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含扩散与无限时滞的竞争型 Lotka-Volterra 模型的周期解与稳定性*

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摘要:研究了一类含扩散与无限分布时滞的竞争型 Lotka-Volterra 生态模型,利用对应特征值问题解的性质和比较原理,通过对应周期抛物系统 $\frac{\partial u_i(t,x)}{\partial t} - A_i u_i(t,x) = u_i(t,x)[a_i(t,x) - b_i(t,x)u_i(t,x)]$,$(i=1,2)$ 的周期解得到模型的上下解$(\tilde{u}_1,\tilde{u}_2)$,$(0,0)$,证明了模型在所对应的特征方程的主特征值$\sigma_1(a_i)\geqslant 0$,$(i=1,2)$ 时存在全局渐近稳定的平凡解,当 $\sigma_1(a_1)<0$,$\sigma_1(a_2)\geqslant 0$ 和 $\sigma_1(a_1)\geqslant 0$,$\sigma_1(a_2)<0$ 时分别存在全局渐近稳定的半平凡解$(\theta_1(t,x),0)$ 和$(0,\theta_2(t,x))$。并采用单调迭代技巧构造恰当的 T-周期序列,证明了对任意的非负初始值,模型存在一对周期正解及其渐近稳定的条件。

关键词:扩散;无限时滞;上下解;全局渐近稳定;周期解
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1 引言及预备知识

含扩散与时滞的 Lotka-Volterra 是一类非常重要的生态学数学模型,许多科学工作者利用这类模型对生物种群问题进行了研究,周期解的存在性与稳定性是生态方程的一个重要研究内容,已受到广泛的重视。特别是近年来,时滞对周期解的影响吸引了越来越多学者的关注,大量的结果被建立[1-7]。然而,这些研究主要涉及有限时滞的情形,而对无限时滞问题却很少涉及[8-9]。基于此,本文将研究一类含扩散与无限时滞的竞争型 Lotka-Volterra 系统,通过特征值问题构造模型的上下解,应用比较原理得到其周期解存在与全局渐近稳定的一个充分条件。

本文考虑如下竞争型 Lotka-Volterra 系统

$$\frac{\partial u_1(t,x)}{\partial t} - A_1 u_1(t,x) = u_1(t,x)[a_1(t,x) - b_1(t,x)u_1(t,x) - \int_{-\infty}^{0} u_2(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)]$$

$$\frac{\partial u_2(t,x)}{\partial t} - A_2 u_2(t,x) = u_2(t,x)[a_2(t,x) - b_2(t,x)u_2(t,x) - \int_{-\infty}^{0} u_1(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)]$$

$$(t,x)\in[0,+\infty)\times\Omega$$

$$B_i[u_i](t,x) = 0,(t,x)\in[0,+\infty)\times\partial\Omega$$

$$u_i(s,x) = u_{i,0}(s,x)$$

$$(s,x)\in(-\infty,0]\times\Omega, i=1,2 \qquad (1)$$

其中 Ω 是 $\mathbf{R}^N$ 中的有界区域,边界为 $\partial\Omega$,算子 A_i 定义为

$$A_i u_i(t,x) = \sum_{s,k=1}^{N}\alpha_{sk}^i(t,x)\frac{\partial^2 u_i}{\partial x_s\partial x_k} + \sum_{s=1}^{N}\beta_s^i(t,x)\frac{\partial u_i}{\partial x_s}$$

且为一致椭圆算子。α_{sk}^i,β_s^i,$a_i(t,x)$,$b_i(t,x)$ 是关于 t 的 T-周期函数,且在$[0,+\infty)\times\overline{\Omega}$ 上 Hölder 连续,$b_i>0$,$\eta_i(t,s,x)$ 是$[0,T]\times\overline{\Omega}$ 上的 T-周期光滑函数,满足$\int_{-\infty}^{0}\mathrm{d}_s\eta_i(t,s,x)=1$,对任意固定的$(t,x)\in[0,T]\times\Omega$,对时滞 s 在$(-\infty,0]$ 上非减。同时假设 $B_i[u_i]=u_i$ 或$\frac{\partial u_i}{\partial v}+\gamma_i(x)u_i$,其中

$$\gamma_i(x)\in C^{1+\alpha}(\partial\Omega) \text{ 且 } \gamma_i(x)\geqslant 0, x\in\partial\Omega$$

