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Periodic Solutions for a Discrete Mutual System with Delays*

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Abstract: In this paper, a discrete-time mutual system $\begin{cases} x(k+1) = x(k)\exp[r_1(k)(1-\dfrac{x(k-\tau(k))}{k_1(k)}) + a(k)y(k)] \\ y(k+1) = y(k)\exp[r_2(k)(1-\dfrac{y(k-\tau(k))}{k_2(k)}) + b(k)x(k)] \end{cases}$, is considered. By using coincidence degree and the related continuation the orem as well as prior estimates, easily verifiable sufficient conditions for the existence of positive periodic solutions are obtained, i. e., if the following conditions i) r_i $(i=1,2)$, $k_j(j=1,2)$, a,b: $\mathbf{Z}\to\mathbf{R}^+$ are ω periodic; ii) $a^L > \left(\dfrac{r_1}{k_1}\right)^M$, $b^L > \left(\dfrac{r_2}{k_2}\right)^M$; iii) $r_1^L > a^M k_1^M$ hold, then system has at least an ω periodic solution. Our results are important complement to earlier results in the literature.

Key words: discrete mutual system; time delay; coincidence degree; periodic solution

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1 Introduction

Facultative mutualism means that two species cohabit a common habit and each species enhances the average growth rate of the other[1]. In 1997, he and Gopalsamy[2] investigated the stability and persistence of the following mutual system with time delay by Liapunov's second method

$$\begin{cases} \dot{x}(t) = r_1 x(t)[1-\dfrac{x(t-\tau)}{k_1}] + ax(t)y(t) \\ \dot{y}(t) = r_2 y(t)[1-\dfrac{y(t-\tau)}{k_2}] + bx(t)y(t) \end{cases} \tag{1}$$

where $r_i, k_i(i=1,2)$, a, b are positive constants, $x(t)$ and $y(t)$ are respectively densities of two species at time t, r_i is the intrinsic growth rate of two species, k_i is the ecosystem support or environmental carrying capacity for two species, a and b are the rates of transmission between two species. For Eq. (1), Meng and Wei[3] considered the stability and bifurcation by analyzing the associated equation and using the normal form method and center manifold theorem. Xu and Liao[4] made a discussion on the existence of positive solution of system with periodic parameters and impulses.

It is well known that any biological or environmental parameters are naturally subject to fluctuation in time. It is necessary and important to consider models with periodic ecological parameters. Thus the assumption of periodicity of the parameters is a way of incorporating the periodicity of the environment. Based on the viewpoint, we modify (1) as follows

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$$\begin{cases}\dot{x}(t) = r_1(t)x(t)[1-\dfrac{x(t-\tau(t))}{k_1(t)}] + a(t)x(t)y(t) \\ \dot{y}(t) = r_2(t)y(t)[1-\dfrac{y(t-\tau(t))}{k_2(t)}] + b(t)x(t)y(t)\end{cases} \tag{2}$$

Furthermore, discrete time models governed by difference equations are more appropriate to describe the dynamics relationship among populations than continuous ones when the populations have non-overlapping generations. Also, discrete time models can provide efficent models of continuous ones for numerical simulation. Therefore, it is reasonable to study time mutual systems governed by difference equations. There are some papers which deal with this topic[5-10].

The principle object of this article is to propose a discrete analogue system (2) and explore its dynamics. That is, following the methods in [10-11], we derive a discrete analog of (2) and apply the Mawhin's continuous theorem[12] to study the existence of positive periodic solutions of discrete analog of (2).

The remainder of the paper is organized as follows: in Section 2, with the help of differential equations with piecewise constant arguments, we first propose a discrete analogue of system (2), modelling the dynamics of time non-autonomous mutual system with time delay. In Section 3, a easily verifiable sufficient condition for the existence of positive solutions of difference equations is obtained.

