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A Modified Hooke-Jeeves Method*

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Abstract: This paper is concerned with the solution method to uncostrained optimization problems without using derivatives. The objective function value may be increasing at the acceleration step in the standard Hooke-Jeeves method with discrete steps (HJMDS) for solving unconstrained optimization problems. The acceleration step of the standard HJMDS is modified such that the nonincreasing in the objective function value can be guaranteed in the acceleration step of the modified Hooke-Jeeves method with discrete steps (MHJMDS) given in this paper. Then, a new algorithm using the MHJMDS is designed. Numerical results show that the MHJMDS is more efficient than the standard HJMDS because of the requirement of a very small number of function evaluations.

Key words: unconstrained optimization; Hooke-Jeeves method; modified Hooke-Jeeves method; discrete step; acceleration step

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Introduction

Consider the following unconstrained optimization problem

$$(\mathrm{P}) \qquad \min_{x\in \mathbf{R}^n} f(x)$$

where $f:\mathbf{R}^n\to\mathbf{R}^1$ is a function of n variables (without the requirement of continuously differentiability). There exist a number of methods that do not use derivative-based information and can be used to solve Problem (P) successfully, for example, the cyclic coordinate method[1], the method of Hooke-Jeeves using line searches[2], the method of Hooke-Jeeves with discrete steps[2], the method of Rosenbrock using line searches[3], the method of Rosenbrock with discrete steps[3], and so on. Among the above methods, the Hooke-Jeeves method is very efficient for solving some optimaztion problems[4-6]. In [4], the Hooke-Jeeves direct search method is implemented and is demonstrated to solve a class of problems in the geometric optimization of yield-line patterns efficiently. In [5], a new Hooke-Jeeves based Memetic Algorithm (HJMA) is given for solving dynamic optimization problems. In [6], a revised Hooke-Jeeves algorithm is proposed to solve the optimization model of the trajectory of horizontal well with perturbation. In addition, a novel hybrid optimization approach, which is based on teaching-learning based optimization (TLBO) algorithm and Taguchi's method, is presented and the results obtained by the proposed approach are compared with those of the Hooke-Jeeves pattern search method, particle swarm optimization algorithm, and so on[7]. A unified convergent theorem is given for a class of direct scarch techniques which is a class of descent methods with fixed step size and contains the Hooke-Jeeves technique, the simplified and varied Hooke-Jeeves techniques, and the axis directional search technique as its special cases[8]. In the present paper, we consider the standard Hooke-Jeeves method with discrete steps (ab-

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breviated as HJMDS). We observed that the objection function value may be increasing at the acceleration step in the standard HJMDS. Basing this observation, we will modifiy the acceleration step of the standard HJMDS such that the non-increasing property in the objective function value can be guaranteed in the acceleration step of our modified method.

1 The Algorithm

The standard Hooke-Jeeves method with discrete steps (HJMDS) is as follows.

Algorithm HJMDS Initialization Step Let $d_1, \cdots, d_n$ be the coordinate directions, choose a scalar $\varepsilon>0$ to be used for terminating the algorithm. Furthermore, choose an initial step size $\Delta>\varepsilon$, and an acceleration factor $\alpha>0$. Choose a starting point x_1, let $y_1=x_1$, let $k=j=1$, and go to the Main Step.

Main Step Step 1 If $f(y_j+\Delta d_j)<f(y_j)$, the trial is termed a success; let $y_{j+1}=y_j+\Delta d_j$, and go to Step 2. If, however, $f(y_j+\Delta d_j)\geqslant f(y_j)$, the trial is deemed a failure. In this case, if $f(y_j-\Delta d_j)<f(y_j)$, let $y_{j+1}=y_j-\Delta d_j$, and go to Step 2; if $f(y_j-\Delta d_j)\geqslant f(y_j)$, let $y_{j+1}=y_j$, and go to Step 2;

Step 2 If $j<n$, replace j by $j+1$, and repeat Step 1. Otherwise, go to Step 3 if $f(y_{n+1})<f(x_k)$, and go to Step 4 if $f(y_{n+1})\geqslant f(x_k)$;

Step 3 Let $x_{k+1}=y_{n+1}$, and let $y_1=x_{k+1}+\alpha(x_{k+1}-x_k)$. Replace k by $k+1$, let $j=1$, and go to Step 1;

Step 4 If $\Delta\leqslant\varepsilon$, stop; x_k is the prescribed solution. Otherwise, replace Δ by $\frac{\Delta}{2}$. Let $y_1=x_k$, $x_{k+1}=x_k$, replace k by $k+1$, let $j=1$, and repeat Step 1.