$$u_{i,0}\in C((-\infty,0],C_0(\overline{\Omega})) \text{ 且 } u_{i,0}\geqslant 0,$$

$$(t,x)\in[-s,0]\times\overline{\Omega}$$

2 主要结果及证明

为证明本文的主要结果,引入以下引理。

引理 1 如果(1)式存在上解$(\tilde{u}_1,\tilde{u}_2)$和下解

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$(\hat{u}_1,\hat{u}_2)$,即有光滑函数$(\tilde{u}_1,\tilde{u}_2)$,$(\hat{u}_1,\hat{u}_2)$满足$\tilde{u}_i \geqslant \hat{u}_i$且

$$\frac{\partial \tilde{u}_1(t,x)}{\partial t} - A_1\tilde{u}_1(t,x) \geqslant \tilde{u}_1(t,x)[a_1(t,x) - b_1(t,x)\tilde{u}_1(t,x) - \int_{-\infty}^{0}\hat{u}_2(t-s,x)\mathrm{d}_s\eta_1(t,s,x)]$$

$$\frac{\partial \tilde{u}_2(t,x)}{\partial t} - A_2\tilde{u}_2(t,x) \geqslant \tilde{u}_2(t,x)[a_2(t,x) - b_2(t,x)\tilde{u}_2(t,x) - \int_{-\infty}^{0}\hat{u}_1(t-s,x)\mathrm{d}_s\eta_2(t,s,x)]$$

$$\frac{\partial \hat{u}_1(t,x)}{\partial t} - A_1\hat{u}_1(t,x) \geqslant \hat{u}_1(t,x)[a_1(t,x) - b_1(t,x)\hat{u}_1(t,x) - \int_{-\infty}^{0}\tilde{u}_2(t-s,x)\mathrm{d}_s\eta_1(t,s,x)]$$

$$\frac{\partial \hat{u}_2(t,x)}{\partial t} - A_2\hat{u}_1(t,x) \geqslant \hat{u}_2(t,x)[a_2(t,x) - b_2(t,x)\hat{u}_2(t,x) - \int_{-\infty}^{0}\tilde{u}_1(t-s,x)\mathrm{d}_s\eta_2(t,s,x)]$$

$$(t,x) \in [0,+\infty)\times\Omega$$

$$B_i[\tilde{u}_i](t,x) \geqslant 0 \geqslant B_i[\hat{u}_i](t,x),(t,x) \in [0,+\infty)\times\partial\Omega$$

$$\tilde{u}_i(s,x) \geqslant u_{i,0}(s,x) \geqslant \hat{u}_i(s,x)$$

$$(s,x) \in (-\infty,0]\times\Omega, i=1,2 \qquad (2)$$

则(1)式有唯一解(u_1,u_2),且满足

$$\tilde{u}_i \geqslant u_i \geqslant \hat{u}_i,(t,x) \in [0,+\infty)\times\bar{\Omega}, i=1,2$$

证明 因为$(\tilde{u}_1,\tilde{u}_2)$,$(\hat{u}_1,\hat{u}_2)$是(1)式的上下解,则由文献[10]的定理3.1有(1)式存在唯一解(u_1,u_2),且满足$\tilde{u}_i \geqslant u_i \geqslant \hat{u}_i,(t,x) \in [0,+\infty)\times\bar{\Omega}, i=1,2$。 证毕

考虑如下微分系统

$$\frac{\partial \theta(t,x)}{\partial t} - A\theta(t,x) = \theta(t,x)[a(t,x) - b(t,x)\theta(t,x)],(t,x) \in [0,+\infty)\times\Omega$$

$$B[\theta](t,x) = 0,(t,x) \in [0,+\infty)\times\partial\Omega \quad (3)$$

其中$A,B,a(t,x),b(t,x)$的定义和要求同上面的$A_i,B_i,a_i(t,x),b_i(t,x)$。对于方程(3)的周期解的存在性和稳定性可由如下引理给出。

引理2[11] 特征值问题

$$\frac{\partial \phi(t,x)}{\partial t} - A\phi(t,x) - a(t,x)\phi(t,x) = \sigma(a)\phi(t,x),(t,x) \in [0,+\infty)\times\partial\Omega$$

$$B[\phi](t,x) = 0$$

$$\phi(t,x) = \phi(t+T,x),(t,x) \in [0,+\infty)\times\partial\Omega \qquad (4)$$

有一个主特征值$\sigma_1(a)$及对应的正主特征函数ϕ,且对任意非负初值函数有

(i) 若$\sigma_1(a) \geqslant 0$,则(3)式的平凡解0是全局渐近稳定的;

