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Normal Families of Meromorphic Functions and Shared Holomorphic Functions*

WU Chun
(College of Mathematics Science, Chongqing Normal University, Chongqing 401331, China)

Abstract: In this paper, we discuss the normality criterion concerning shared holomorphic functions, and prove that if $\mathscr{F}$ be a family of meromorphic functions in a domain D, $L[f]=a_0 f' + a_1 f$ $(a_0 \neq 0)$, and a, b, c, d be four holomorphic functions in D. For each $f \in \mathscr{F}$, if $a(z) \neq d(z)$, $b(z)+a_1(z)a(z)+a_0(z)a'(z) \neq 2c(z)$, $c(z)-a_0(z)a'(z)-a_1(z)a(z) \neq 0$, $f(z)=a(z) \Leftrightarrow L[f](z)=b(z)$ and $L[f](z)=c(z) \Rightarrow f(z)=d(z)$, then $\mathscr{F}$ is normal in D.

Key words: meromorphic functions; normal families; shared function
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1 Introduction and main results

Let D be a domain in **C**, and $\mathscr{F}$ be a family of meromorphic functions defined in a domain D. $\mathscr{F}$ is said to be normal in D, in the sense of Montel, if for any sequence $\{f_n\} \subset \mathscr{F}$, there exists a subsequence $\{f_{n_j}\}$ such that f_{n_j} converges spherically locally uniformly in D, to a meromorphic function or ∞[1-3].

Let $g(z)$ be a meromorphic function, a be a finite complex number. If $f(z)$ and $g(z)$ assume the same zeros, then we say that share a IM (ignoring multiplicity) [1,4-5].

In 2005, Chang, Fang and Zalcman[6] got the following results.

Theorem A Let $\mathscr{F}$ be a family of meromorphic functions defined in a domain D. and let a, b, c and d be complex numbers such that $b \neq 0, c \neq a$ and $d \neq b$. If for each $f \in \mathscr{F}$, $f=a \Leftrightarrow f'=b$ and $f=c \Rightarrow f'=d$, then $\mathscr{F}$ is normal in D.

In 2012, Lü[7] replaced the condition $f=a \Leftrightarrow f'=b$ and $f=c \Rightarrow f'=d$ by $f(z)=a(z) \Leftrightarrow L[f](z)=b(z)$ and $f(z)=c(z) \Rightarrow L[f](z)=d(z)$ and proved the following result.

Theorem B Let $\mathscr{F}$ be a family of meromorphic functions in a domain D, let $L[f]=a_0 f' + a_1 f (a_0 \neq 0)$, and let a, b, c, d be four holomorphic functions in D. For each $f \in \mathscr{F}$, if 1) $a(z) \neq c(z)$; 2) $b(z)-a_1(z)a(z)-a_0(z)c'(z) \neq 0$; 3) $b(z)-a_1(z)c(z)-a_0(z)c'(z) \neq 0$; 4) $f(z)=a(z) \Leftrightarrow L[f](z)=b(z)$ and $f(z)=c(z) \Rightarrow L[f](z)=d(z)$, then $\mathscr{F}$ is normal in D.

A natural question is: whether $\mathscr{F}$ is normal if we replace the condition $f(z)=c(z) \Rightarrow L[f](z)=d(z)$ in Theorem B by $L[f](z)=c(z) \Rightarrow f(z)=d(z)$.

In this paper, we answer this question by the following result.

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The first author biography: WU Chun, male, associate professor, doctor, engaged in the study of value distribution theory, E-mail: wuchun@cqnu.edu.cn
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作者简介:吴春,男,副教授,博士,研究方向为值分布理论,E-mail:wuchun@cqnu.edu.cn
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Theorem 1　Let $\mathscr{F}$ be a family of meromorphic functions in a domain D, let $L[f]=a_0f'+a_1f(a_0\neq0)$, and let a,b,c,d be four holomorphic functions in D. For each $f\in\mathscr{F}$, if 1) $a(z)\neq d(z)$; 2) $b(z)+a_1(z)a(z)+a_0(z)a'(z)\neq 2c(z)$; 3) $c(z)-a_0(z)a'(z)-a_1(z)a(z)\neq0$; 4) $f(z)=a(z)\Leftrightarrow L[f](z)=b(z)$ and $L[f](z)=c(z)\Rightarrow f(z)=d(z)$, then $\mathscr{F}$ is normal in D.