2 Discrete analogue of system (2)

In the following, we will discrete the system (2). Following the lines of [10-11], we assume that the average growth rates in system (2) change at regular intervals of time, we can incorporate this aspect in (2) and obtain the following modified system

$$\begin{cases}\dfrac{1}{x(t)}\dot{x}(t) = r_1([t])[1-\dfrac{x([t]-\tau([t]))}{k_1([t])}] + a([t])y([t]) \\ \dfrac{1}{y(t)}\dot{y}(t) = r_2([t])[1-\dfrac{y([t]-\tau([t]))}{k_2([t])}] + b([t])x([t])\end{cases} \tag{3}$$

where $t\neq 0,1,2,\cdots$, $[t]$ denotes the integer part of t, $t\in(0,+\infty)$. Equations of type (3) are known as differential equations with piecewise constant arguments and these equations occupy a position midway between differential equations and difference equations. By a solution of (3), we mean a function $\bar{x}=(x,y)^{\mathrm{T}}$, which is defined for $t\in[0,+\infty)$ and has the following properties:

1) $\bar{x}$ is continuous on $[0,+\infty)$.

2) The derivatives $\dfrac{\mathrm{d}x(t)}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}y(t)}{\mathrm{d}t}$ exist at each point $t\in[0,+\infty)$ with the possible exception of the points $t\in\{0,1,2,\cdots\}$, where left-sided derivatives exist.

3) The equations in (3) are satisfied on each interval $[k,k+1)$ with $k=0,1,2,\cdots$.

We integrate (3) on any interval of the form $[k,k+1)$, $k=0,1,2,\cdots$, and obtain for $k\leqslant t<k+1$, $k=0,1,2,\cdots$

$$\begin{cases}x(t) = x(k)\exp\{[r_1(k)(1-\dfrac{x(k-\tau(k))}{k_1(k)}) + a(k)y(k)](t-k)\} \\ x(t) = y(k)\exp\{[r_2(k)(1-\dfrac{y(k-\tau(k))}{k_2(k)}) + b(k)x(k)](t-k)\}\end{cases} \tag{4}$$

Let $t\to k+1$, then (4) takes the following form

$$\begin{cases}x(k+1) = x(k)\exp[r_1(k)(1-\dfrac{x(k-\tau(k))}{k_1(k)}) + a(k)y(k)] \\ y(k+1) = y(k)\exp[r_2(k)(1-\dfrac{y(k-\tau(k))}{k_2(k)}) + b(k)x(k)]\end{cases} \tag{5}$$

which is a discrete time analogue of system (2). Here $k = 0, 1, 2, \cdots$.

3 Existence of positive periodic solutions

For convenience and simplicity in the following discussion, we always use the notations below throughout the paper: $I_\omega := \{0, 1, 2, \cdots, \omega - 1\}$, $\bar{f} := \frac{1}{\omega}\sum_{k=0}^{\omega-1} f(k)$, $f^L := \min_{k\in\mathbf{Z}} \{f(k)\}$, $f^M := \max_{k\in\mathbf{Z}} \{f(k)\}$, where $f(k)$ is an ω-periodic sequence of real numbers defined for $k \in \mathbf{Z}$. We always assume that

(H1) $r_i(i = 1, 2)$, $k_j(j = 1, 2)$, a, b: $\mathbf{Z}\to\mathbf{R}^+$ are ω periodic, i. e., $r_i(k + \omega) = r_i(t)(i = 1, 2)$, $k_j(k + \omega) = k_j(k)(j = 1, 2)$, $a(k + \omega) = a(k)$, $b(k + \omega) = b(k)$ for any $k \in \mathbf{Z}$.

(H2) i) $a^L > \left(\frac{r_1}{k_1}\right)^M$; ii) $b^L > \left(\frac{r_2}{k_2}\right)^M$.

In order to explore the existence of positive periodic solutions of (5) and for the reader's convenience, we shall first introduce a few concepts and results without proof, borrowing from [12].

