+\int_0^{\lambda_2}\mathrm{d}\lambda_2\int_0^{\lambda_2}(\mathrm{Re}f_{22}+\mathrm{Im}f_{22})\mathrm{d}\lambda_2 + \lambda_2\int_0^{\lambda_1}(\mathrm{Re}f_{21}+\mathrm{Im}f_{21})\Big|_{\lambda_2=0}\mathrm{d}\lambda_1\}e_1 + \{(\mu_2-2R_1)\int_{2R_1}^{\mu_1}(\mathrm{Re}f_{21}-\mathrm{Im}f_{21})\Big|_{\mu_2=2R_1}\mathrm{d}\mu_1 + \sum_{i=1}^{2}(\mu_i-2R_1)[g_{i2}-\int_0^{\lambda_1}(\mathrm{Re}f_{i1}+\mathrm{Im}f_{i1})\Big|_{\substack{\lambda_2=0\\ \mu_1=\mu_2=2R_1}}\mathrm{d}\lambda_1-\int_0^{\lambda_2}(\mathrm{Re}f_{i2}+\mathrm{Im}f_{i2})\Big|_{\substack{\mu_1=2R_1\\ \mu_2=2R_1}}\mathrm{d}\lambda_1] + \int_{2R_1}^{\mu_1}\mathrm{d}\mu_1\int_{2R_1}^{\mu_1}(\mathrm{Re}f_{11}-\mathrm{Im}f_{11})\Big|_{\mu_2=2R_1}\mathrm{d}\mu_1+\int_{2R_1}^{\mu_2}\mathrm{d}\mu_2\int_{2R_1}^{\mu_2}(\mathrm{Re}f_{22}-\mathrm{Im}f_{22})\mathrm{d}\mu_2\}e_2 \quad (20)$$

而$\widetilde{W}(z)$是多双曲正则函数，满足下列边界条件

$$\mathrm{Re}[\Lambda(z)\widetilde{W}(z)] = -\mathrm{Re}[\Lambda(z)\Psi(z)], z\in L, \mathrm{Im}[\Lambda(z_0)\widetilde{W}(z_0)] = 0$$

于是$\tilde{g}_1(\mu_1,\mu_2)$，$\tilde{g}_2(\lambda_1,\lambda_2)$可表示为

$$\tilde{g}_1(\mu_1,\mu_2) = \frac{b-a}{a+b}\Big|_{\substack{\lambda_1=0\\ \lambda_2=0}}\cdot[\sum_{i=1}^{2}(r_i(z_0)-b_{i0})(\mu_i-2R_1)+(\mu_2-2R_1)\int_{2R_1}^{\mu_1}(\mathrm{Re}f_{21}-\mathrm{Im}f_{21})\Big|_{\substack{\lambda_1=\lambda_2=0\\ \mu_2=2R_1}}\mathrm{d}\mu_1 + \int_{2R_1}^{\mu_1}\mathrm{d}\mu_1\int_{2R_1}^{\mu_1}(\mathrm{Re}f_{11}-\mathrm{Im}f_{11})\Big|_{\substack{\lambda_1=\lambda_2=0\\ \mu_2=2R_1}}\mathrm{d}\mu_1+\int_{2R_1}^{\mu_2}\mathrm{d}\mu_2\int_{2R_1}^{\mu_2}(\mathrm{Re}f_{22}-\mathrm{Im}f_{22})\Big|_{\substack{\lambda_1=0\\ \lambda_2=0}}\mathrm{d}\mu_2] \quad (21)$$

$$\tilde{g}_2(\lambda_1,\lambda_2) = \frac{b+a}{b-a}\Big|_{\substack{\mu_1=2R_1\\ \mu_2=2R_1}}\cdot[\sum_{i=1}^{2}\lambda_i(r_i(z_0)+b_{i0})+\lambda_2\int_0^{\lambda_1}(\mathrm{Re}f_{21}+\mathrm{Im}f_{21})\Big|_{\substack{\lambda_2=0\\ \mu_1=\mu_2=2R_1}}\mathrm{d}\lambda_1 + \int_0^{\lambda_1}\mathrm{d}\lambda_1\int_0^{\lambda_1}(\mathrm{Re}f_{11}+\mathrm{Im}f_{11})\Big|_{\substack{\lambda_2=0\\ \mu_1=\mu_2=2R_1}}\mathrm{d}\lambda_1+\int_0^{\lambda_2}\mathrm{d}\lambda_2\int_0^{\lambda_2}(\mathrm{Re}f_{22}+\mathrm{Im}f_{22})\Big|_{\substack{\mu_1=2R_1\\ \mu_2=2R_1}}\mathrm{d}\lambda_2] \quad (22)$$

