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Research on Electricity Demand Forecasting Based on Improved Grey Prediction Model*

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Abstract: Simulation precision of conventional grey prediction models is poor when modeling sequence has the feature of oscillation. Actually, the smoother the sequence is, the higher the simulation precision is. With the purpose of perfecting the smoothness of oscillation sequence and improving the simulation precision of grey models, this paper researches a smoothing algorithm which can compress the amplitude of oscillation sequence, and by this algorithm, deduces a novel grey prediction model based on oscillation sequence, that is $\hat{x}(t) = F\beta_1^{t-3} - (-1)^t E - T$. Finally, we employ the new model to forecast the electricity demand of a city in western China, and compare the simulation precision with other grey models (the simulation error of the new model is 7%, others are all 12%), the results shows the new model has the best simulation effect. Research findings in this paper have an important significance to enrich and perfect grey system theory and construct a more reasonable electricity demand forecasting model.

Key words: grey prediction model; smoothing algorithm; oscillation sequence; electricity demand forecasting

中图分类号: N941.5; TQ083$^+$.1　　**文献标志码**: A　　**文章编号**: 1672-6693(2012)06-0099-06

Grey system theory, proposed originally by Deng Julong[1], is one of the major methods for studying and solving uncertain problems, and grey prediction model, represented by GM (1, 1) and DGM(1, 1) [2], is an important component of the theory, after nearly 30 years of development, it has obtained many encouraging research achievements[3]. However, whether GM (1, 1) or DGM(1, 1) model, the final formulas of them are all homogeneous exponential functions. When modeling sequence has the characteristic of monotonic increasing (or decreasing), we can often get a satisfactory simulation or prediction accuracy, but when modeling sequence shows an oscillation characteristics, the simulation or prediction accuracy of GM(1, 1) or DGM(1, 1) are all unsatisfactory[4]. Actually in the real world, we meet the sequence is more of an oscillation sequence, not monotonic increasing (or decreasing) sequence, so it is very important to build grey prediction models based on oscillation sequence, it can enrich and perfect the theory system and expand the application range of grey prediction models.

Electricity demand is one of the most important variables required for estimating the amount of additional capacity required to ensure a sufficient supply of energy. Electricity demand forecasting can be used to control the generation and distribution of electricity more efficiently. From an operational point of view, the key question is whether there will be problems in meet-

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ing the peak demand; failure to meet this peak demand could result in blackouts. In addition, electricity demand forecasting has become even more important because it is also required for estimating future electricity spot prices[5]. Since the early 1970s, various studies of electricity demand forecasting have been undertaken using various estimation methods, including semiparametric regression[6], time series modeling[7], exponential smoothing[8], Bayesian statistics[9], time-varying splines, neural networks[10], decomposition techniques[11], transfer functions[12], grey dynamic models[13], and judgmental forecasting[14].

Actually, there are many factors maybe affect electricity demand. It is very difficult to completely analyze all the influence factors about electricity demand, on the other hand, even in the same district, electricity demand in different season is not the same, and it often manifests the characteristic of oscillation. On this occasion, traditional forecasting methods cannot do it.

In this paper, we will employ a smoothing algorithm to compress the amplitude of oscillation sequence, weaken the randomness of modeling sequence, and on this basis build a prediction model of oscillation sequence. Furthermore, we use this method to build a prediction model of electricity demand, and the prediction effect about this new model and traditional grey prediction models[1-2] is compared, and the result shows that the new model can significantly improve the simulation accuracy.

1 Oscillation sequence and smoothness operator

Definition 1 Assume that a sequence

$$X=(x(1),x(2),\cdots,x(n))$$

is given, then

1) If for $k=2,3,\cdots,n$, $x(k)-x(k-1)>0$, then X is called a monotonic increasing sequence; and

2) If for $k=2,3,\cdots,n$, $x(k)-x(k-1)<0$, then X is called a monotonic decreasing sequence; and

3) If exist k and $k'=2,3,\cdots,n$, $x(k)-x(k-1)>0$ and $x(k')-x(k'-1)<0$, then X is called an oscillation sequence.

Assume that

$$M=\max\{x(k)\mid k=1,2,\cdots,n\}$$
$$m=\min\{x(k)\mid k=1,2,\cdots,n\}$$

Then $T=M-m$ is called the amplitude of X.

