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Stability Analysis of Singular Systems with Interval Time-Varying Delay*

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Abstract: In this article, we concern with the problem of delay-dependent stability for singular systems with interval time-varying delay. The purpose of the problem is to design stability criteria such that the singular system is regular, impulse free and asymptotically stable. Some new delay-dependent stability criteria are derived by taking new Lyapunov-Krasovskii functional and free weighting matrices. The introduced functional makes use of the information of not only both the lower and upper bounds of the interval time-varying delay, but also the interval of the two bounds. The proposed stability criteria are given in terms of linear matrix inequality and it is accordingly easy to check by use of Matlab. Numerical examples are given to demonstrate that the proposed method can obtain larger allowable delay bounds than the methods in reference, which illustrates the effectiveness of the approach.

Key words: stability; Lyapunonv-Krasovskii functional; singular systems; linear matrix inequality (LMI)
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Introduction

Since time-delay which exists in many applications is often causes instability and poor performance of systems, considerable attention has been devoted to the stability analysis of time-delay systems during the last decades, for example [1-20], and the references therein.

Recently, the stability analysis of interval time-varying delay system has been a focused topic of theoretical and practical importance[3-8]. A typical example of dynamic systems with interval time-varying delays is networked control systems[3]. He et al. investigated H filter design for systems with interval time-varying delays[3]. Very recently, the stability analysis of system with a delay varying in an interval was studied in [8].

On the other hand, the stability analysis of singular systems, which are known as descriptor systems, implicit systems, generalized state-space systems or semi-state systems, have widely investigated by many researchers[9-17]. The singular model can preserve the structure of practical systems and can better describe a large class of physical systems than state-space ones[16-17]. It should be pointed out that when the stability problem for singular systems is investigated; the regularity and absence of impulses (for continuous systems) and causality (for discrete systems) are required to be considered simultaneously[18]. Hence, the stability analysis of singular systems with interval time-varying delay is much more complicated than that for state-space ones. To the best of the authors' knowledge, no results have been reported in the literature concerning the problem of stability of the singular systems with interval time-varying delay.

In this paper, we will deal with the problem of stability of a class of singular systems with interval time-

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varying delay. Neither model transformation nor redundant matrix variables will be employed. By employing a new Lyapunov-Krasovskii functional, which considers not only the lower and upper bounds of the interval time-varying delay, but also the interval of the two bounds, sufficient conditions are given in the form of linear matrix inequality (LMI) such that time delay system is asymptotically stable. Compared with some existing results, our conditions are shown to be less conservative.

Notion Through this paper, the superscripts "T" stand for the transpose of a matrix and the inverse of a matrix; $\mathbf{R}^n$ denotes n-dimensional Euclidean space; $\mathbf{R}^{n\times m}$ denotes the set of all real matrices with m rows and n columns; $P>0$ means that P is positive definite; I is the identity matrix of appropriate dimension; $*$ denotes the matrix entries implied by symmetry.

1 System description and preliminaries

Consider the following uncertain singular system with interval time-varying delay

$$\begin{cases} E\dot{x}(t)=Ax(t)+A_\tau x(t-\tau(t)) \\ x(t)=\varphi(\theta),\theta\in[-\tau_M,0] \end{cases} \tag{1}$$

where $x(t)\in\mathbf{R}^n$ is the state vector; $\tau(t)$ is the time-varying delay satisfying

$$0\leqslant\tau_m\leqslant\tau(t)\leqslant\tau_M,\ \dot{\tau}(t)\leqslant\tau_D\leqslant 1 \tag{2}$$

$\varphi(\theta)\in C([-\tau_M,0],\mathbf{R}^n)$ is the initial function with the norm $\|\varphi\|=\sup_{s\in[-\tau_M,0]}\|\varphi(s)\|$; E, A and A_τ are known matrices of appropriate dimensions, where E may be singular and we assume that $rank(E)=r\leqslant n$.

Definition 1[16-17] 1) The pair (E,A) is said to be regular if $\det(sE-A)$ is not identically zero; 2) The pair (E,A) is said to be impulse-free if $\deg(\det(sE-A))=rank\,E$.