Let X, Y be normed vector spaces, L: Dom $L \subset X \to Y$ is a linear mapping, $N: X\to Y$ is a continuous mapping. The mapping L will be called a fredholm mapping of index zero if dimKer L = codim Im $L < +\infty$ and Im L is closed in Y. If L is a fredholm mapping of index zero and there exist continuous projectors $P: X\to X$ and $Q: Y\to Y$ such that Im P = Ker L, Im L = Ker Q = Im$(I - Q)$. It follows that $L\,|\,$Dom $L \cap$ Ker $P: (I - P)X\to$ Im L is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N: \bar{\Omega}\to X$ is compact. Since Im Q is isomorphic to Ker L, there exist isomorphisms J: Im $Q\to$ Ker L.

Lemma 1[12] (Continuation Theorem) Let L be a fredholm mapping of index zero and let N be L-compact on $\bar{\Omega}$. Suppose

a) For each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;

b) $QNx \neq 0$ for each $x \in$ Ker $L \cap \partial\Omega$, and deg $\{JQN, \Omega \cap \partial$ Ker $L, 0\} \neq 0$;

Then the equation $Lx = Nx$ has at least one solution lying in Dom $L \cap \bar{\Omega}$.

Lemma 2[10] Let $g: \mathbf{Z}\to \mathbf{R}$ be ω periodic, i. e., $g(k + \omega) = g(k)$; then for any fixed $k_1, k_2 \in I_\omega$ and any $k \in \mathbf{Z}$, one has $g(k) \leqslant g(k_1) + \sum_{s=0}^{\omega-1} |g(s + 1) - g(s)|$, $g(k) \geqslant g(k_2) + \sum_{s=0}^{\omega-1} |g(s + 1) - g(s)|$.

Lemma 3 $(\hat{x}(k), \hat{y}(k))^{\mathrm{T}}$ is an ω periodic solution of (5) with strictly positive components if and only if $(\ln\{\hat{x}(k)\}, \ln\{\hat{y}(k)\})^{\mathrm{T}}$ is an ω periodic solution of

$$\begin{cases} x(k + 1) - x(k) = r_1(k)\left[1 - \dfrac{\exp[x(k - \tau(k))]}{k_1(k)}\right] + a(k)\exp[y(k)] \\ y(k + 1) - y(k) = r_2(k)\left[1 - \dfrac{\exp[y(k - \tau(k))]}{k_2(k)}\right] + b(k)\exp[x(k)] \end{cases} \tag{6}$$

Define $$l_2 = \{z = \{z(k)\}: z(k) \in \mathbf{R}^2, k \in \mathbf{Z}\}$$

For $a = (a_1, a_2)^{\mathrm{T}} \in \mathbf{R}^2$, define $|a| = \max\{|a_1|, |a_2|\}$. Let $l^\omega \subset l_2$ denote the subspace of all ω periodic sequences equipped with the usual supremum norm $\|\cdot\|$, i. e., $\|z\| = \max_{k\in I_\omega}|z(k)|$, for any $z = \{z(k): k \in \mathbf{Z}\} \in l^\omega$. It is easy to show that l_ω is a finite-dimensional Banach space.

Let

$$l_0^\omega = \{z = \{z(k)\} \in l^\omega: \sum_{k=0}^{\omega-1} z(k) = 0\} \tag{7}$$

$$l_0^\omega = \{z = \{z(k)\} \in l^\omega: z(k) = h \in \mathbf{R}^2, k \in \mathbf{Z}\} \tag{8}$$

then it follows that l_0^ω and l_c^ω are both closed linear subspaces of l^ω and $l^\omega = l_0^\omega + l_c^\omega$, dim $l_c^\omega = 2$. In the following, we will be ready to establish our results.

Theorem 1 In addition to conditions (H1) and (H2), suppose that (H3) $r_1^L > a^M k_1^M$ holds, then the system (5) has at least an ω periodic solution.

