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二阶非线性时标动态方程的振动准则*

张 少 艳
(广东金融学院 应用数学系,广州 510520)

摘要:时标理论在同时处理连续系统和离散系统方面具有非常广泛的应用。近年来有非常多的关于二阶中立型时标动态方程的振动性的结论,但已有结论均要求特殊的时标集,或 $r(t)$ 函数递增。本文运用时标上积分及不等式的性质,得出 $x(t)/x(\delta(t))\leqslant\alpha(t,T)$ 的结论。利用该结论、Riccati 变换技巧及配方法,得到了方程解的振动准则,即若方程能使得 $\limsup\limits_{x\to\infty}\int_T^t\left[Q(s)q(s)-\frac{r(s)\,(z^\Delta(s))^2}{4C(s)z(s)}\right]\Delta s=\infty$ 或 $\limsup\limits_{t\to\infty}\int_{t_3}^t\left[q(s)Q_1(s)-(z^\Delta(s))^2(r(s))\,\frac{1}{\gamma}\,(R_T(s)r^{\frac{1}{\gamma}}(s))^{1-\gamma}\right]\Delta s=\infty$ 成立,则方程的解释振动所得到的结果去掉了时标集是特殊的及函数是递增的条件,其应用范围更为广泛。

关键词:二阶时标动态方程;中立型;振动性;非线性;广义 Riccati 技巧
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时标理论最初由 Stefan Hilger 在其博士论文中提出,其目的是为了统一离散与连续这 2 种情形,为同时处理连续系统和离散系统提出了基本的方法。因此,时标理论引起越来越多学者的关注[1-8]。一个时标 T 是指实数集 **R** 的一个任意非空闭子集,它具有由 **R** 诱导的拓扑及顺序关系[1]。

近年来,人们对二阶非线性中立型时标动态方程

$$(r(t)((y(t)+p(t)y(\tau(t)))^\Delta)^\gamma)^\Delta+f(t,y(\delta(t)))=0 \tag{1}$$

其中 γ 为不可约正奇数的商的振动性进行了广泛的研究。

2004 年,Wu H. W. 等人在文献[2]中给出了在特殊时滞项 $\delta(t)$ 下,要求 $(\delta\circ\sigma)(t)=(\delta\circ\sigma)(t)$ 成立时方程(1)的振动准则。2007 年,S. H. Saker 在文献[3]中将上述条件替换成了 $\int_{t_0}^\infty\delta^\gamma(s)q(s)\,(1-p(\delta(s)))^\gamma\Delta s=\infty$,给出了方程(1)的振动准则。但这 2 篇文献所给出的结果都只适用于指数 $\gamma\geqslant1$ 时。2010 年,Zhang S. Y. 等人在文献[4]中去掉了对时滞项的要求,增加了条件 $r^\Delta(t)\geqslant0$,得到了方程(1)的新振动准则,其中有部分结果是适用于 $0<\gamma<1$。

本文同样假设下列条件成立,设 T 为时标集,并记 $\mathbf{R}^+=(0,+\infty)$,$\mathbf{R}_0^+=[0,+\infty)$:

(H1) $\tau\in C_{rd}(T,T)$,$\tau(t)\leqslant t$,$\lim\limits_{t\to\infty}\tau(t)=\infty$,$\delta\in C_{rd}(T,T)$,$\delta(t)\leqslant t$,$\lim\limits_{t\to\infty}\delta(t)=\infty$;

(H2) $r\in C_{rd}(T,\mathbf{R}^+)$ 且 $\int_a^\infty\left(\frac{1}{r(s)}\right)^{\frac{1}{\gamma}}\Delta s=\infty$,$p\in C_{rd}(T,\mathbf{R}_0^+)$,$0\leqslant p<1$;

(H3) $f(t,u)\in C(T\times\mathbf{R},\mathbf{R})$,存在 $q(t)\in C_{rd}(T,\mathbf{R}_0^+)$ 且 $q(t)$ 不最终恒等于 0,使得 $uf(t,u)\geqslant q(t)\,|u^{\gamma+1}|$。