Definition 2 Assume that

$$X=(x(1),x(2),\cdots,x(n))$$

is an oscillation sequence, and another sequence

$$XD=(x(1)d,x(2)d,\cdots,x(n-1)d)$$

Where

$$x(k)d=\frac{[x(k)+T]+[x(k+1)+T]}{4} \qquad (1)$$

In Formula (1), $k=1,2,\cdots,n-1$, and T is the amplitude of X. D is a sequence operator and called a (first-order) smoothness operator of X, and XD is called a smoothness sequence of X.

Theorem 1 the amplitude of oscillation sequence $T(X)$ is no less than two times of the one of its smooth sequence $T(XD)$. That is $T(X)>2T(XD)$.

Proof Assume that

$$\max\{x(k)\mid k=1,2,\cdots,n\}=x(p)$$
$$\min\{x(k)\mid k=1,2,\cdots,n\}=x(q)$$

Then

$$T(X)=x(p)-x(q)$$

Assume that

$$\max\{x(k)d\mid k=1,2,\cdots,n-1\}=x(i)d$$
$$\min\{x(k)d\mid k=1,2,\cdots,n-1\}=x(j)d$$

Then

$$T(XD)=x(i)d-x(j)d$$

According to Definition 2

$$x(i)d=\frac{[x(i)+T]+[x(i+1)+T]}{4}$$

$$x(j)d=\frac{[x(j)+T]+[x(j+1)+T]}{4}$$

Then

$$T(XD)=\frac{[x(i)+T]+[x(i+1)+T]}{4}-\frac{[x(j)+T]+[x(j+1)+T]}{4}=\frac{[x(i)-x(j)]+[x(i+1)-x(j+1)]}{4}$$

For

$$T(X)=x(p)-x(q)>x(i)-x(j)=x(p)-x(q)>x(i+1)-x(j+1)$$

So

$$T(XD)<\frac{T(X)+T(X)}{4}=\frac{T(X)}{2}$$

That is $T(X)>2T(XD)$.

It can be seen from Theorem 1, smoothness operators have the functions of compressing amplitude of oscillation sequence, by it, and we can improve the degree of smoothness of oscillation sequence, and build a more reasonable grey prediction model.

2 Original DGM (1, 1) model

GM (1, 1) is an exponential model, but when data sequence completely satisfies exponent characteristic, the model still exists simulation error. In order to solve this problem, Xie Naiming[2] proposed a novel model whose name is Discrete GM (1, 1) or DGM (1, 1) in abbreviation; this model can realize the complete simulation for exponential sequences. In this paper, we will use DGM (1, 1) model to build a novel grey prediction of oscillation sequence.

Definition 3 Assume that

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n))$$

is a sequence, where

$$x^{(0)}(k) \geqslant 0,\ k = 1, 2, \cdots, n$$

and $X^{(1)}$ is the 1-AGO sequence of $X^{(0)}$

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n))$$

where

$$X^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$$

Then

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2 \qquad (2)$$

Eq. (2) is called a DGM (1, 1) model, or the discrete form of a GM (1, 1) model.

Theorem 2 if $\hat{\beta} = (\beta_1, \beta_2)^{\mathrm{T}}$ is a sequence parameters and

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \square \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \square & \square \\ x^{(1)}(n-1) & 1 \end{bmatrix}$$

Then the least square estimate sequence of the grey differential Eq. (2) satisfies $\hat{\beta} = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}Y$.

Theorem 3 if $B, Y, \hat{\beta}$ as stated in Theorem 2, then

1) Let $x^{(1)}(1) = x^{(0)}(1), k = 1, 2, \cdots, n-1$ then

$$\hat{x}^{(1)}(k+1) = \beta_1^k x^0(1) + \frac{1-\beta_1^k}{1-\beta_1}\beta_2 \qquad (3)$$

2) The restored values, $k = 1, 2, \cdots, n-1$ then

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = c_m \beta_1^{k-1} \qquad (4)$$

Where

$$c_m = x^{(0)}(1)(\beta_1 - 1) + \beta_2$$

3 A novel grey prediction model based on oscillation sequence

The degree of smoothness of oscillation sequence is poor, so we cannot directly build grey prediction model with it. It can be seen from Theorem 1, smoothness sequence is smoother than its oscillation sequence, so we can firstly build model through smoothness sequence, and then deduce the prediction model of oscillation sequence according to Definition 2. The modeling procedure is as Fig. 1.