(ii) 若$\sigma_1(a) < 0$,且初值不恒为零,则(3)式存在唯一的全局渐近稳定的T-周期正解$\theta(t,x)$。

其中$\sigma(a)$是方程(4)的关于a有关的特征值函数。若$A,B,a(t,x),b(t,x)$分别取$A_i,B_i,a_i(t,x),b_i(t,x)$时,(4)式中的主特征值相应地记为$\sigma_1(a_i)$,(3)式中的$T$-周期正解相应地记作$\theta_i(t,x)$。显然$(0,0),(\theta_1,0),(0,\theta_2)$是(1)式的解。

定理1 (i) 若$\sigma_1(a_1) \geqslant 0,\sigma_1(a_2) \geqslant 0$,则对于任意非负值$(u_{1,0},u_{2,0})$,(1)式的平凡解$(0,0)$是全局渐近稳定的;

(ii) 若$\sigma_1(a_1) < 0,\sigma_1(a_2) \geqslant 0$,则对于任意非负初值$(u_{1,0},u_{2,0})$,$u_{2,0} \neq 0$,(1)式的半平凡解$(\theta_1(t,x),0)$是全局渐近稳定的;

(iii) 若$\sigma_1(a_1) \geqslant 0,\sigma_1(a_2) < 0$,则对于任意非负初值$(u_{1,0},u_{2,0})$,$u_{1,0} \neq 0$,(1)式的半平凡解$(0,\theta_2(t,x))$是全局渐近稳定的。

证明 设u_i^*是如下周期抛物型系统的周期解[11]

$$\frac{\partial u_i^*(t,x)}{\partial t} - A_i u_i^*(t,x) = u_i^*(t,x)[a_i(t,x) - b_i(t,x)u_i^*(t,x)]$$

$$(t,x) \in [0,+\infty)\times\Omega$$

$$B_i[u_i^*](t,x) = 0,(t,x) \in [0,+\infty)\times\partial\Omega$$

$$u_i^*(0,x) = u_{i,0}(0,x),x \in \Omega, i=1,2 \qquad (5)$$

令 $\tilde{u}_i = u_{i,0}(t,x),(t,x) \in (-\infty,0]\times\bar{\Omega}$

$$\tilde{u}_i(t,x) = u_i^*(t,x),(t,x) \in [0,\infty)\times\bar{\Omega}, i=1,2$$

则$(\tilde{u}_1,\tilde{u}_2)$,$(0,0)$是(1)式的上、下解,由引理1知(1)式存在唯一解(u_1,u_2)且满足

$$(0,0) \leqslant (u_1,u_2) \leqslant (\tilde{u}_1,\tilde{u}_2)$$

$$(t,x) \in [0,\infty)\times\bar{\Omega} \qquad (6)$$

当$\sigma_1(a_1) \geqslant 0,\sigma_1(a_2) \geqslant 0$时,由引理2及(6)式知$\lim\limits_{t\to\infty}\|u_i(t,\cdot)\|_{C(\bar{\Omega})} \leqslant \lim\limits_{t\to\infty}\|\tilde{u}_i(t,\cdot)\|_{C(\bar{\Omega})} = 0$

故$\lim\limits_{t\to\infty}\|u_i(t,\cdot)\|_{C(\bar{\Omega})} = 0, i=1,2$,即(i)得证。

$\sigma_1(a_1) < 0,\sigma_1(a_2) \geqslant 0$,由引理2及(6)式知

$$\lim_{t\to\infty}\|u_2(t,\cdot)\|_{C(\bar{\Omega})} \leqslant \lim_{t\to\infty}\|\tilde{u}_2(t,\cdot)\|_{C(\bar{\Omega})} = 0$$

$$\limsup_{t\to\infty}[u_1(t,\cdot) - \theta_1(t,\cdot)] \leqslant \lim_{t\to\infty}[\tilde{u}_1(t,\cdot) - \theta_1(t,\cdot)] = 0 \qquad (7)$$