所得到的结果去掉了文献[4]中要求的 $r^\Delta(t)\geqslant0$ 条件,改进了其部分结果,给出方程(1)在较弱条件下的振动准则。

1 基本引理

为了书写的方便,记 $x(\sigma(t))=x^\sigma(t)=x^\sigma$,$x^\Delta(\sigma(t))=(x^\Delta)^\sigma$,并假设 $x(t)=y(t)+p(t)y(\tau(t))$。

方程(1)的解 $y(t)$ 称为振动,若 $y(t)$ 既不是最终正解,也不是最终负解,否则称 $y(t)$ 为非振动的。方程(1)称为振动的,如果它的所有解都是振动的。

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作者简介:张少艳,女,讲师,硕士,研究方向为微分方程,E-mail:zsyhjf@yhahoo.com.cn
网络出版地址:http://www.cnki.net/kcms/detail/50.1165.N.20130917.1738.201305.80_013.html

本文研究的是方程(1)的解的振动性，因此总假设时标集 T 满足 inf T$=t_0$，sup T$=\infty$，令 $T\in$ T，$[T,\infty)_T=\{t\in$ T$:=t\geqslant T\}$。令 $\tau^*(t)=\min\{\tau(t),\delta(t)\}$，$\tau_{-1}^*(t)=\sup\{s\geqslant t_0:\tau(t)\leqslant t\}$。显然 $\tau_{-1}^*(t)\geqslant t$。

令 $\alpha(t,T)=1+\int_T^t\frac{\Delta s}{(r(s))^{\frac{1}{\gamma}}}\Big/\int_T^{\delta(t)}\frac{\Delta s}{(r(s))^{\frac{1}{\gamma}}}$，$R_T(t)=\int_T^t\frac{1}{r^{\frac{1}{\gamma}}(s)}\Delta s$ 。

首先给出主要定理证明中需要的几个引理。

引理 1[4]　设(H1)～(H3)成立，若方程(1)存在最终正解，则 $\exists T\in$ T，使得 $x(t)\geqslant 0$，$x^{\Delta}(t)\geqslant 0$，$(r(t)(x^{\Delta}(t))^{\gamma})^{\Delta}\leqslant 0$，$t\geqslant T$ 且等号不最终成立。

引理 2　设(H1)～(H3)成立，若方程(1)存在最终正解，则 $\exists T\in$ T，使得 $\frac{x(t)}{x(\delta(t))}\leqslant\alpha(t,T)$，$x(t)\geqslant R_T(t)r^{\frac{1}{\gamma}}(t)x^{\Delta}(t)$，$t\geqslant T$ 且等号不最终成立。

证明　因为方程(1)存在最终正解，故引理 1 成立。

因为 $(r(t)(x^{\Delta}(t))^{\gamma})^{\Delta}\leqslant 0$ 及(H1)成立，故存在 $T\in[t_0,\infty)$，使得 $\delta(t)\geqslant T$，对于所有的 $t\geqslant T$ 都成立，于是有 $x(t)=x(t)+\int_{\delta(t)}^t\frac{(r(s)\,(x^{\Delta}(s))^{\gamma})^{\frac{1}{\gamma}}}{r^{\frac{1}{\gamma}}(s)}\Delta s\leqslant x(\delta(t))+(r(\delta(t))\,(x^{\Delta}(\delta(t)))^{\gamma})^{\frac{1}{\gamma}}\int_{\delta(t)}^t\frac{\Delta s}{r^{\frac{1}{\gamma}}(s)}$，故有 $\frac{x(t)}{x(\delta(t))}\leqslant 1+\frac{(r(\delta(t))\,(x^{\Delta}(\delta(t)))^{\gamma})^{\frac{1}{\gamma}}}{x(\delta(t))}\int_{\delta(t)}^t\frac{\Delta s}{r^{\frac{1}{\gamma}}(s)}$。又因为