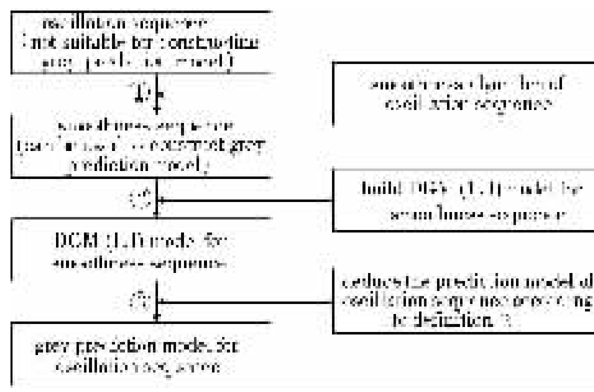

Fig. 1 **Modeling procedure of oscillation sequence**

Now, we build the grey prediction model of oscillation sequence according to Fig. 1

3.1 Generation of smoothness sequence from oscillation sequence

Assume that an oscillation sequence

$$X = (x(1), x(2), \cdots, x(n), x(n+1))$$

According to Definition 2, the smoothness sequence XD of X is

$$XD = Y = (y(1), y(2), \cdots, y(n))$$

Where

$$y(k) = x(k)d = \frac{[x(k) + T] + [x(k+1) + T]}{4}$$

$k = 1, 2, \cdots, n$ and T is the amplitude of sequence X.

3.2 Building DGM (1, 1) model based on smoothness sequence

Now, we build the DGM (1, 1) model of sequence $Y = (y(1), y(2), \cdots, y(n))$

$$y^{(1)}(k+1) = \beta_1 y^{(1)}(k) + \beta_2 \qquad (5)$$

$$\hat{\beta} = (\beta_1, \beta_2)^{\mathrm{T}} = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}Y \qquad (6)$$

where

$$Y=\begin{bmatrix} y^{(1)}(2) \\ y^{(1)}(3) \\ \square \\ y^{(1)}(n) \end{bmatrix}, B=\begin{bmatrix} y^{(1)}(1) & 1 \\ y^{(1)}(2) & 1 \\ \square & \square \\ y^{(1)}(n-1) & 1 \end{bmatrix}$$

According to Eq. (4)

$$\hat{y}(k+1)=\hat{y}^{(1)}(k+1)-\hat{y}^{(1)}(k)=c_m\beta_1^{k-1} \quad (7)$$

Where

$$c_m=y(1)(\beta_1-1)+\beta_2$$

Eq. (7) is called a DGM (1, 1) model of smoothness sequence Y.

3.3 Deducing grey prediction for oscillation sequence

According definition 2

$$\hat{y}(k)=\frac{[\hat{x}(k)+T]+[\hat{x}(k+1)+T]}{4}$$

$$\hat{x}(k+1)=4\hat{y}(k)-\hat{x}(k)-2T \quad (8)$$

When $k=1$, let

$$C=\hat{x}(2)=x(2)$$

and C is called the initial value of grey prediction model of oscillation sequence. When $k=t-1$, we will get a different equation according the pairity of t, so it need to do the following discussions.

1) When t is an odd number

$$\hat{x}(t)=4\hat{y}(t-1)-4\hat{y}(t-2)+\cdots+4\hat{y}(2)-C-2T \quad (9)$$

According to Eq. (4), we can know that the former $(t-2)$ terms of Eq. (9) is a geometric progression, and its common ratio is $q=-\beta_1^{-1}$, according to the summation formula of geometric progression, we can get

$$\hat{x}(t)=\frac{4\hat{y}(t-1)[1-(-\beta_1^{-1})^{t-2}]}{1+\beta_1^{-1}}-C-2T \quad (10)$$

For

$$\hat{y}(t-1)=[y(1)(\beta_1-1)+\beta_2]\times\beta_1^{t-3}$$

So

$$\hat{x}(t)=\frac{4\beta_1^{t-3}[y(1)(\beta_1-1)+\beta_2][1-(-\beta_1^{-1})^{t-2}]}{1+\beta_1^{-1}}-C-2T \quad (11)$$