Definition 2[18] 1) For some given scalars τ_m, τ_M and τ_D, the singular system (1) is said to be regular and impulse free for any time delay $\tau(t)$ satisfying (2), if the pairs (E,A) is regular and impulse free; 2) The singular system (1) is said to be stable if for any $\varepsilon>0$, there exists a scalar $\delta(\varepsilon)>0$ such that for any compatible initial conditions $\varphi(t)$ satisfying $\sup\limits_{-\tau(t)\leqslant t\leqslant 0}\|\varphi(t)\|\leqslant\delta(\varepsilon)$, the solution $x(t)$ of the system (1) satisfies $\|x(t)\|\leqslant\varepsilon$ for $t\geqslant 0$. Furthermore, $\lim\limits_{t\to\infty}x(t)=0$.

Before ending this section, we introduce the following lemmas which are useful in deriving stability criteria for system (1).

Lemma 1[2] For any constant matrix $W\in\mathbf{R}^{n\times n}$, $W=W^{\mathrm T}>0$, scalar $\gamma>0$, and vector function $\dot{x}:[-\gamma,0]\to\mathbf{R}^n$ such that the following integration is well defined, then

$$-\gamma\int_{t-\gamma}^{t}\dot{x}^{\mathrm T}(\xi)W\dot{x}(\xi)\mathrm{d}\xi\leqslant\begin{bmatrix}x(t)\\x(t-\gamma)\end{bmatrix}^{\mathrm T}\begin{bmatrix}-W & W\\ * & -W\end{bmatrix}\begin{bmatrix}x(t)\\x(t-\gamma)\end{bmatrix}$$

Lemma 2[19] Consider the function $\varphi:\mathbf{R}^+\to\mathbf{R}$, if $\dot{\varphi}$ is bounded on $[0,\infty)$, then $\varphi(t)$ is uniformly continuous on $[0,\infty)$.

Lemma 3[19] (Barbalat's Lemma) Consider the function $\varphi:\mathbf{R}^+\to\mathbf{R}$, if $\varphi(t)$ is uniformly continuous and $\int_0^\infty\varphi(s)\mathrm{d}s<\infty$, then $\lim\limits_{t\to\infty}\varphi(t)=0$.

2 Main results

In this section, we present stability criteria for the singular systems with interval time-varying delays (1). Now, we have the following main results.

Theorem 1 For some given scalars τ_m, τ_M and τ_D, the system (1) is asymptotically stable if there exist some matrices $P>0$, $Q_{11}>0$, $Q=\begin{bmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{bmatrix}>0$, $Z_1>0$, $Z_2>0$, $S_i>0$, $i=1,2,3$, and matrix N of appropriate dimensions such that

$$\Omega=\begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13} & 0 & \Omega_{15}\\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0\\ * & * & \Omega_{33} & \Omega_{34} & 0\\ * & * & * & \Omega_{44} & 0\\ * & * & * & * & \Omega_{55}\end{bmatrix}<0 \tag{3}$$

where $\Omega_{11}=E^{\mathrm{T}}PA+A^{\mathrm{T}}PE+NR^{\mathrm{T}}A+A^{\mathrm{T}}RN^{\mathrm{T}}+Q_{11}+Z_1+Z_2-E^{\mathrm{T}}S_1E-E^{\mathrm{T}}S_2E+A^{\mathrm{T}}\Theta A$, $\Omega_{12}=E^{\mathrm{T}}PA_\tau+NR^{\mathrm{T}}A_\tau+A^{\mathrm{T}}\Theta A_\tau$, $\Omega_{13}=E^{\mathrm{T}}S_1E$, $\Omega_{14}=E^{\mathrm{T}}S_2E$, $\Omega_{15}=Q_{12}$, $\Omega_{22}=-2E^{\mathrm{T}}S_3E+A_\tau^{\mathrm{T}}\Theta A_\tau$, $\Omega_{23}=E^{\mathrm{T}}S_3^{\mathrm{T}}E$, $\Omega_{24}=E^{\mathrm{T}}S_3E$, $\Omega_{34}=-Q_{12}$, $\Omega_{33}=-Q_{11}-E^{\mathrm{T}}S_1E-E^{\mathrm{T}}S_3E$, $\Omega_{44}=-Q_{22}-E^{\mathrm{T}}S_2E-E^{\mathrm{T}}S_3E$, $\Omega_{55}=Q_{22}-(1-\tau_D)Z_2$, $\delta=\tau_M-\tau_m$, $\Theta=\tau_m^2S_1+\tau_M^2S_2+\delta^2S_3$, and $R\in\mathbf{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TR=0$.