$$x(\delta(t))\geqslant x(\delta(t))-x(T)=\int_T^{\delta(t)}\frac{(r(s)\,(x^{\Delta}(s))^{\gamma})^{\frac{1}{\gamma}}}{r^{\frac{1}{\gamma}}(s)}\Delta s\geqslant$$

$$(r(\delta(t))\,(x^{\Delta}(\delta(t)))^{\gamma})^{\frac{1}{\gamma}}\int_T^{\delta(t)}\frac{\Delta s}{r^{\frac{1}{\gamma}}(s)},\ \frac{x(\delta(t))}{(r(\delta(t))\,(x^{\Delta}(\delta(t)))^{\gamma})^{\frac{1}{\gamma}}}\geqslant\int_T^{\delta(t)}\frac{\Delta s}{r^{\frac{1}{\gamma}}(s)},$$

故

$$\frac{x(t)}{x(\delta(t))}\leqslant 1+\int_{\delta(t)}^t\frac{\Delta s}{(r(s))^{\frac{1}{\gamma}}}\Big/\int_T^{\delta(t)}\frac{\Delta s}{(r(s))^{\frac{1}{\gamma}}}\leqslant 1+\int_T^t\frac{\Delta s}{(r(s))^{\frac{1}{\gamma}}}\Big/\int_T^{\delta(t)}\frac{\Delta s}{(r(s))^{\frac{1}{\gamma}}}$$

则有 $\frac{x(t)}{x(\delta(t))}\leqslant\alpha(t,T)$。又因为

$$x(t)\geqslant x(t)-x(T)=\int_T^t x^{\Delta}(s)\Delta s=\int_T^t\frac{[r(s)\,(x^{\Delta}(s))^{\gamma}]^{\frac{1}{\gamma}}}{r^{\frac{1}{\gamma}}(s)}\Delta s\geqslant$$

$$[r(t)\,(x^{\Delta}(t))^{\gamma}]^{\frac{1}{\gamma}}\int_T^t\frac{1}{r^{\frac{1}{\gamma}}(s)}\Delta s=x^{\Delta}(t)r^{\frac{1}{\gamma}}(t)\int_T^t\frac{1}{r^{\frac{1}{\gamma}}(s)}\Delta s=R_T(t)r^{\frac{1}{\gamma}}(t)x^{\Delta}(t)$$

故引理 2 成立。　　证毕

注 1　当 $t\in[t_0,\infty)$ 时，$y(t)=x(t)-p(t)y(\tau(t))$，由(H3)有

$$0\geqslant(r(t)(x^{\Delta}(t))^{\gamma})^{\Delta}+q(t)(y(\delta(t)))^{\gamma}=(r(t)(x^{\Delta}(t))^{\gamma})^{\Delta}+q(t)\,[x(\delta(t))-p(\delta(t))y(\tau(\delta(t)))]^{\gamma}$$

因为 $x(t)\geqslant y(t)$，故 $0\geqslant(r(t)(x^{\Delta}(t))^{\gamma})^{\Delta}+q(t)[x(\delta(t))-p(\delta(t))x(\tau(\delta(t)))]^{\gamma}$，又因为 $\tau(t)\leqslant t$，$x^{\Delta}(t)>0$，故

$$0\geqslant(r(t)(x^{\Delta}(t))^{\gamma})^{\Delta}+q(t)\,[x(\delta(t))-p(\delta(t))x(\delta(t))]^{\gamma}=(r(t)(x^{\Delta}(t))^{\gamma})^{\Delta}+q(t)(1-p(\delta(t)))^{\gamma}\,[x(\delta(t))]^{\gamma}\quad(2)$$