Let

$$F=\frac{4[y(1)(\beta_1-1)+\beta_2]}{1+\beta_1^{-1}}=\text{const}$$

and

$$y(1)=\frac{x(1)+x(2)+2T}{4}$$

Then

$$\hat{x}(t)=F\beta_1^{t-3}+F\beta_1^{-1}-C-2T \quad (12)$$

2) When t is an even number, similarly

$$\hat{x}(t)=F\beta_1^{t-3}-F\beta_1^{-1}+C \quad (13)$$

Combine (12) and (13), we can get

$$\begin{cases}\hat{x}(t)=F\beta_1^{t-3}+F\beta_1^{-1}-C-2T \\ \hat{x}(t)=F\beta_1^{t-3}-F\beta_1^{-1}+C\end{cases} \quad (14)$$

Integrate the two equations,

$$\hat{x}(t)=F\beta_1^{t-3}-(-1)^tF\beta_1^{-1}+(-1)^tC+[(-1)^t-1]T=F\beta_1^{t-3}-(-1)^t(F\beta_1^{-1}-C-T)-T$$

Let

$$E=F\beta_1^{-1}-C-T$$

Then

$$\hat{x}(t)=F\beta_1^{t-3}-(-1)^tE-T \quad (15)$$

Eq. (15) is called a grey prediction model of oscillation sequence.

4 Electricity demand forecasting

Assume that the electricity consumption of a city in western China from January 2010 to July 2010 is shown in Tab. 1. Now use the grey prediction model of oscillation sequence (GPROS(1, 1) model) to build a model of electricity demand forecasting for this city, and then compare its simulation error with GM(1, 1) and DGM (1, 1) model.

Tab. 1 Electricity consumption from January 2010 to July 2010

Month	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.
Electricity consumption /(10^6 kW·h)	439	320	584	481	640	635	790

Step1 According to Tab. 1, the original sequence X is

$$X=(439, 320, 584, 481, 640, 635, 790)$$

Step2 According to Definition 2, the smoothness sequence Y of X is

$$Y=(424.75, 461.0, 501.25, 515.25, 553.75, 591.25)$$

The geometric figure of oscillation sequence and smoothness sequence is shown as Fig. 2.

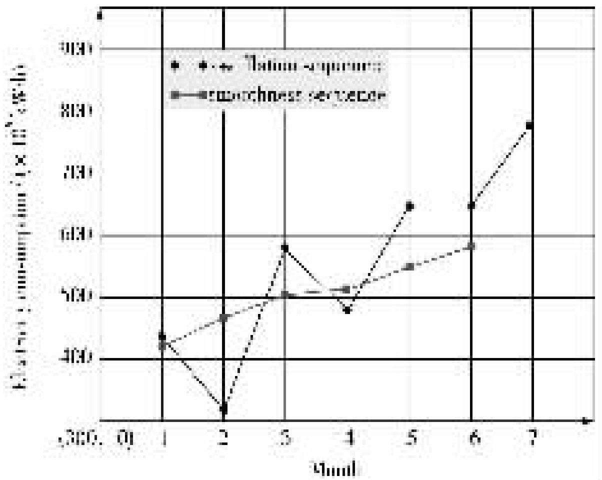

Fig. 2 Oscillation sequence and smoothness sequence

It can be seen from Fig. 2 that the smoothness sequence is smoother than that of the oscillation sequence, and this is very important for improving the simulation or prediction precision of grey models.

Step3 Compute model parameters, as follows:

$$C = x(2) = 320,\ T = M - m = 470.0$$

$$y(1) = \frac{x(1) + x(2) + 2T}{4} = 424.75$$

$$\beta_1 = 1.0616,\ \beta_2 = 437.5177$$

$$F = \frac{4[y(1)(\beta_1 - 1) + \beta_2]}{1 + \beta_1^{-1}} = 955.07$$

$$E = F\beta_1^{-1} - C - T = 109.65$$

Step4 According to Eq. (15), the model of electricity demand forecasting is

$$\hat{x}(t) = F\beta_1^{t-3} - (-1)^t E - T = 955.07 \times 1.0616^{t-3} - 109.65 \times (-1)^t - 470 \quad (16)$$

Eq. (16) is called a grey prediction model of electricity demand.