又对$\forall \varepsilon > 0,\exists T_\varepsilon > 0$,当$(t,x) \in (T_\varepsilon,\infty)\times\Omega$时

$$\frac{\partial u_1(t,x)}{\partial t} - A_1u_1(t,x) \geqslant u_1(t,x)[a_1(t,x) - b_1(t,x)u_1(t,x) - \varepsilon]$$

由比较原理知

$$u_1(t,x) \geqslant U_1(t,x),(t,x) \in [T_\varepsilon,\infty) \times \overline{\Omega}$$

其中 U_1 为如下方程的解

$$\frac{\partial U_1(t,x)}{\partial t} - A_1 U_1(t,x) = U_1(t,x)[a_1(t,x) - b_1(t,x)U_1(t,x) - \varepsilon],(t,x) \in [0,\infty) \times \Omega$$

$$B_1[U_1](t,x) = 0,(t,x) \in [T_\varepsilon,+\infty) \times \partial\Omega$$

$$U_1(T_\varepsilon,x) = u_1(T_\varepsilon,x),x \in \Omega \tag{8}$$

由引理 2 及 ε 的任意性有

$$\liminf_{t\to\infty}[u_1(t,x) - \theta_1(t,x)] \geqslant \lim_{t\to\infty}[U_1(t,x) - \theta_1(t,x)] = 0 \tag{9}$$

由(7)式和(9)式知 $\lim\limits_{t\to\infty}[u_1(t,x) - \theta_1(t,x)] = 0$。即(ii)得证,同理可证(iii)。　　证毕

定理 2　记 T-正周期函数 θ_1^*,θ_2^* 为

$$\theta_1^*(t,x) = \int_{-\infty}^0 \theta_1(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)$$

$$\theta_2^*(t,x) = \int_{-\infty}^0 \theta_2(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)$$

若 $\sigma_1(a_1 - \theta_2^*) < 0,\sigma_1(a_2 - \theta_1^*) < 0$,则(1)式存在以 T-周期的正解 $(\bar\theta_1,\underline{\theta}_2),(\underline{\theta}_1,\bar\theta_2)$。且对任意非负初始条件 $(u_{1,0},u_{2,0})$ 对应的解 (u_1,u_2),满足

$$\liminf_{t\to\infty}[u_i(t,\cdot) - \underline{\theta}_i(t,\cdot)] \geqslant 0 \geqslant \limsup_{t\to\infty}[u_i(t,\cdot) - \bar\theta_i(t,\cdot)],x \in \overline{\Omega},i = 1,2$$

证明　若 $\sigma_1(a_1 - \theta_2^*) < 0,\sigma_1(a_2 - \theta_1^*) < 0$,则 $\sigma_1(a_1) < 0,\sigma_1(a_2) < 0$。同定理 1 结论(ii)的证明过程,当 $\sigma_1(a_i) < 0$ 时有

$$\limsup_{t\to\infty}[u_i(t,\cdot) - \theta_i(t,\cdot)] \leqslant 0,i = 1,2 \tag{10}$$

故对 $\forall\varepsilon > 0,\exists T_\varepsilon > 0$,当 $(t,x) \in (T_\varepsilon,\infty) \times \Omega$ 时

$$\frac{\partial u_1(t,x)}{\partial t} - A_1 u_1(t,x) \geqslant u_1(t,x)[a_1(t,x) - b_1(t,x)u_1(t,x) - \theta_2^*(t,x) - \varepsilon]$$

$$\frac{\partial u_2(t,x)}{\partial t} - A_2 u_2(t,x) \geqslant u_2(t,x)[a_2(t,x) - b_2(t,x)u_2(t,x) - \theta_1^*(t,x) - \varepsilon]$$

令 V_1,V_2 是方程组

$$\frac{\partial V_1(t,x)}{\partial t} - A_1 V_1(t,x) = V_1(t,x)[a_1(t,x) - b_1(t,x)V_1(t,x) - \theta_2^*(t,x) - \varepsilon]$$

$$\frac{\partial V_2(t,x)}{\partial t} - A_2 V_2(t,x) = V_2(t,x)[a_2(t,x) - b_2(t,x)V_2(t,x) - \theta_1^*(t,x) - \varepsilon]$$

$$(t,x) \in (T_\varepsilon,\infty) \times \Omega$$

$$B_i[V_i](t,x) = 0,(t,x) \in [T_\varepsilon,+\infty) \times \partial\Omega$$

$$V_i(T_\varepsilon,x) = u_i(T_\varepsilon,x),x \in \Omega,i = 1,2 \tag{11}$$

的解,由比较原理知

$$u_i(t,x) \geqslant V_i(t,x),(t,x) \in [T_\varepsilon,\infty) \times \overline{\Omega},i = 1,2$$