If $a_0=1$ and $a_1=0$ in $L[f](z)$, we find that the condition 2) can reduce to $b(z)+a'(z)\neq 2c(z)$.

Thus, the following corollary is an immediately consequence of Theorem 1.

Corollary 1　Let $\mathscr{F}$ be a family of meromorphic functions in a domain D, let a,b,c,d be four holomorphic functions in D. For each $f\in\mathscr{F}$, if 1) $a(z)\neq d(z)$; 2) $b(z)+a'(z)\neq 2c(z)$; 3) $c(z)-a'(z)\neq0$; 4) $f(z)=a(z)\Leftrightarrow f'=b(z)$ and $f'=c(z)\Rightarrow f(z)=d(z)$, then $\mathscr{F}$ is normal in D.

2 Some Lemmas

Lemma 1　(Zalcman's Lemma)[8] Let $\mathscr{F}$ be a family of meromorphic functions in the unit disc Δ, all of whose zeros have multiplicity at least k, and suppose that there exists $A\geqslant1$ such that $|f^{(k)}(z)|\leqslant A$ whenever $f(z)=0$. If $\mathscr{F}$ is not normal at z_0 in the unit disc, then there exist, for each $0\leqslant\alpha\leqslant k$.

1) a real number r, $r<1$; 2) points z_n, $|z_n|<r$; 3) positive numbers ρ_n, $\rho_n\to0^+$; 4) functions f_n, $f_n\in\mathscr{F}$, such that $g_n(\xi)=\dfrac{f_n(z_n+\rho_n\xi)}{\rho_n^{\alpha}}\to g(\xi)$ locally uniformly, where g is a non-constant meromorphic function in **C**, all of whose zeros have multiplicity at least k, such that $g^{\#}(\xi)\leqslant g^{\#}(0)=kA+1$. In particular, g has order at most two.

Lemma 2[9]　Let g be a meromorphic function with finite order on **C**. If g has only finitely many critical values, then it has only finitely many asymptotic values.

Lemma 3[10]　Let $g(z)$ be a transcendental meromorphic function such that $g(0)\neq\infty$ and the set of finite on **C** such that the set of finite critical and asymptotic values of $g(z)$ is bounded. Then, there exists $R>0$, such that $|g'(z)|\geqslant\dfrac{|g(z)|}{2\pi|z|}\log\dfrac{|g(z)|}{R}$.

Lemma 4[4]　Suppose that $f(z)$ is meromorphic and transcendental in the plane. Then as $r\to\infty$, $T(r,f)\leqslant\left(2+\dfrac{1}{l}\right)N\left(r,\dfrac{1}{f}\right)+\left(2+\dfrac{2}{l}\right)\overline{N}\left(r,\dfrac{1}{f^{(l)}-1}\right)+S(r,f)$.

Lemma 5[11]　Let $f(z)=a_nz^n+a_{n-1}z^{n-1}+\cdots+a_0+\dfrac{q(z)}{p(z)}$, where $a_0,a_1,\cdots,a_n$ are constants with $a_n\neq0$, $q(z)$ and $p(z)$ are two coprime polynomials with $\deg q(z)<\deg p(z)$, k be a positive integer. If $f^{(k)}\neq1$, then we have: i) $n=k$, and $k!\ a_k=1$; ii) $f(z)=\dfrac{1}{k!}z^k+\cdots+a_0+\dfrac{1}{(az+b)^m}$; iii) If the zeros of f are of order $\geqslant k+1$, then $m=1$ in ii) and $f(z)=\dfrac{(cz+d)^{k+1}}{az+b}$, where $c(\neq0)$, d are constants.

Lemma 6[12]　Let f be a rational function such that $f'\neq0$ on **C**. Then, either $f=az+b$ or $f=\dfrac{a}{(z+z_0)^n}+b$, where $a(\neq0)$, b and z_0 are constants, and n is a positive integer.

Lemma 7　Let $\mathscr{F}$ be a family of meromorphic functions in a domain D, a and b be distinct complex numbers and $a\neq 2b$. If for $f\in\mathscr{F}$, $f(z)=0\Leftrightarrow f'(z)=a$ and $f'(z)\neq b$, then $\mathscr{F}$ is normal in D.