Step5 According to Eq. (16), calculate simulation values and errors, and compare the simulation accuracy with GM(1, 1) and DGM(1, 1) model.

Tab. 2 Simulation values and errors with various models

Month	Real value $x(k)$	Improved model		GM(1, 1) model		DGM(1, 1) model	
		Simulation value $\hat{x}(k)$	Simulation error Δ_k/%	Simulation value $\hat{x}(k)$	Simulation error Δ_k/%	Simulation value $\hat{x}(k)$	Simulation error Δ_k/%
Feb	320	320.0	0.00	405.2	26.63	409.4	27.94
Mar	584	594.7	1.83	461.5	20.97	465.1	20.36
Apr	481	434.3	9.71	525.6	9.27	528.5	9.88
May	640	716.0	11.88	598.7	6.45	600.5	6.17
Jun	635	563.0	11.34	681.9	7.39	682.4	7.46
Jul	790	852.7	7.94	776.7	1.68	775.3	1.86
Mean relative simulation error Δ/%			7.12	–	12.07	–	12.29

Computational formula of simulation error is given by

$$\Delta_k = \frac{|\hat{x}(k) - x(k)|}{x(k)}$$

Computational formula of mean relative simulation error is given by

$$\Delta = \frac{1}{6}\sum_{k=2}^{7}\Delta_k$$

Where $k = 2, 3, 4, 5, 6, 7$.

It can be seen from Tab. 2 and Fig. 3 that a GPROS(1, 1) model has the best simulation precision among those grey prediction models.

5 Results

GM (1, 1) and DGM (1, 1) are all exponent models, when modeling data has the characteristic of monotonic increasing or monotonic decreasing, we can often get a satisfactory simulation or prediction.

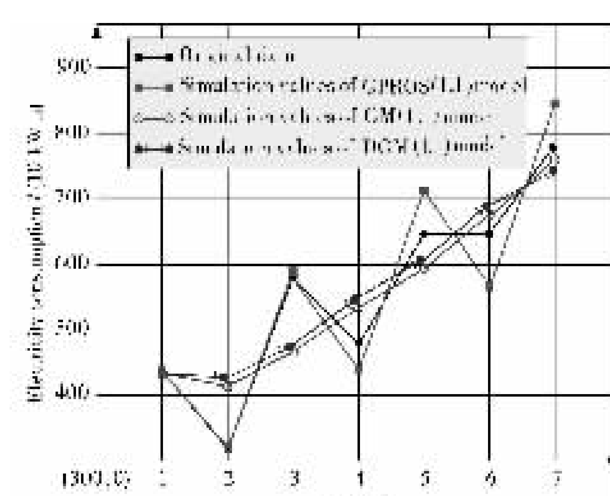

Fig. 3 broken line graphs of simulation values with various models

accuracy by using the two models, but when modeling data sequence shows the feature of oscillation, the simulation or prediction accuracy are all unsatisfactory. In this paper, we employ a smoothing algorithm to compress the amplitude of oscillation sequence, weaken the randomness of modeling sequence, and build a prediction model of oscillation sequence. Finally, we use this model to build a prediction model of electricity demand, and comparison its simulation accuracy with GM (1, 1) and DGM (1, 1) model; the results show that the novel model has the best simulation precision. Research findings of this paper have an important significance to enrich and perfect grey system theory and build more reasonable electricity demand forecasting model.

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基于改进灰色预测模型的电力需求预测研究

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摘要：当建模序列具有随机振荡特征时灰色预测模型的模拟及预测精度较差，实际上序列越光滑模型的模拟精度就越高；本文通过改善建模序列的光滑性以提高灰色预测模型的模拟精度，研究了一种压缩随机振荡序列振幅的算法，推导了基于随机振荡序列的灰色预测模型 $\hat{x}(t) = F\beta_1^{t-3} - (-1)^t E - T$；最后应用该模型预测我国西部某城市的电力需求，并与其他灰色预测模型的模拟精度进行了比较（新模型的模拟精度为 7%，其他模型的精度均为 12%），表明新模型具有更好的模拟效果，研究成果对丰富与完善灰色预测理论体系，促进灰色模型与电力需求预测的对接，具有积极意义。

关键词：灰色预测模型；平滑算法；振荡序列；电力需求预测

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