Proof. Let $X = Y = l^\omega$,

$$(Lz)(k) = z(k+1) - z(k) = \begin{pmatrix} x(k+1) - x(k) \\ y(k+1) - y(k) \end{pmatrix} \tag{9}$$

$$(Nz)(k) = \begin{pmatrix} r_1(k) - \dfrac{r_1(k)\exp[x(k-\tau(k))]}{k_1(k)} + a(k)\exp[y(k)] \\ r_2(k) - \dfrac{r_2(k)\exp[y(k-\tau(k))]}{k_2(k)} + b(k)\exp[x(k)] \end{pmatrix} \tag{10}$$

where $z \in X, k \in \mathbf{Z}$. Then it is trivial to see that L is a bounded linear operator and $\text{Ker } L = l_c^\omega$, $\text{Im } L = l_0^\omega$, and $\dim \text{Ker } L = 2 = \text{codim Im } L$.

then it follows that L is a fredholm mapping of index zero. Define

$$P\tilde{y} = \frac{1}{\omega}\sum_{s=0}^{\omega-1}\tilde{y}(s), \tilde{y} \in X.\ Q\tilde{z} = \frac{1}{\omega}\sum_{s=0}^{\omega-1}\tilde{z}(s), \tilde{z} \in Y$$

It is not difficult to show that P and Q are continuous projectors such as

$$\text{Im } P = \text{Ker } L,\ \text{Im } L = \text{Ker } Q = \text{Im}(I-Q)$$

Furthermore, the generalized inverse (to L) k_P: $\text{Im } L \to \text{Ker } P \cap \text{Dom } L$ exists and is given by

$$K_P(\tilde{z}) = \sum_{s=0}^{\omega-1}\tilde{z}(s) - \frac{1}{\omega}\sum_{s=0}^{\omega-1}(\omega - s)\tilde{z}(s)$$

Obviously, QN and $K_P(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, using the Ascoli-Arzela theorem, it is not difficult to show that $\overline{K_P(I-Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\bar{\Omega})$ is bounded. Thus, N is L-compact on $\bar{\Omega}$ with any open bounded set $\bar{\Omega} \subset X$.

Now we are at the point to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $Lz = \lambda Nz, \lambda \in (0,1)$, we have

$$\begin{cases} x(k+1) - x(k) = \lambda\left[r_1(k) - \dfrac{r_1(k)\exp[x(k-\tau(k))]}{k_1(k)} + a(k)\exp[y(k)]\right] \\ y(k+1) - y(k) = \lambda\left[r_2(k) - \dfrac{r_2(k)\exp[y(k-\tau(k))]}{k_2(k)} + b(k)\exp[x(k)]\right] \end{cases} \tag{11}$$

Suppose that $z(k) = (x(k), y(k))^T \in X$ is an arbitrary solution of system (11) for a certain $\lambda \in (0, 1)$, summing both sides of (11) from 0 to $\omega - 1$ with respect to k respectively, we obtain

$$\begin{cases} \displaystyle\sum_{k=0}^{\omega-1}\left[\frac{r_1(k)\exp(x(k-\tau(k)))}{k_1(k)}\right] - \sum_{k=0}^{\omega-1}a(k)\exp(y(k)) = \bar{r}_1\omega \\ \displaystyle\sum_{k=0}^{\omega-1}\left[\frac{r_2(k)\exp(y(k-\tau(k)))}{k_2(k)}\right] - \sum_{k=0}^{\omega-1}b(k)\exp(x(k)) = \bar{r}_2\omega \end{cases} \tag{12}$$

It follows from (12) and assumes (H3) that

$$\sum_{k=0}^{\omega-1}|x(k+1) - x(k)| \leqslant \lambda\left\{\sum_{k=0}^{\omega-1}\left[r_1(k) - \frac{r_1(k)\exp(x(k-\tau(k)))}{k_1(k)} + a(k)\exp(y(k))\right]\right\} \leqslant 2\bar{r}_1\omega \tag{13}$$

$$\sum_{k=0}^{\omega-1}|y(k+1) - y(k)| \leqslant \lambda\left\{\sum_{k=0}^{\omega-1}\left[r_2(k) - \frac{r_2(k)\exp(y(k-\tau(k)))}{k_2(k)} + b(k)\exp(x(k))\right]\right\} \leqslant 2\bar{r}_2\omega \tag{14}$$