It is clear that the objective function value may be increasing at step 3 (i. e. , the acceleration step) of the Main Step in Algorithm HJMDS and it may lead to reduce the effectiveness of algorithm. Based on this observation, we focus on modifying the Step 3 in Algorithm HJMDS. Our modified Hooke-Jeeves method with discrete steps (MHJMDS) is as follows.

Algorithm MHJMDS Initialization Step This step is same as the Initialization Step of Algorithm HJMDS.

Main Step Step 1 This step is also same as the Step 1 of Algorithm HJMDS;

Step 2 If $j<n$, replace j by $j+1$, and repeat Step 1. Otherwise, go to Step 3 if $f(y_{n+1})<f(x_k)$, and go to Step 7 if $f(y_{n+1})\geqslant f(x_k)$;

Step 3 Let $i=1$, $x_{k+1}=y_{n+1}$, and let $y_1=x_{k+1}+\alpha(x_{k+1}-x_k)$, go to Step 4 if $f(y_1)<f(x_{k+1})$, and go to Step 5 if $f(y_1)\geqslant f(x_{k+1})$;

Step 4 If $i\geqslant m$, go to Step 8 and let $y_1=y_{11}$, $x_{k+1}=y_1$; Otherwise, if $i<m$, let $y_{11}=2y_1-x_{k+1}$; in this case, if $f(y_{11})\leqslant f(y_1)$, let $y_1=y_{11}$; replace i by $i+1$, and repeat Step 4; if $f(y_{11})>f(y_1)$, go to Step 8;

Step 5 If $i\geqslant m$, let $y_1=x_{k+1}$, go to step 8. Otherwise, if $i<m$, let $y_{11}=\frac{1}{2}(y_1+x_{k+1})$. In this case, if $f(y_{11})<f(x_{k+1})$, let $y_1=y_{11}$, and go to Step 8. However, if $f(y_{11})\geqslant f(x_{k+1})$, go to Step 6;

Step 6 If $f(y_{11})\leqslant f(y_1)$, let $y_1=y_{11}$ and replace i by $i+1$, go to Step 5. Otherwise, if $f(y_{11})>f(y_1)$, let $y_1=x_{k+1}$, go to Step 8.

Step 7 If $\Delta\leqslant\varepsilon$, stop; x_k is the prescribed solution. Otherwise, replace Δ by $\frac{\Delta}{2}$. Let $y_1=x_k$, $x_{k+1}=x_k$, and go to Step 8.

Step 8 Replace k by $k+1$, let $j=1$, and go to Step 1.

It is easy to see that the objective function value can not increase in the acceleration Steps 3～6 of Algorithm MHJMDS.

Remark The proof of Algorithm MHJMDS's convergence is similar to that of Theorems 7. 1 and 8. 1 in Ref. [8]. Here we omit this proof.

2 Numerical experiments

Consider the following problem[2]

(TP) minimize $(x_1-2)^4+(x_1-2x_2)^2$

We first solve the Problem (TP) using Algorithm HJMDS. The parameters α and Δ are chosen as 1.0 and 0.2, respectively. Tab.1 summarizes the computations starting from the initial point (2.0, 3.0). Here (S) denotes that the trial is a success and (F) denotes that the trial is a failure. The procedure is stopped here with the termination parameter $\varepsilon=0.1$.