所以

$$\liminf_{t\to\infty}[u_i(t,\cdot) - \Theta_i(t,\cdot)] \geqslant \limsup_{t\to\infty}[V_i(t,\cdot) - \Theta_i(t,\cdot)] = 0 \tag{12}$$

其中 Θ_1,Θ_2 为如下边值问题的正解。

$$\frac{\partial V_1(t,x)}{\partial t} - A_1 V_1(t,x) = V_1(t,x)[a_1(t,x) - b_1(t,x)V_1(t,x) - \theta_2^*(t,x)]$$

$$\frac{\partial V_2(t,x)}{\partial t} - A_2 V_2(t,x) = V_2(t,x)[a_2(t,x) - b_2(t,x)V_2(t,x) - \theta_1^*(t,x)]$$

$$(t,x) \in (0,\infty) \times \Omega$$

$$B_i[V_i](t,x) = 0,(t,x) \in [0,+\infty) \times \partial\Omega \tag{13}$$

令 $\bar\theta_i^{(0)} = \theta_i,\underline{\theta}_i^{(0)} = \Theta_i$,由(10)式和(12)式有

$$\limsup_{t\to\infty}[u_i(t,x) - \theta_i(t,x)] \leqslant 0 \leqslant \liminf_{t\to\infty}[u_i(t,x) - \Theta_i(t,x)]$$

即有 $\bar\theta_i^{(0)} \geqslant \underline{\theta}_i^{(0)}$。

下面构造 T-周期函数序列 $\{\bar\theta_i^{(m)}\},\{\underline{\theta}_i^{(m)}\}$,满足

$$\frac{\partial\bar\theta_1^{(m)}}{\partial t} - A_1\bar\theta_1^{(m)} = \bar\theta_1^{(m)}\left[a_1 - b_1\bar\theta_1^{(m)} - \int_{-\infty}^0 \underline{\theta}_2^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)\right]$$

$$\frac{\partial\bar\theta_2^{(m)}}{\partial t} - A_2\bar\theta_2^{(m)} = \bar\theta_2^{(m)}\left[a_2 - b_2\bar\theta_2^{(m)} - \int_{-\infty}^0 \underline{\theta}_1^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)\right]$$

$$\frac{\partial\underline{\theta}_1^{(m)}}{\partial t} - A_1\underline{\theta}_1^{(m)} = \underline{\theta}_1^{(m)}\left[a_1 - b_1\underline{\theta}_1^{(m)} - \int_{-\infty}^0 \bar\theta_2^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)\right]$$

$$\frac{\partial\underline{\theta}_2^{(m)}}{\partial t} - A_2\underline{\theta}_2^{(m)} = \underline{\theta}_2^{(m)}\left[a_2 - b_2\underline{\theta}_2^{(m)} - \int_{-\infty}^0 \bar\theta_1^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)\right]$$

$$(t,x) \in [0,+\infty) \times \Omega$$

$$B_i[\bar\theta_i^{(m)}] = B_i[\underline{\theta}_i^{(m)}] = 0,(t,x) \in [0,+\infty) \times \partial\Omega \tag{14}$$

假设对任意整数 $k \leqslant m$ 有

$$\bar\theta_i^{(k-1)} \geqslant \bar\theta_i^{(k)} \geqslant \underline{\theta}_i^{(k)} \geqslant \underline{\theta}_i^{(k-1)},(t,x) \in [0,T] \times \overline{\Omega}$$

且

$$\liminf_{t\to\infty}[u_i(t,\cdot) - \underline{\theta}_i^{(k)}(t,\cdot)] \geqslant 0 \geqslant \limsup_{t\to\infty}[u_i(t,\cdot) - \bar\theta_i^{(k)}(t,\cdot)],\quad x \in C(\Omega),i = 1,2 \tag{15}$$