In view of the hypothesis that $z = \{z(k)\} \in X$, there exist $\xi_i, \eta_i \in I_\omega$ such as

$$x(\xi_1) = \min_{k \in I_\omega}\{x(k)\},\ x(\eta_1) = \max_{k \in I_\omega}\{x(k)\},\ y(\xi_2) = \min_{k \in I_\omega}\{y(k)\},\ y(\eta_2) = \max_{k \in I_\omega}\{y(k)\} \tag{15}$$

From the first equation and the second equation of (12), we have

$$\bar{r}_1\omega < \sum_{k=0}^{\omega-1}\left[\frac{r_1(k)\exp(x(k-\tau(k)))}{k_1(k)}\right] \leqslant \left(\overline{\frac{r_1}{k_1}}\right)\omega\ \exp(x(\eta_1))$$

$$\bar{r}_2\omega < \sum_{k=0}^{\omega-1}\left[\frac{r_2(k)\exp(y(k-\tau(k)))}{k_2(k)}\right] \leqslant \overline{\left(\frac{r_2}{k_2}\right)}\omega \exp(y(\eta_2))$$

which lead to

$$x(\eta_1) > \ln\left[\frac{\bar{r}_1}{\overline{\overline{\left(\frac{r_1}{k_1}\right)}}}\right], y(\eta_2) > \ln\left[\frac{\bar{r}_2}{\overline{\overline{\left(\frac{r_2}{k_2}\right)}}}\right] \tag{16}$$

In view of (11), it is clear that

$$\nabla x(\eta_1) \geqslant 0, \nabla y(\eta_2) \geqslant 0, \nabla x(\xi_1) \leqslant 0, \nabla y(\xi_2) \leqslant 0 \tag{17}$$

where ∇ denotes the backward difference operator, i. e. , $\nabla x(k) = x(k) - x(k-1)$, $\nabla y(k) = y(k) - y(k-1)$.

It follows from (11) that

$$r_1(\eta_1) - \frac{r_1(\eta_1)\exp[x(\eta_1-\tau(\eta_1))]}{k_1(\eta_1)} + a(\eta_1)\exp[y(\eta_1)] \geqslant 0 \tag{18}$$

$$r_2(\eta_2) - \frac{r_2(\eta_2)\exp[y(\eta_2-\tau(\eta_2))]}{k_2(\eta_2)} + b(\eta_2)\exp[x(\eta_2)] \geqslant 0 \tag{19}$$

$$r_1(\xi_1) - \frac{r_1(\xi_1)\exp[x(\xi_1-\tau(\xi_1))]}{k_1(\xi_1)} + a(\xi_1)\exp[y(\xi_1)] \leqslant 0 \tag{20}$$

$$r_2(\xi_2) - \frac{r_2(\xi_2)\exp[y(\xi_2-\tau(\xi_2))]}{k_2(\xi_2)} + b(\xi_2)\exp[x(\xi_2)] \leqslant 0 \tag{21}$$

In the sequel, we consider two cases.

a) If $x(\xi_1) \geqslant y(\eta_2)$, then it follows from (18) that

$$r_1(\eta_1) + a(\eta_1)\exp[y(\eta_1)] \geqslant \frac{r_1(\eta_1)\exp[x(\eta_1-\tau(\eta_1))]}{k_1(\eta_1)} \geqslant \frac{r_1(\eta_1)\exp[x(\xi_1)]}{k_1(\eta_1)}$$

Then
$$r_1(\eta_1) + a(\eta_1)\exp[x(\xi_1)] \geqslant \frac{r_1(\eta_1)\exp[x(\xi_1)]}{k_1(\eta_1)}$$

which leads to
$$x(\xi_1) \leqslant \ln\left[\frac{k_1(\eta_1)r_1(\eta_1)}{r_1(\eta_1) - a(\eta_1)k_1(\eta_1)}\right] \leqslant \ln\left[\frac{k_1r_1}{r_1 - ak_1}\right]^M \tag{22}$$

Then

$$y(\xi_2) \leqslant y(\eta_2) \leqslant x(\xi_1) \leqslant \ln\left[\frac{k_1(\eta_1)r_1(\eta_1)}{r_1(\eta_1) - a(\eta_1)k_1(\eta_1)}\right] \leqslant \ln\left[\frac{k_1r_1}{r_1 - ak_1}\right]^M \tag{23}$$