Proof Since $rank(E)=r\leqslant n$, there must exist two invertible matrices G and $H\in\mathbf{R}^{n\times n}$ such that

$$\overline{E}=GEH=\begin{bmatrix}I_r & 0\\ 0 & 0\end{bmatrix} \tag{4}$$

Similar to (4), we define $\overline{A}=GAH=\begin{bmatrix}\overline{A}_{11} & \overline{A}_{12}\\ \overline{A}_{21} & \overline{A}_{22}\end{bmatrix}$, $\overline{P}=G^{-\mathrm{T}}PG^{-1}=\begin{bmatrix}\overline{P}_{11} & \overline{P}_{12}\\ \overline{P}_{21} & \overline{P}_{22}\end{bmatrix}$, $\overline{S}_1=G^{-\mathrm{T}}S_1G^{-1}=\begin{bmatrix}\overline{S}_{111} & \overline{S}_{112}\\ \overline{S}_{121} & \overline{S}_{122}\end{bmatrix}$, $\overline{S}_2=G^{-\mathrm{T}}S_2G^{-1}=\begin{bmatrix}\overline{S}_{211} & \overline{S}_{212}\\ \overline{S}_{221} & \overline{S}_{222}\end{bmatrix}$, $\overline{N}=H^{\mathrm{T}}N=\begin{bmatrix}\overline{N}_{11}\\ \overline{N}_{21}\end{bmatrix}$, $\overline{R}=G^{-\mathrm{T}}R=\begin{bmatrix}0\\ \overline{\Phi}\end{bmatrix}$.

Since $\Omega_{11}<0$, $Q_{11}>0$, $Z_1>0$, $Z_2>0$ and $\Theta=\tau_m^2S_1+\tau_M^2S_2+\delta^2S_3>0$, we can formulate the following inequality easily, $\Psi=E^{\mathrm{T}}PA+A^{\mathrm{T}}PE+NR^{\mathrm{T}}A+A^{\mathrm{T}}RN^{\mathrm{T}}-E^{\mathrm{T}}S_1E-E^{\mathrm{T}}S_2E<0$.

Pre-and post-multiplying $\Psi<0$ by H^{T} and H, respectively, yields

$$\overline{\Psi}=H^{\mathrm{T}}\Psi H=\overline{E}^{\mathrm{T}}\overline{P}\,\overline{A}+\overline{A}^{\mathrm{T}}\overline{P}\,\overline{E}+\overline{N}\,\overline{R}^{\mathrm{T}}\overline{A}+\overline{A}^{\mathrm{T}}\overline{R}\,\overline{N}^{\mathrm{T}}-\overline{E}^{\mathrm{T}}\overline{S}_1\overline{E}-\overline{E}^{\mathrm{T}}\overline{S}_2\overline{E}=\begin{bmatrix}\overline{\Psi}_{11} & \overline{\Psi}_{12}\\ * & \overline{A}_{22}^{T}\overline{\Phi}\,\overline{N}_{21}^{T}+\overline{N}_{21}\overline{\Phi}^{\mathrm{T}}\overline{A}_{22}\end{bmatrix}<0 \tag{5}$$

Since $\overline{\Psi}_{11}$ and $\overline{\Psi}_{12}$ are irrelevant to the results of the following discussion, the real expression of these two variables are omitted here. From (5), it is easy to see that

$$\overline{A}_{22}^{\mathrm{T}}\overline{\Phi}\,\overline{N}_{21}^{\mathrm{T}}+\overline{N}_{21}\overline{\Phi}^{\mathrm{T}}\overline{A}_{22}<0 \tag{6}$$

and thus $\overline{A}_{22}$ is nonsingular. Otherwise, supposing $\overline{A}_{22}$ is singular, there must exist a non-zero vector $\xi\in\mathbf{R}^{n-r}$, which ensures $\overline{A}_{22}\xi=0$. And then we can conclude that $\xi^{\mathrm{T}}(\overline{A}_{22}^{\mathrm{T}}\overline{\Phi}\,\overline{N}_{21}^{\mathrm{T}}+\overline{N}_{21}\overline{\Phi}^{\mathrm{T}}\overline{A}_{22})\xi=0$, and this contradicts (6). So $\overline{A}_{22}$ is nonsingular. Then, the pair of (E,A) is regular and impulse-free, which implies from Definition 2 that the system (1) is regular and impulse-free.