再由(3)，(4)式及条件C，易得$C_\alpha^1[\omega,D]\leqslant M_4$。 证毕

定理2 当条件C满足时，方程组(1)式的问题RH至多有一解。

证明 设方程组(1)式的问题RH有解$\omega_1(z)$，$\omega_2(z)$，令$\omega(z)=\omega_1(z)-\omega_2(z)$，则它是方程组(1)式在$f_{ik}=0, i,k=1,2$且$r_i(t)=0, r(t)=0, t\in L; b_{i0}=0, b_0=0, i=1,2$时问题RH的解。由定理1知道其解可表示为$\omega(z)=0$，即$\omega_1(z)=\omega_2(z)$。 证毕

参考文献：

[1] Wen G C. Linear and quasilinear complex equations of hyperbolic and mixed type[M]. London:Taylor and Francis,2002.

[2] 杨丕文，杨硕. 可换四元数空间中某些双曲型方程的Riemann-Hilbert边值问题[J]. 数学学报,2008,51(1):171-180.

[3] 杨丕文，李曼荔，杨硕. 在可交换四元数空间中的双曲方程的特征边值问题[J]. 数学学报,2007,50(6):1249-1256.

[4] 鄢盛勇，杨柳，杨丕文. 一般柱形域上广义双曲正则函数的Riemann-Hilbert边值问题 [J]. 四川师范大学学报(自然科学版),2006,29(2):143-146.

[5] Begehr H,Dzhuraev A. An introduction to several complex variables and partial differential equations[M]. Harlow:Longman,1997.

[6] Cheng J, Li M Z. Boundary value problems for complex overdetermined partial differential equations of second order[A]. Wen G C, Zhao Z. Proceeding of the Integral Equations and Value Problems[C]. Singapore: World Sci, 1991: 11-17.
[7] 李觉友. 非齐次 Moisil-Theodorsco 方程组的 Riemann 边值问题[J]. 重庆师范大学学报(自然科学版), 2007, 24(4): 26-29.
[8] 李觉友. Moisil-Theodorsco 方程组的一个非线性边值问题[J]. 重庆师范大学学报(自然科学版), 2008, 25(1): 42-46.

Riemann-Hilbert Boundary Value Problem in a Class of Overdetermined Hyperbolic Equations of Second Order

YAN Sheng-yong

(Dept. of Mathematics, Sichuan College of Education, Chengdu 610041, China)

Abstract: In this paper, we study the Riemann-Hilbert boundary value problems in the general bicylinder in a class of overdetermined hyperbolic equations of second order $\left(\frac{\partial^2\omega}{\partial\bar{z}_i\partial\bar{z}_k}\right)=(f_{ik})$, $i,k=1,2$, $z\in D$ in the several hyperbolic complex space. Firstly, with new functions we translate the Riemann-Hilbert boundary value problems to two Riemann-Hilbert boundary value problems in overdetermined hyperbolic equations of first order, or the generalized hyperbolic regular functions in the general bicylinder, and obtain the representations of the solutions, because the problems have been solvedly. Secondly, we must deal with the Riemann-Hilbert boundary value problems in a class of overdetermined hyperbolic equations of first order that being equivalent to the Riemann-Hilbert boundary value problems in the general bicylinder in that class of overdetermined hyperbolic equations of second order to discuss the question. In the general bicylinder, with theory of functions method, we get conditions for its solvability, prove its solvability of the problem, and give the integral representations of the solution and its uniqueness.

Key words: Riemann-Hilbert boundary value problem; overdetermined hyperbolic complex equations; several hyperbolic complex

(责任编辑 黄 颖)