Proof　Suppose that $\mathscr{F}$ is not normal in D, then there exists $z_0\in D$, such that $\mathscr{F}$ is not normal at z_0. By Lemma 1, for $A=\max\{|a|,|b|\}$, there exist a sequence of function $f_n\in\mathscr{F}$, a sequence of complex numbers $z_n\to z_0$ and a sequence of positive numbers $\rho_n\to0$, such that $g_n(\xi)=\rho_n^{-1}f_n(z_n+\rho_n\xi)\to g(\xi)$, spherically locally uniformly in **C**, where $g(\xi)$ is a non-constant meromorphic function. Moreover, $g(\xi)$ is of order at most 2, and $g^{\#}(\xi)\leqslant g^{\#}(0)=A+1$ for all $\xi\in$ **C**.

We claim: i) $g(\xi)=0\Leftrightarrow g'(\xi)=a$, ii) $g'(\xi)\neq b$.

Suppose that $g(\xi_0)=0$. Then by Hurwitz's theorem, there exist ξ_n, $\xi_n\to\xi_0$, such that $g_n(\xi)=\rho_n^{-1}f_n(z_n+\rho_n\xi_n)=0$ (for n sufficiently large). Thus $f_n'(z_n+\rho_n\xi_n)=a$, in the limit as $n\to\infty$ we obtain $g'(\xi_0)=a$, This is

$g(\xi)=0\Rightarrow g'(\xi)=a$.

Suppose now that $g'(\xi)=a$, We claim that $g'(\xi)\not\equiv a$, Indeed, If $g'(\xi)\equiv a$, we have $g(\xi)=a\xi+C$, where C is a constant. By a simple calculation, we obtain $g^{\#}(0)\leqslant|a|\leqslant A<A+1$, which contradict that $g^{\#}(0)=A+1$.

Since $g'(\xi)=a$ but $g'(\xi)\not\equiv a$ and $f'_n(z_n+\rho_n\xi)-a\Rightarrow g'(\xi)-a$ on some neighborhood of the point ξ_0, $\xi_n\rightarrow\xi_0$, such that $f'_n(z_n+\rho_n\xi)=a$. Thus, $f_n(z_n+\rho_n\xi_n)=0$. It is easy to deduce that $g(\xi_0)=0$. Thus, we prove i).

Next we prove ii). Suppose $g'(\xi_0)=b$, We claim that $g'(\xi)\not\equiv b$. Indeed, If $g'(\xi)\equiv b$, we have $g(\xi)=b\xi+C$, where C is a constant. By a simple calculation, we obtain $g^{\#}(0)\leqslant|b|\leqslant A<A+1$, which contradict that $g^{\#}(0)=A+1$.

Thus, by Hurwitz's theorem, there exists a sequence $\xi_n\rightarrow\xi_0$, such that $f'_n(z_n+\rho_n\xi_n)=b$. Since $f'_n(z)\neq b$, we have $f'_n(z_n+\rho_n\xi_n)\neq b$, which is a contradiction. Thus, ii) is proved.

Suppose that g is not a rational function, by Lemma 4 and $g'(\xi)\neq b$, we know g must have infinitely many zeros $\{\xi_n\}$, and $\xi_n\rightarrow\infty(n\rightarrow\infty)$. Let $h(\xi)=g(\xi)-b\xi$, then $h'(\xi)=g'(\xi)-b\neq 0$. It is easy to see that $h(\xi)$ is a meromorphic function with finite order. Thus, from Lemma 2 and Lemma 3, there exists $R>0$ such that $\frac{|\xi_n h'(\xi_n)|}{|h(\xi_n)|}\geqslant\frac{1}{2\pi}\log\frac{|h(\xi_n)|}{R}=\frac{1}{2\pi}\log\frac{|b\xi_n|}{R}\rightarrow\infty, n\rightarrow\infty$, which contradicts with $\frac{|\xi_n h'(\xi_n)|}{|h(\xi_n)|}=\left|\frac{a-b}{b}\right|$.

Thus, $g(\xi)$ is a rational function.

Next, we consider the following two cases.

Case 1 If $b=0$, from $g'\neq b$ and Lemma 6, we have $g=\alpha\xi+\beta$ or $g=\frac{\alpha}{(\xi+\xi_0)^n}+\beta$, where α,β,ξ_0 are constants, and n is a positive integer. Suppose $g=\alpha\xi+\beta$, then $g=0$ has a single zero $\xi=\frac{-\beta}{\alpha}$, meanwhile $g'=\alpha$ has infinitely many zeros. It contradicts with $g(\xi)=0\Leftrightarrow g'(\xi)=a$. Thus, $g=\frac{\alpha}{(\xi+\xi_0)^n}+\beta$. If $a=0$, which contradicts with that a and b are distinct constants. If $a\neq 0$, then the number of zeros of $g=0$ is at most n, meanwhile the number of zeros of $g'=\frac{-n\alpha}{(\xi+\xi_0)^{n+1}}=a$ is $n+1$, which contradicts with $g(\xi)=0\Leftrightarrow g'(\xi)=a$.