引理 3[5]　若 x,z 是 Δ 可导的函数，则对 $x\neq 0$，$\forall t\in$ T，有 $x^{\Delta}\left(\frac{z^2}{x}\right)^{\Delta}=(z^{\Delta})^2-xx^{\sigma}\left[\left(\frac{z}{x}\right)^{\Delta}\right]^2$。

2 主要定理

定理 1　设(H1)～(H3)成立，若存在一个 Δ 可导的正函数 z，对于 γ 为不可约正奇数的商 $\gamma\geqslant 1$ 使得 $\limsup_{x\to\infty}\int_T^t\left[Q(s)q(s)-\frac{r(s)\,(z^{\Delta}(s))^2}{4C(s)z(s)}\right]\Delta s=\infty$ 成立，其中

$$C(t)=(R_T(t)r^{\frac{1}{\gamma}}(t))^{\gamma-1},\ Q(t)=z(t)(1-p\circ\delta(t))^{\gamma}\alpha^{-\gamma}(t,T)$$

则方程(1)的解振动。

证明　假设方程(1)的解非振动，不失一般性，假设 $\exists t_0\in[0,\infty)$，$y(t)>0$，$t\geqslant t_0$，则引理 1～2 成立。定义 Riccati 变换 $w(t)=\frac{z(t)r(t)(x^{\Delta}(t))^{\gamma}}{x^{\gamma}(t)}$。对其求导可得

$$w^{\Delta}=(z)^{\Delta}\left[\frac{r\,(x^{\Delta})^{\gamma}}{x^{\gamma}}\right]^{\sigma}+z\left[\frac{r\,(x^{\Delta})^{\gamma}}{x^{\gamma}}\right]^{\Delta}=\frac{z^{\Delta}}{z^{\sigma}}w^{\sigma}+z\left[\frac{[r\,(x^{\Delta})^{\gamma}]^{\Delta}x^{\gamma}-r\,(x^{\Delta})^{\gamma}\,(x^{\gamma})^{\Delta}}{x^{\gamma}\,(x^{\sigma})^{\gamma}}\right]$$

又因为在时标中　$(fg)^{\Delta}(t)=f^{\Delta}(t)g(t)+f(\sigma(t))g^{\Delta}(t)=f^{\Delta}(t)g(\sigma(t))+f(t)g^{\Delta}(t)$

即可得　$f^{\Delta}(t)g(t)-f(t)g^{\Delta}(t)=f^{\Delta}(t)g(\sigma(t))-f(\sigma(t))g^{\Delta}(t)$

将上式代入化简可得　$w^{\Delta}=\frac{z^{\Delta}}{z^{\sigma}}w^{\sigma}+z\frac{[r\,(x^{\Delta})^{\gamma}]^{\Delta}}{x^{\gamma}}-z\frac{r^{\sigma}\,((x^{\Delta})^{\sigma})^{\gamma}\,(x^{\gamma})^{\Delta}}{x^{\gamma}\,(x^{\sigma})^{\gamma}}$

又因为由链式法则可得

$$((x(t))^{\gamma})^{\Delta}=\gamma\left\{\int_0^1 (x(t)+h\mu(t)x^{\Delta}(t))^{\gamma-1}\mathrm{d}h\right\}x^{\Delta}(t)=\gamma\left\{\int_0^1 (x(t)+h\,(x(\sigma(t))-x(t))^{\gamma-1}\mathrm{d}h\right\}x^{\Delta}(t)\geqslant$$

$$\gamma\left\{\int_0^1((1-h)x(t)+hx(t))^{\gamma-1}\mathrm{d}h\right\}x^{\Delta}(t)=\gamma\,(x(t))^{\gamma-1}x^{\Delta}(t)\tag{3}$$