Next we will show that system (1) is stable. To this end, we define the following Lyapunov-Krasovskii functional

$$V(x_t)=V_1(x_t)+V_2(x_t)+V_3(x_t) \tag{7}$$

where $V_1(x_t)=x^{\mathrm{T}}(t)E^{\mathrm{T}}PEx(t)$, $V_2(x_t)=\int_{t-\tau_m}^{t}X^{\mathrm{T}}(s)QX(s)\mathrm{d}s+\int_{t-\delta}^{t}x^{\mathrm{T}}(s)Z_1x(s)\mathrm{d}s+\int_{t-\tau(t)}^{t}x^{\mathrm{T}}(s)Z_2x(s)\mathrm{d}s$, $V_3(x_t)=\tau_m\int_{-\tau_m}^{0}\mathrm{d}s\int_{t+s}^{t}\dot{x}^{\mathrm{T}}(\theta)E^{\mathrm{T}}S_1E\dot{x}(\theta)\mathrm{d}\theta+\tau_M\int_{-\tau_M}^{0}\mathrm{d}s\int_{t+s}^{t}\dot{x}^{\mathrm{T}}(\theta)E^{\mathrm{T}}S_2E\dot{x}(\theta)\mathrm{d}\theta+\delta\int_{-\tau_M}^{-\tau m}\mathrm{d}s\int_{t+s}^{t}\dot{x}^{\mathrm{T}}(\theta)E^{\mathrm{T}}S_3E\cdot\dot{x}(\theta)\mathrm{d}\theta$, where $X^{\mathrm{T}}(s)=[x^{\mathrm{T}}(s)\quad x^{\mathrm{T}}(s-\delta)]$, $\delta=\tau_M-\tau_m$ and $Q=\begin{bmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{bmatrix}>0$.

Taking the derivative of $V(x_t)$ with respect to t along the trajectory of (1) yields

$$\dot{V}_1(x_t)=2x^{\mathrm{T}}(t)P\dot{x}(t)=x^{\mathrm{T}}(t)(E^{\mathrm{T}}PA+A^{\mathrm{T}}PE)x(t)+2x^{\mathrm{T}}(t)E^{\mathrm{T}}PA_\tau x(t-\tau(t))$$

$$\dot{V}_2(x_t)=X^{\mathrm{T}}(t)QX(t)-X^{\mathrm{T}}(t-\tau_m)QX(t-\tau_m)+x^{\mathrm{T}}(t)Z_1x(t)-x^{\mathrm{T}}(t-\delta)Z_1x(t-\delta)+x^{\mathrm{T}}(t)Z_2x(t)-(1-\dot{\tau}(t))x^{\mathrm{T}}(t-\tau(t))Z_2x(t-\tau(t))$$

$$\dot V_3(x_t)=\dot x^{\mathrm T}(t)E^{\mathrm T}(\tau_m^2S_1+\tau_M^2S_2+\delta^2S_3)E\dot x(t)-\tau_m\int_{t-\tau_m}^{t}\dot x^{\mathrm T}(s)E^{\mathrm T}S_1E\dot x(s)\mathrm ds-$$

$$\tau_M\int_{t-\tau_M}^{t}\dot x^{\mathrm T}(s)E^{\mathrm T}S_2E\dot x(s)\mathrm ds-\delta\int_{t-\tau_M}^{t-\tau_m}\dot x^{\mathrm T}(s)E^{\mathrm T}S_3E\dot x(s)\mathrm ds$$

Use Lemma 1 to obtain

$$-\tau_m\int_{t-\tau_m}^{t}\dot x^{\mathrm T}(s)E^{\mathrm T}S_1E\dot x(s)\mathrm ds\leqslant\begin{bmatrix}x(t)\\x(t-\tau_m)\end{bmatrix}^{\mathrm T}\begin{bmatrix}-E^{\mathrm T}S_1E & E^{\mathrm T}S_1E\\ * & -E^{\mathrm T}S_1E\end{bmatrix}\begin{bmatrix}x(t)\\x(t-\tau_m)\end{bmatrix}$$

$$-\tau_M\int_{t-\tau_M}^{t}\dot x^{\mathrm T}(s)E^{\mathrm T}S_2E\dot x(s)\mathrm ds\leqslant\begin{bmatrix}x(t)\\x(t-\tau_M)\end{bmatrix}^{\mathrm T}\begin{bmatrix}-E^{\mathrm T}S_2E & E^{\mathrm T}S_2E\\ * & -E^{\mathrm T}S_2E\end{bmatrix}\begin{bmatrix}x(t)\\x(t-\tau_M)\end{bmatrix}$$