Tab.1 Summary of computations for algorithm HJMDS

k	Δ	X_k / $f(x_k)$	j	y_j / $f(y_j)$	d_j	$y_j+\Delta d_j$ / $f(y_j+\Delta d_j)$	$y_j-\Delta d_j$ / $f(y_j-\Delta d_j)$
1	0.2	(2.00,3.00)	1	(2.00,3.00)	(1.0,0.0)	(2.20,3.00)	(1.80,3.00)
		16		16		14.441 6 (S)	17.641 6 (F)
			2	(2.20,3.00)	(0.0,1.0)	(2.20,3.20)	(2.20,2.80)
				14.441 6		17.641 6 (F)	11.561 6 (S)
2	0.2	(2.20,2.80)	1	(2.40,2.60)	(1.0,0.0)	(2.60,2.60)	(2.20,2.60)
		11.561 6		7.865 6		6.889 6 (S)	9.001 6 (F)
			2	(2.6,2.6)	(0.0,1.0)	(2.60,2.80)	(2.60,2.40)
				6.889 6		9.129 6 (F)	4.969 6 (S)
3	0.2	(2.60,2.40)	1	(3.00,2.00)	(1.0,0.0)	(3.20,2.00)	(2.80,2.00)
		4.969 6		2		2.713 6 (F)	1.849 6 (S)
			2	(2.80,2.00)	(0.0,1.0)	(2.80,2.20)	(2.80,1.80)
				1.849 6		2.969 6(F)	1.049 6 (S)
4	0.2	(2.80,1.80)	1	(3.00,1.20)	(1.0,0.0)	(3.20,1.20)	(2.80,1.20)
		1.049 6		1.36		2.713 6 (F)	0.569 6 (S)
			2	(2.80,1.20)	(0.0,1.0)	(2.80,1.40)	(2.80,1.00)
				0.569 6		0.409 6 (S)	1.049 6 (F)
5	0.2	(2.80,1.40)	1	(2.80,1.00)	(1.0,0.0)	(3.00,1.00)	(2.60,1.00)
		0.409 6		1.049 6		2.00 (F)	0.489 6 (S)
			2	(2.60,1.00)	(0.0,1.0)	(2.60,1.20)	(2.60,0.80)
				0.489 6		0.169 6 (S)	1.129 6 (F)
6	0.2	(2.60,1.20)	1	(2.40,1.00)	(1.0,0.0)	(2.60,1.00)	(2.20,1.00)
		0.169 6		0.185 6		0.489 6 (F)	0.004 16 (S)
			2	(2.20,1.00)	(0.0,1.0)	(2.20,1.2)	(2.20,0.80)
				0.041 6		0.041 6 (S)	0.361 6 (F)
7	0.2	(2.20,1.2)	1	(1.80,1.20)	(1.0,0.0)	(2.00,1.20)	(1.60,1.20)
		0.041 6		0.361 6		0.16 (S)	0.665 6 (F)
			2	(2.00,1.20)	(0.0,1.0)	(2.00,1.40)	(2.00,1.00)
				0.16		0.64 (F)	2.1743 * e−29(S)
8	0.2	(2.00,1.00)	1	(1.80,0.8)	(1.0,0.0)	(2.00,0.80)	(1.60,0.80)
		2.174 3 * e−29		0.041 6		0.16 (F)	0.025 6 (S)
			2	(1.60,0.80)	(0.0,1.0)	(1.60,1.00)	(1.60,0.600)
				0.025 6		0.185 6 (F)	0.185 6 (F)
9	0.1	(2.00,1.00)	1	(2.00,1.00)	(1.0,0.0)	(2.10,1.00)	(1.90,1.00)
		2.174 3 * e−29		2.174 3 * e−29		0.010 1 (F)	0.010 1 (F)
			2	(2.00,1.01)	(0.0,1.0)	(2.00,1.10)	(2.00,0.90)
				2.174 3 * e−30		0.04 (F)	0.04 (F)

Now we solve the problem (TP) using Algorithm MHJMDS. The parameters α and Δ are also chosen as 1.0 and 0.2, respectively, and the parameter m is chosen as 4. Tab.2 summarizes the computations starting from the initial point(2.0, 3.0). The meanings of the notations "S", "F" are the same as the above. The termination scalar ε is also chosen as 0.1.