由(2)、(3)式及引理1,可得

$$w^{\Delta}\leqslant-z\frac{q\,(1-p\circ\delta)^{\gamma}\,(x\circ\delta)^{\gamma}}{x^{\gamma}}+\frac{z^{\Delta}}{z^{\sigma}}w^{\sigma}-z\frac{r^{\sigma}\,((x^{\Delta})^{\sigma})^{\gamma}\gamma x^{\gamma-1}x^{\Delta}}{x^{\gamma}\,(x^{\sigma})^{\gamma}}\leqslant$$

$$-zq\,(1-p\circ\delta)^{\gamma}\alpha^{-\gamma}(t,T)+\frac{z^{\Delta}}{z^{\sigma}}w^{\sigma}-zr^{\sigma}\left[\frac{((x^{\Delta})^{\sigma})^{\gamma}}{(x^{\sigma})^{\gamma}}\right]^{2}\frac{(x^{\sigma})^{\gamma}\gamma x^{\gamma-1}x^{\Delta}}{x^{\gamma}\,((x^{\Delta})^{\sigma})^{\gamma}}$$

令 $Q(t)=z(t)(1-p\circ\delta(t))^{\gamma}\alpha^{-\gamma}(t,T)$,又由引理1可得 $w^{\Delta}\leqslant-qQ+\frac{z^{\Delta}}{z^{\sigma}}w^{\sigma}-\gamma zr^{\sigma}\left(\frac{w^{\sigma}}{z^{\sigma}r^{\sigma}}\right)^{2}\frac{(x^{\sigma})^{\gamma}x^{\Delta}}{x\,((x^{\Delta})^{\sigma})^{\gamma}}$。因为 $\sigma(t)\geqslant t$,$(r\,(x^{\Delta})^{\gamma})^{\Delta}\leqslant 0$,故 $r\,(x^{\Delta})^{\gamma}\geqslant(r\circ\sigma)(x^{\Delta}(\sigma(t)))^{\gamma}$,即

$$(x^{\Delta})^{\gamma}\geqslant\frac{r\circ\sigma}{r}(x^{\Delta}(\sigma(t)))^{\gamma}\tag{4}$$

故 $w^{\Delta}\leqslant-zQ+\frac{z^{\Delta}}{z^{\sigma}}w^{\sigma}-\gamma zr^{\sigma}\left(\frac{w^{\sigma}}{z^{\sigma}r^{\sigma}}\right)^{2}\frac{r^{\sigma}x^{\gamma}x^{\Delta}}{rx\,(x^{\Delta})^{\gamma}}=-zQ+\frac{z^{\Delta}}{z^{\sigma}}w^{\sigma}-\gamma z\,\frac{1}{r}\left(\frac{w^{\sigma}}{z^{\sigma}}\right)^{2}\frac{x^{\gamma-1}}{(x^{\Delta})^{\gamma-1}}$。

由引理2且 $\gamma\geqslant 1$ 可得到 $w^{\Delta}\leqslant-zQ+\frac{z^{\Delta}}{z^{\sigma}}w^{\sigma}-\gamma z\,\frac{1}{r}\left(\frac{w^{\sigma}}{z^{\sigma}}\right)^{2}(R_T(t)r^{\frac{1}{\gamma}}(t))^{\gamma-1}$。又令 $C(t)=(R_T(t)r^{\frac{1}{\gamma}}(t))^{\gamma-1}$,故 $w^{\Delta}(t)\leqslant-Q(t)q(t)+\frac{z^{\Delta}(t)}{z^{\sigma}(t)}w^{\sigma}-\frac{z(t)C(t)}{(z^{\sigma}(t))^{2}r(t)}(w^{\sigma})^{2}$。配方得

$$w^{\Delta}(t)\leqslant-Q(t)q(t)-\left[\frac{z(t)C(t)}{(z^{\sigma})^{2}r(t)}(w^{\sigma})^{2}-\frac{z^{\Delta}(t)}{z^{\sigma}}w^{\sigma}+\frac{r(t)(z^{\Delta}(t))^{2}}{4C(t)z(t)}\right]+\frac{r(t)(z^{\Delta}(t))^{2}}{4C(t)z(t)}\leqslant-Q(t)q(t)+\frac{r(t)(z^{\Delta}(t))^{2}}{4C(t)z(t)}$$