$$-\delta\int_{t-\tau_M}^{t-\tau_m}\dot x^{\mathrm T}(s)E^{\mathrm T}S_3E\dot x(s)\mathrm ds=-\delta\int_{t-\tau(t)}^{t-\tau_m}\dot x^{\mathrm T}(s)E^{\mathrm T}S_3E\dot x(s)\mathrm ds-\delta\int_{t-\tau_M}^{t-(t)}\dot x^{\mathrm T}(s)E^{\mathrm T}S_3E\dot x(s)\mathrm ds\leqslant$$

$$\begin{bmatrix}x(t-\tau_m)\\x(t-\tau(t))\end{bmatrix}^{\mathrm T}\begin{bmatrix}-E^{\mathrm T}S_3E & E^{\mathrm T}S_3E\\ * & -E^{\mathrm T}S_3E\end{bmatrix}\begin{bmatrix}x(t-\tau_m)\\x(t-\tau(t))\end{bmatrix}+\begin{bmatrix}x(t-\tau(t))\\x(t-\tau_M)\end{bmatrix}^{\mathrm T}\begin{bmatrix}-E^{\mathrm T}S_3E & E^{\mathrm T}S_3E\\ * & -E^{\mathrm T}S_3E\end{bmatrix}\begin{bmatrix}x(t-\tau(t))\\x(t-\tau_M)\end{bmatrix}$$

Noting that (1), the following holds: $\dot x^{\mathrm T}(t)E^{\mathrm T}\Theta E\dot x(t)=\zeta^{\mathrm T}(t)\widetilde A^{\mathrm T}\Theta\widetilde A\zeta(t)$, where $\widetilde A=[A\quad A_\tau\quad 0\quad 0\quad 0]$, $\zeta^{\mathrm T}(t)=[x^{\mathrm T}(t)\quad x^{\mathrm T}(t-\tau(t))\quad x^{\mathrm T}(t-\tau_m)\quad x^{\mathrm T}(t-\tau_M)\quad x^{\mathrm T}(t-\delta)]$ and $\Theta=\tau_m^2S_1+\tau_M^2S_2+\delta^2S_3$. Furthermore, noting $E^{\mathrm T}R=0$, we can deduce

$$0=2\dot x^{\mathrm T}(t)E^{\mathrm T}R(N^{\mathrm T}x(t))=2(Ax(t)+A_\tau x(t-\tau(t)))^{\mathrm T}R(N^{\mathrm T}x(t))\tag{8}$$

Thus it follows from above, we have $\dot V(x_t)\leqslant\zeta^{\mathrm T}(t)\,\Omega\zeta(t)$, where Ω is defined in (6). One can see that if $\Omega<0$, then $\dot V(x_t)<0$ and

$$\lambda_1\|x(t)\|^2-V(x(0))\leqslant x^{\mathrm T}(t)E^{\mathrm T}PEx(t)-V(x(0))\leqslant V(x(t))-V(x(0))=$$

$$\int_0^t\dot V(x(s))\mathrm ds\leqslant-\lambda_2\int_0^t\|x(s)\|^2\mathrm ds<0\tag{9}$$

where $\lambda_1=\lambda_{\min}(E^{\mathrm T}PE)>0$, $\lambda_1=-\lambda_{\max}(\Omega)>0$.

From (9), it is easy to obtain that $\lambda_1\|x(t)\|^2+\lambda_2\int_0^t\|x(s)\|^2\mathrm ds\leqslant V(x(0))$. Then $0<\|x(t)\|^2\leqslant\frac{1}{\lambda_1}V(x(0))$, $0<\int_0^t\|x(s)\|^2\mathrm ds\leqslant\frac{1}{\lambda_2}V(x(0))$. Thus, $\|x(t)\|^2$ and $\int_0^t\|x(s)\|^2\mathrm ds$ are bounded. Using same method, we have that $\|\dot x(t)\|$ is bounded. According to Lemma 2 and Lemma 3, we get $\lim_{t\to\infty}x(t)=0$, according to Definition 2, the singular system (1) is stable. This completes the proof.

Remark 1 From the proof process of Theorem 1, the most attractive contribution is that in Theorem 1 we have made the best use of not only the lower bound of the interval time-varying delay, but also the interval of the upper bound and the lower bound of the time delay. To reduce the conservatism, we employ a new Lyaponov-Krasovskii functional (7), which is mainly based on the information about τ_m, τ_M and $\tau_M-\tau_m$.