成立,则由

$$\sigma_1\left(a_1-\int_{-\infty}^{0}\underline{\theta}_2^{(m)}(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)\right)\leqslant$$
$$\sigma_1\left(a_1-\int_{-\infty}^{0}\bar{\theta}_2^{(m)}(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)\right)\leqslant$$
$$\sigma_1\left(a_1-\int_{-\infty}^{0}\bar{\theta}_2^{(0)}(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)\right)$$
$$\sigma_1\left(a_2-\int_{-\infty}^{0}\underline{\theta}_1^{(m)}(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)\right)\leqslant$$
$$\sigma_1\left(a_2-\int_{-\infty}^{0}\bar{\theta}_1^{(m)}(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)\right)\leqslant$$
$$\sigma_1\left(a_2-\int_{-\infty}^{0}\bar{\theta}_1^{(0)}(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)\right)$$

(14)式有 T-周期解 $\bar{\theta}_i^{(m)}$,$\underline{\theta}_i^{(m)}$。又由(15)式知对 $\forall\varepsilon>0$,$\exists T_\varepsilon>0$,使得 $(t,x)\in(T_\varepsilon,\infty)\times\Omega$ 时有

$$\frac{\partial u_1}{\partial t}-A_1u_1\leqslant u_1\left[a_1-b_1u_1-\int_{-\infty}^{0}\underline{\theta}_2^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)+\varepsilon\right]$$
$$\frac{\partial u_1}{\partial t}-A_1u_1\geqslant u_1\left[a_1-b_1u_1-\int_{-\infty}^{0}\bar{\theta}_2^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)-\varepsilon\right]$$
$$\frac{\partial u_2}{\partial t}-A_2u_2\leqslant u_2\left[a_2-b_2u_2-\int_{-\infty}^{0}\underline{\theta}_1^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)+\varepsilon\right]$$
$$\frac{\partial u_2}{\partial t}-A_2u_2\geqslant u_2\left[a_2-b_2u_2-\int_{-\infty}^{0}\bar{\theta}_1^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)-\varepsilon\right]$$

故由比较原理得

$$W_i(t,x)\geqslant u_i(t,x)\geqslant Z_i(t,x),\quad (t,x)\in[T_\varepsilon,\infty]\times\bar{\Omega},i=1,2$$

其中 $W_i(t,x)$,$Z_i(t,x)$ 分别为如下系统的正解

$$\frac{\partial W_1}{\partial t}-A_1W_1=W_1\left[a_1-b_1W_1-\int_{-\infty}^{0}\underline{\theta}_2^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)+\varepsilon\right]$$
$$\frac{\partial W_2}{\partial t}-A_2W_2=W_2\left[a_2-b_2W_2-\int_{-\infty}^{0}\underline{\theta}_1^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)+\varepsilon\right]$$
$$(t,x)\in(T_\varepsilon,\infty)\times\Omega$$
$$B_i[W_i](t,x)=0,(t,x)\in[T_\varepsilon,+\infty)\times\partial\Omega$$
$$W_i(T_\varepsilon,x)=u_i(T_\varepsilon,x),x\in\Omega,i=1,2\quad(16)$$

$$\frac{\partial Z_1}{\partial t}-A_1Z_1=Z_1\left[a_1-b_1Z_1-\int_{-\infty}^{0}\bar{\theta}_2^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_1(t,s,x)-\varepsilon\right]$$
$$\frac{\partial Z_2}{\partial t}-A_2Z_2=Z_2\left[a_2-b_2Z_2-\int_{-\infty}^{0}\bar{\theta}_1^{(m-1)}(t-s,x)\,\mathrm{d}_s\eta_2(t,s,x)-\varepsilon\right]$$
$$(t,x)\in(T_\varepsilon,\infty)\times\Omega$$
$$B_i[Z_i](t,x)=0,(t,x)\in[T_\varepsilon,+\infty)\times\partial\Omega$$
$$Z_i(T_\varepsilon,x)=u_i(T_\varepsilon,x),x\in\Omega,i=1,2\quad(17)$$

由引理2及 ε 的任意性有

$$\liminf_{t\to\infty}[u_i(t,\cdot)-\underline{\theta}_i^{(m)}(t,\cdot)]\geqslant\lim_{t\to\infty}[Z_1(t,\cdot)-\underline{\theta}_i^{(m)}(t,\cdot)]=0\quad(18)$$
$$\liminf_{t\to\infty}[u_i(t,\cdot)-\bar{\theta}_i^{(m)}(t,\cdot)]\leqslant\lim_{t\to\infty}[W_1(t,\cdot)-\bar{\theta}_i^{(m)}(t,\cdot)]=0\quad(19)$$