对上式从 T 到 $t(t\geqslant T)$ 积分,可得 $w(T)\geqslant\int_T^t\left[Q(s)q(s)-\frac{r(s)\,(z^{\Delta}(s))^{2}}{4C(s)z(s)}\right]\Delta s$。再取 $t\to\infty$ 的极限,可发现与条件矛盾,故方程的解振动。　　证毕

定理2　设(H1)～(H3)成立,若存在一个 Δ 可导函数 z,对于 γ 为不可约正奇数的商 $\gamma\geqslant 1$,使得

$$\limsup_{t\to\infty}\int_{t_3}^{t}\left[q(s)Q_1(s)-(z^{\Delta}(s))^{2}(r(s))\,\frac{1}{\gamma}\,(R_T(s)r^{\frac{1}{\gamma}}(s))^{1-\gamma}\right]\Delta s=\infty$$

成立,其中 $Q_1(t)=z^{2}(\sigma(t))(1-(p\circ\delta)(t))^{\gamma}\alpha^{-\gamma}(t,T)$,则方程(1)的解振动。

证明　假设方程(1)的解非振动,不失一般性,假设 $\exists\, t_0\in[0,\infty)$,$y(t)>0$,$t\geqslant t_0$,则引理1、2成立。

定义Riccati变换 $w(t)=\frac{z^{2}(t)r(t)(x^{\Delta}(t))^{\gamma}}{x^{\gamma}(t)}$。对上式求导可得 $w^{\Delta}=\left[r\,(x^{\Delta})^{\gamma}\cdot\frac{z^{2}}{x^{\gamma}}\right]^{\Delta}=[r\,(x^{\Delta})^{\gamma}]^{\Delta}\left[\frac{z^{2}}{x^{\gamma}}\right]+[r\,(x^{\Delta})^{\gamma}]^{\sigma}\left[\frac{z^{2}}{x^{\gamma}}\right]^{\Delta}$。再由(4)式可得 $w^{\Delta}\leqslant-q\,(1-p\circ\delta)^{\gamma}\,(x\circ\delta)^{\gamma}\left[\frac{z^{2}}{x^{\gamma}}\right]+\frac{[r\,(x^{\Delta})^{\gamma}]}{(x^{\gamma})^{\Delta}}\cdot((x)^{\gamma})^{\Delta}\cdot\left[\frac{z^{2}}{x^{\gamma}}\right]^{\Delta}$。令 $Q_1(t)=z^{2}(\sigma(t))(1-(p\circ\delta)(t))^{\gamma}\alpha^{-\gamma}(t,T)$。由引理1及引理3可得

$$w^{\Delta}=-qQ_1+\frac{[r\,(x^{\Delta})^{\gamma}]}{(x^{\gamma})^{\Delta}}\left[(z^{\Delta})^{2}-x^{\gamma}\,(x^{\sigma})^{\gamma}\left(\left(\frac{z}{x}\right)^{\Delta}\right)^{2}\right]\leqslant-qQ_1+\frac{[r\,(x^{\Delta})^{\gamma}]}{(x^{\gamma})^{\Delta}}(z^{\Delta})^{2}$$

又由引理2,$\gamma\geqslant 1$ 及 $((x(t))^{\gamma})^{\Delta}\geqslant\gamma\,(x(t))^{\gamma-1}x^{\Delta}(t)$,可得

$$w^{\Delta}=-qQ_1+\frac{r\,(z^{\Delta})^{2}}{\gamma}\left(\frac{x^{\Delta}}{x}\right)^{\gamma-1}\leqslant-qQ_1+\frac{r\,(z^{\Delta})^{2}}{\gamma}(R_T(t)r^{\frac{1}{\gamma}}(t))^{1-\gamma}$$