Remark 2 If τ_m is zero, we can obtain a stability criteria of system (1) using the corresponding Lyapunov-Krasovskii functional reduces to

$$V(x_t)=x^{\mathrm T}(t)E^{\mathrm T}PEx(t)+\int_{t-\tau_M}^{t}x^{\mathrm T}(s)Qx(s)\mathrm ds+\int_{t-\tau(t)}^{t}x^{\mathrm T}(s)Zx(s)\mathrm ds+\tau_M\int_{-\tau_M}^{0}\mathrm ds\int_{t+s}^{t}\dot x^{\mathrm T}(\theta)E^{\mathrm T}SE\dot x(\theta)\mathrm d\theta$$

Similar to the proof of Theorem 1, one can easily derive a less conservative result than some existing ones, which will be shown through numerical examples in the next section. Due to page limit the result is omitted.

3 Numerical examples

In this section, some examples are provided to demonstrate the effectiveness of our results.

Example 1 Consider the linear time delay system

$$\dot x(t)=\begin{bmatrix}-2 & 0\\0 & -0.9\end{bmatrix}x(t)+\begin{bmatrix}-1 & 0\\-1 & -1\end{bmatrix}x(t-\tau)\tag{10}$$

In this example, we choose $R=[0\quad 0]^{\mathrm{T}}$. Tab. 1 lists the maximum allowable upper bound (MAUB) of the time-varying delay by using Theorem 1 and those in [4-6,8] for different τ_m. From the table, one can see that the derived results in this paper are less conservative than those in [4-6,8].

Example 2 Consider the system (1) with

$$\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\dot{x}(t)=\begin{bmatrix}0.5 & 0\\ 0 & -1\end{bmatrix}x(t)+\begin{bmatrix}-1.1 & 1\\ 0 & 0.5\end{bmatrix}x(t-\tau(t)) \tag{11}$$

We choose $R=[0\quad 1]^{\mathrm{T}}$. When $\tau_m=0$, the upper bounds on the time delay from Theorem 1 are shown in Tab. 2. For comparison, the table also lists the upper bounds obtained from the criteria in [9-15]. It can be seen that our method is less conservative.

But when the time-delay $\tau(t)$ is interval delay, the criteria in [9-15] fail to make any decision for this case. According to Theorem 1, the MAUB on the time-varying delay is shown in Tab. 3 for different τ_m.

Tab. 1 MAUB of the time-varying delay for different τ_m

τ_m	1	2	3	4
[6]	1.64	2.39	3.20	4.06
[4]	1.74	2.43	3.22	4.06
[8]	1.873 7	2.504 9	3.259 1	4.074 4
[5]	1.804 3	2.521 3	3.331 1	4.188 0
Theorem 1	4.472 1	4.472 1	4.472 1	4.472 1

Tab. 2 Comparison of MAUB using different methods ($\tau_m=0$)

Methods	[10]	[11]	[12]	[13,14]
MAUB	0.556 7	0.870 8	0.909 1	0.968 0
Methods	[9]	[15]	Theorem 1	
MAUB	1.042 3	1.066 0	1.066 0	

Tab. 3 MAUB of the time-varying delay for different τ_m using Theorem 1

τ_m	0	0.5	0.8	1
MAUB 1	1.066 0	1.066 0	1.066 0	1.066 0
MAUB 2	1.020 4	1.038 0	1.051 6	1.062 2

where MAUB 1 : $\tau_D=0$, MAUB 2: $\tau_D=0.5$

4 Conclusions

We have addressed the stability of singular systems with interval time-varying delay. Sufficient conditions are given in terms of strict LMI by employing a new Lyapunov-Krasovskii functional. The functional is based on not only the lower and upper bounds of time-varying, but also their interval. Neither model transformation nor bounding technique for some cross terms are introduced to the paper. Examples show the advantages of the theoretic results obtained, and show that our results are much less conservative than some existing results in the literature.

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区间变时滞广义系统的稳定性分析

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摘要:本文研究了区间变时滞广义系统的时滞依赖稳定性问题,即寻求稳定判据保证广义系统的正则性、无脉冲性和渐近稳定性。通过利用一个新的 Lyapunov-Krasovskii 泛函和自由权矩阵方法,得到了该系统渐近稳定的时滞依赖新判据。所提出的泛函,能充分利用区间时滞的上界、下界和这两个界中间值的信息。新判据以线性矩阵不等式形式给出,很容易利用 MATLAB 验证。数值算例表明,与参考文献中的方法相比,本文结果可获得较大的允许时滞上界,这也验证了方法的有效性。

关键词:稳定;Lyapunov-Krasovskii 泛函;广义系统;线性矩阵不等式(LMI)

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