由(18)式和(19)式有 $\bar{\theta}_i^{(m)}\geqslant\underline{\theta}_i^{(m)}$,$(t,x)\in[0,T]\times\bar{\Omega}$。同理可得 $\underline{\theta}_i^{(m)}\geqslant\underline{\theta}_i^{(m-1)}$,$\bar{\theta}_i^{(m)}\leqslant\bar{\theta}_i^{(m-1)}$,所以对任意的正整数 m 都有

$$\bar{\theta}_i^{(m)}\geqslant\bar{\theta}_i^{(m+1)}\geqslant\underline{\theta}_i^{(m+1)}\geqslant\underline{\theta}_i^{(m)},(t,x)\in[0,T]\times\bar{\Omega}$$

且

$$\liminf_{t\to\infty}[u_i(t,\cdot)-\underline{\theta}_i^{(m)}(t,\cdot)]\geqslant0\geqslant\limsup_{t\to\infty}[u_i(t,\cdot)-\bar{\theta}_i^{(m)}(t,\cdot)]$$

因此 $\{\bar{\theta}_i^{(m)}\}$,$\{\underline{\theta}_i^{(m)}\}$ 为单调有界序列。令

$$\lim_{t\to\infty}\bar{\theta}_i^{(m)}(t,x)=\bar{\theta}_i(t,x),\lim_{t\to\infty}\underline{\theta}_i^{(m)}(t,x)=\underline{\theta}_i(t,x)$$

则由(14)式知 $(\bar{\theta}_1,\underline{\theta}_2)$,$(\underline{\theta}_1,\bar{\theta}_2)$ 满足(1)式,即 $(\bar{\theta}_1,\underline{\theta}_2)$,$(\underline{\theta}_1,\bar{\theta}_2)$ 为(1)式的 T-周期正解,且当 $m\to\infty$ 时有

$$\liminf_{t\to\infty}[u_i(t,\cdot)-\underline{\theta}_i(t,\cdot)]\geqslant0\geqslant\limsup_{t\to\infty}[u_i(t,\cdot)-\bar{\theta}_i(t,\cdot)]$$

故定理2得证。 证毕

推论1 在定理2的条件下,若 $\bar{\theta}_i=\underline{\theta}_i\equiv\phi_i(t,x)$,则 $(\phi_1(t,x),\phi_2(t,x))$ 为(1)式的全局渐近稳定解。

证明 由定理2知,若 $\bar{\theta}_i=\underline{\theta}_i\equiv\phi_i(t,x)$,则

$$\liminf_{t\to\infty}[u_i(t,\cdot)-\phi_i(t,\cdot)]=\limsup_{t\to\infty}[u_i(t,\cdot)-\phi_i(t,\cdot)]$$

故 $\lim\limits_{t\to\infty}[u_i(t,\cdot)-\phi_i(t,\cdot)]=0$。 证毕

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Stability and Periodic Solution to Competitive Lotka-Volterra System with Diffusion and Infinite Distributed Delay

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Abstract: A competitive Lotka-volterra system with diffusion and infinite distributed delays is investigated. It is shown that the globally asymptotically stable trivial solution, when $\sigma_1(a_i)\geqslant 0,(i=1,2)$, the globally asymptotically stable semi-trivial periodic solution $(\theta_1(t,x),0)$, and $(0,\theta_2(t,x))$ when $\sigma_1(a_1)<0,\sigma_1(a_2)\geqslant 0$ and $\sigma_1(a_1)\geqslant 0,\sigma_1(a_2)<0$ of the models by construction of a pair of upper and lower solution $(\tilde{u}_1,\tilde{u}_2)$, $(0,0)$ of parabolic periodic system $\frac{\partial u_i(t,x)}{\partial t}-A_i u_i(t,x)=u_i(t,x)[a_i(t,x)-b_i(t,x)u_i(t,x)]$ and in the use of eigenvalue theory and comparison principle. A T-periodic series are established by using the monotone iteration technique. It was obtained that the systems have a pair of periodic positive solutions with respect to every nonnegative initial function.

Key words: diffusion; infinite delays; upper and lower solution; globally asymptotic stable; periodic solutions

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