Tab. 2 Summary of computations for Algorithm MHJMDS

k	Δ	x_k / $f(x_k)$	j	y_j / $f(y_j)$	d_j	$y_j+\Delta d_j$ / $f(y_j+\Delta d_j)$	$y_j-\Delta d_j$ / $f(y_j-\Delta d_j)$
1	0.2	(2.00,3.00)	1	(2.00,3.00)	(1.0,0.0)	(2.20,3.00)	(1.80,3.00)
		16		16		14.441 6 (S)	17.641 6 (F)
			2	(2.20,3.00)	(0.0,1.0)	(2.20,3.20)	(2.20,2.80)
				14.441 6		17.641 6 (F)	11.561 6 (S)
2	0.2	(3.00,2.00)	1	(3.00,2.00)	(1.0,0.0)	(3.20,2.00)	(2.80,2.00)
		2		2		2.713 6 (F)	1.849 6 (S)
			2	(2.80,2.00)	(0.0,1.0)	(2.80,2.20)	(2.80,1.80)
				1.849 6		2.969 6 (F)	1.049 6 (S)
3	0.2	(2.00,1.00)	1	(2.00,1.00)	(1.0,0.0)	(2.20,1.00)	(1.80,1.00)
		1.177 49 e−30		1.177 49 * e−30		0.041 6 (F)	0.041 6 (F)
			2	(2.00,1.00)	(0.0,1.0)	(2.00,1.20)	(2.00,0.80)
				1.177 49 * e−30		0.16 (F)	0.16 (F)
4	0.1	(2.00,1.00)	1	(2.00,1.00)	(1.0,0.0)	(2.10,1.00)	(1.90,1.00)
		1.177 49 e−30		1.177 49 * e−30		0.010 1 (F)	0.010 1 (F)
			2	(2.00,1.00)	(0.0,1.0)	(2.00,1.10)	(2.00,0.90)
				1.177 49 * e−30		0.04 (F)	0.04 (F)

The number of function evaluations is 38 and 24 using Algorithm HJMDS and Algorithm MHJMDS, respectively. It is clear that Algorithm MHJMDS is more efficient than Algorithm HJMDS.

References:

[1] Vassilios S V, Raúl C. Cyclic coordinate method[C]//Christodoulos A F, Panos M P. Encyclopedia of Optimization. US: Springer, 2009: 595-596.

[2] Mokhtar S B, Hanif D S, Shetty C M. Nonlinear programming: theory and algorithms[M]. 3rd ed. New Jersey: John Wiley & Sons Inc, 2006: 368-375.

[3] Vassiliadis V S, Conejeros R. Rosenbrock method [C]//Christodoulos A F, Panos M P. Encyclopedia of Optimization. US: Springer, 2009: 3343-3345.

[4] Ramsay A C A, Johnson D. Geometric optimization of yield-line patterns using a direct search method[J]. Structural Optimization, 1997, 14(2/3): 108-115.

[5] Irene M, Raymond C. A Hooke-Jeeves based memetic algorithm for solving dynamic optimisation problems[C]//Corchado E. Hybrid Artificial Intelligence Systems: Lecture Notes in Computer Science 5572. Berlin: Springer-Verlag, 2009: 301-309.

[6] Li A, Feng E. Optimization model and algorithm of the trajectory of horizontal well with perturbation[J]. J Appl Math & Computing, 2006, 20(1/2): 391-399.

[7] Yildiz A R. Optimization of multipass turning operations using hybrid teaching learning-based approach[J]. Int J Adv Manuf Technol, 2013, 66(9/10/11/12): 1319-1326.

[8] Yu W C. Positive basis and a class of direct search techniques[J]. Science in China Series A, 1979(S1): 53-68.

运筹学与控制论

一个修正的 Hooke-Jeeves 方法

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摘要：本文考虑不用导数信息求解无约束优化问题的方法。对于求解无约束优化问题的带有离散步的标准 Hooke-Jeeves 方法，目标函数值有可能在其加速步中增大。本文修正了标准 HJMDS 的加速步，保证了目标函数值在修正的带离散步 Hooke-Jeeves 方法的加速步中不增。然后，采用修正的带离散步 Hooke-Jeeves 方法设计了一个新算法。数值试验结果表明，修正的带离散步 Hooke-Jeeves 方法与带离散步的标准 Hooke-Jeeves 方法相比，其函数值计算次数明显减少，因而本文给出的修正的带离散步 Hooke-Jeeves 方法比带离散步的标准 Hooke-Jeeves 方法更为有效。

关键词：无约束优化；Hooke-Jeeves 方法；修正 Hooke-Jeeves 方法；离散步；加速步

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