In view of Lemma 3, (16), (22) and (23) that

$$x(k) \leqslant x(\xi_1) + \sum_{s=0}^{\omega-1}|x(s+1) - x(s)| \leqslant \ln\left[\frac{k_1r_1}{r_1 - ak_1}\right]^M + 2\bar{r}_1\omega := m_1 \tag{24}$$

$$x(k) \geqslant x(\eta_1) - \sum_{s=0}^{\omega-1}|x(s+1) - x(s)| \geqslant \ln\left[\frac{\bar{r}_1}{\overline{\overline{\left(\frac{r_1}{k_1}\right)}}}\right] - 2\bar{r}_1\omega := M_1 \tag{25}$$

$$y(k) \leqslant y(\xi_2) + \sum_{s=0}^{\omega-1}|y(s+1) - y(s)| \leqslant \ln\left[\frac{k_1r_1}{r_1 - ak_1}\right]^M + 2\bar{r}_2\omega := m_2 \tag{26}$$

$$y(k) \geqslant y(\eta_2) - \sum_{s=0}^{\omega-1}|y(s+1) - y(s)| \geqslant \ln\left[\frac{\bar{r}_2}{\overline{\overline{\left(\frac{r_2}{k_2}\right)}}}\right] - 2\bar{r}_2\omega := M_2 \tag{27}$$

Thus

$$\max_{k\in I_\omega}\{x(k)\} < \max\{|m_1|, |M_1|\} := S_1, \max_{k\in I_\omega}\{y(k)\} < \max\{|m_2|, |M_2|\} := S_2 \tag{28}$$

b) If $x(\xi_1) < y(\eta_2)$, then it follows from (20) and (21) that

$$a(\xi_1)\exp[y(\xi_1)] \leqslant \frac{r_1(\xi_1)\exp[x(\xi_1-\tau(\xi_1))]}{k_1(\xi_1)}$$

$$b(\xi_2)\exp[x(\xi_2)] \leqslant \frac{r_2(\xi_2)\exp[y(\xi_2 - \tau(\xi_2))]}{k_2(\xi_2)}$$

Then
$$a(\xi_1)\exp[y(\xi_2)] \leqslant \frac{r_1(\xi_1)\exp[x(\eta_1))]}{k_1(\xi_1)}$$

and
$$b(\xi_2)\exp[x(\xi_1)] \leqslant \frac{r_2(\xi_2)\exp[y(\eta_2))]}{k_2(\xi_2)}$$

which lead to
$$a^L\exp[y(\xi_2)] \leqslant \left[\frac{r_1}{k_1}\right]^M \exp[x(\eta_1)] \tag{29}$$

and
$$b^L\exp[x(\xi_1)] \leqslant \left[\frac{r_2}{k_2}\right]^M \exp[y(\eta_2)] \tag{30}$$

If $y(\xi_2) \geqslant x(\eta_1)$, then by the condition (H2) i), (29) can not hold true. If $y(\xi_2) < x(\eta_1)$, then by the condition (H2) ii), (30) can not hold true. Then case b) can not occur.

Obviously, S_1, S_2 are independent of the choice of $\lambda \in (0, 1)$. Take $M = S_1 + S_2 + S_0$, where S_0 is taken sufficiently large such that $\max\{|\ln\{x^*\}|, |\ln\{y^*\}|\} < S_0$, where $(x^*, y^*)^{\mathrm{T}}$ is the unique solution of the following equation

$$\begin{cases} \bar{r}_1 - \left(\overline{\dfrac{r_1}{k_1}}\right)\exp(x) + \bar{a}\exp(y) = 0 \\ \bar{r}_2 - \left(\overline{\dfrac{r_2}{k_2}}\right)\exp(y) + \bar{a}\exp(x) = 0 \end{cases} \tag{31}$$

Now we have proved that any solution $z = \{z(k)\} = \{(x(k), y(k))^{\mathrm{T}}\}$ of (11) in X satisfies $\|z\| < M, k \in \mathbf{Z}$.