对上式从 T 到 $t(t\geqslant T)$ 积分,可得 $w(T)\geqslant\int_T^t\left[q(s)Q_1(s)-(z^{\Delta}(s))^{2}r(s)\,\frac{1}{\gamma}\,(R_T(s)r^{\frac{1}{\gamma}}(s))^{1-\gamma}\right]\Delta s$,再取 $t\to\infty$ 的

极限,可知与条件矛盾,故方程的解振动。　　证毕

3 举例

例 1　$\left(\frac{1}{(t+\sigma(t))^{\gamma}}((y(t)+p(t)y(\tau(t)))^{\Delta})^{\gamma}\right)^{\Delta}+\frac{t^{2\gamma}}{t\,(1-p\circ\delta(t))^{\gamma}\delta^{2\gamma}(t)}y^{\gamma}(\delta(t))=0(\gamma\geqslant 1,t\in[1,+\infty)_{T})$

解　由上式,令 $q(t)=\frac{t^{2\gamma}}{t\,(1-p\circ\delta(t))^{\gamma}\delta^{2\gamma}(t)}$,$z(t)=1$,则 $\int_{T}^{t}\frac{\Delta s}{r^{\frac{1}{\gamma}}(s)}=t^{2}-T^{2}\,(T\geqslant 1)$,故存在某个足够大的常数 b,使得 $\alpha^{-\gamma}(t,T)=\left(\frac{\delta^{2}(t)-T^{2}}{\delta^{2}(t)-T^{2}+t^{2}-T^{2}}\right)^{\gamma}\geqslant\left(\frac{\delta^{2}(t)}{2bt^{2}}\right)^{\gamma}$。根据定理 1,有

$$\int_{1}^{t}\left[Q(s)q(s)-\frac{r(s)\,(z^{\Delta}(s))^{2}}{4C(s)z(s)}\right]\Delta s=\int_{1}^{t}(1-p\circ\delta(s))^{\gamma}\alpha^{-\gamma}(s,T)q(s)\Delta s\geqslant$$

$$\int_{1}^{t}(1-p\circ\delta(s))^{\gamma}\left(\frac{\delta^{2}(s)}{2bt^{2}}\right)^{\gamma}\frac{t^{2\gamma}}{s\,(1-p\circ\delta(s))^{\gamma}\delta^{2\gamma}(s)}\Delta s=\int_{1}^{t}\frac{1}{2bs}\Delta s\to\infty$$

故方程的解振动。

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Oscillation Criteria for a Second-order Nonlinear Neutral Dynamic Equations on Time Scales

ZHANG Shao-yan

(Department of Mathematics, Guangdong University of Finance, Guangdzhou 510630, China)

Abstract: The theory of time scales has been widely used in the simultaneous processing of continuous system and discrete system. Therefore, many conclusions on the oscillation for second-order nonlinear neutral delay dynamic equations on time scales have been put forward in recent years. However, all these conclusions establish on special time scales or in the condition of increasing $r(t)$. Now, by using the characters of the integral and inequalities we get a conclusion, $x(t)/x(\delta(t))\leqslant\alpha(t,T)$. With generalized Riccati technique and completing the square, we find the oscillation criterias for the equation. That is, if the equation can make $\limsup_{x\to\infty}\int_{T}^{t}\left[Q(s)q(s)-\frac{r(s)\,(z^{\Delta}(s))^{2}}{4C(s)z(s)}\right]\Delta s=\infty$ or $\limsup_{t\to\infty}\int_{t_3}^{t}\left[q(s)Q_1(s)-(z^{\Delta}(s))^{2}(r(s))\frac{1}{\gamma}(R_T(s)r^{\frac{1}{\gamma}}(s))^{1-\gamma}\right]\Delta s=\infty$to hold, the equation is oscillation. It is satisfactory that these results have a wider range of application because it needs no special time sales or increasing functions required in the former studies.

Key words: second order dynamic equations on time scales; neutral; oscillation; nonlinear; generalized Riccati technique

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