Let $\Omega := \{z = \{z(k)\} \in X : \|z\| < M\}$, then it is easy to see that Ω is an open, bounded set in X and verifies requirement a) of Lemma 1. When $z \in \partial\Omega \cap \operatorname{Ker} L, z = \{(x(k), y(k))^{\mathrm{T}}\}$ is a constant vector in $\mathbf{R}^3$ with $\|z\| = \max\{|x|, |y|\} = M$. Then

$$QNz = \begin{pmatrix} \bar{r}_1 - \left(\overline{\dfrac{r_1}{k_1}}\right)\exp(x) + \bar{a}\exp(y) \\ \bar{r}_2 - \left(\overline{\dfrac{r_2}{k_2}}\right)\exp(y) + \bar{a}\exp(x) \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{32}$$

Now let us consider homotopic $\phi(x, y, \mu) = \mu QNz + (1-\mu)Gz, \mu \in [0,1]$, where $Gz = \begin{pmatrix} \bar{r}_1 - \left(\overline{\dfrac{r_1}{k_1}}\right)\exp(x) \\ \bar{r}_2 - \left(\overline{\dfrac{r_2}{k_2}}\right)\exp(y) \end{pmatrix}$

Letting J be the identity mapping and by direct calculation, we get

$$\begin{aligned} &\deg\{JQN(x,y)^{\mathrm{T}}; \partial\Omega \cap \ker L, 0\} = \deg\{QN(x,y)\}^{\mathrm{T}}; \partial\Omega \cap \ker L, 0\} = \\ &\deg\{\phi(x,y,1); \partial\Omega \cap \ker L, 0\} = \deg\{\phi(x,y,0); \partial\Omega \cap \ker L, 0\} = \\ &\operatorname{sign}\left\{\det\begin{pmatrix} -\left(\overline{\dfrac{r_1}{k_1}}\right)\exp(x) & 0 \\ 0 & -\left(\overline{\dfrac{r_2}{k_2}}\right)\exp(y) \end{pmatrix}\right\} = \operatorname{sign}\left[\left(\overline{\frac{r_1}{k_1}}\right)\right]\left[\left(\overline{\frac{r_2}{k_2}}\right)\right]\exp\{x + y\} = 1 \neq 0 \end{aligned}$$

By now, we have proved that Ω verifies all requirements of Lemma 1, then it follows that $Lz = Nz$ has at least one solution in $\operatorname{Dom} L \cap \bar{\Omega}$, that is to say, (6) has at least one ω periodic solution in $\operatorname{Dom} L \cap \bar{\Omega}$, say $z^* = \{z^*(k)\} = \{(x^*(k), y^*(k))^{\mathrm{T}}\}$. Let $u^*(k) = (\exp\{x^*(k)\}, \exp\{y^*(k)\})$, then by Lemma 3, we know that $u^*(k)$ is an ω periodic solution of system (5) with strictly positive components. The proof is complete.

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具有时滞的离散互惠系统的周期解

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摘要：本文研究了一类散互惠系统 $\begin{cases} x(k+1) = x(k)\exp[r_1(k)(1-\frac{x(k-\tau(k))}{k_1(k)}) + a(k)y(k)] \\ y(k+1) = y(k)\exp[r_2(k)(1-\frac{y(k-\tau(k))}{k_2(k)}) + b(k)x(k)] \end{cases}$，运用迭合度和与其相关的连续性定理及先验估计，得到了系统存在正周期解的易于验证的充分条件，也就是，若下列条件 i) r_i $(i=1,2)$，$k_j(j=1,2)$，a,b：$\mathbf{Z}\to\mathbf{R}^+$ 是 ω 周期的；ii) $a^L > \left(\frac{r_1}{k_1}\right)^M$，$b^L > \left(\frac{r_2}{k_2}\right)^M$；iii) $r_1^L > a^M k_1^M$ 满足，则系统至少有一个正的 ω 周期解，所得结果是前人工作的重要的补充。

关键词：离散互惠系统；时滞；迭合度；周期解

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