Case2 If $b\neq 0$, we distinguish three cases.

Case2.1 If g is a polynomial. From $g'\neq b$, we have $g=\alpha\xi+\beta$, where $\alpha(\neq 0)$, $\beta(\neq b)$ are constants, in the same manner as above, we get a contradiction.

Case2.2 If $g=\frac{q(\xi)}{p(\xi)}$, where $q(\xi)$ and $p(\xi)$ are coprime polynomials, and $\deg q(\xi)<\deg p(\xi)$. By a simple calculation, we obtain that 0 is the only one deficiency value of $g'=\frac{q'p-p'q}{p^2}$, which contradicts with $g'\neq b$, $b\neq 0$.

Case2.3 If $g(z)=a_n\xi^n+a_{n-1}\xi^{n-1}+\cdots+\xi_0+\frac{q(\xi)}{p(\xi)}$, where $a_0,a_1,\cdots,a_n$ are constants with $a_n\neq 0$, q and p are two coprime polynomials with $\deg q<\deg p$, and n is a positive integer. Because g is a rational function, by Lemma 5, we have

$$g(z)=b\xi+a_0+\frac{A}{(c\xi+d)^m}, g'(\xi)=b-\frac{Acm}{(c\xi+d)^{m+1}}, \tag{1}$$

where $a_0, c(\neq 0), d, A\neq 0$ are constants, m is a positive integer. Suppose $a=0$, then from i), we obtain that the multiplicity of zeros of g is at least 2. Thus, the roots number of $g=0$ is at most $\frac{m+1}{2}$, and the roots are different from each other, meanwhile $g'=0$ has $m+1$ distinct roots, which contradicts with i). Thus $a\neq 0$, from i), we know that the roots of $g=0$ and $g'=a$ are all simple. Therefore, $\frac{g}{g'-a}$ is a entire function and has unique zero $-\frac{d}{c}$ which is the pole of g. So

$$\frac{g}{g'-a}\equiv k\,(cz+d)^l, \tag{2}$$

where k is a nonconstant and l is a positive integer.

From (1) and (2), we get

$$\frac{b}{c}(c\xi+d)^{m+2}+\left(a_0-\frac{bd}{c}\right)(c\xi+d)^{m+1}+A(c\xi+d)\equiv(b-a)k\,(c\xi+d)^{m+l+1}-Acmk\,(c\xi+d)^{l}. \tag{3}$$

According to (3), we obtain $l=1, a_0-\frac{bd}{c}=0$, and $(m+1)b=a(m=1,2,\cdots)$. Especially, if $m=1$, we have $a=2b$, which contradicts the assumption.

Therefore, $\mathscr{F}$ is normal in D. Lemma 7 is proved completely.

3 Proof of Theorems

3.1. Proof of Theorem 1

Since normality is a local property, we assume that $D=\Delta$, the unit disc. Set $\mathscr{F}_1=\{F:F=f-a, f\in\mathscr{F}\}$, In view of the form of $L[f]$ and the assumption, we can easily deduce $F(z)=0\Leftrightarrow F'(z)=\varphi(z)$, where $\varphi(z)=\frac{b-a_1a-a_0a'}{a_0}$.

Suppose that $\mathscr{F}$ is not normal at $z_0\in\Delta$. Let $M=\max_{z\in D_1}\{|\varphi(z)|\}$, and $A=\max\{M,1\}$, where $r>0$ is a constant and $D_1=\{z:|z-z_0|\leqslant r\}\subset\Delta$. Then, in domain $D_2=\left\{z:|z-z_0|<\frac{r}{2}\right\}$, we have $|F'(z)|=|\varphi(z)|\leqslant A$ when $f(z)=0$. Since $\mathscr{F}$ is not normal at $z_0\in\Delta$, we have $\mathscr{F}_1$ is not normal at $z_0\in\Delta$ as well. Then by Lemma 1, there exist a sequence of function $f_n\in\mathscr{F}$, a sequence of complex numbers $z_n\to z_0$ and a sequence of positive numbers $\rho_n\to 0$, such that $g_n(\xi)=\rho_n^{-1}[f_n(z_n+\rho_n\xi)-a(z_n+\rho_n\xi)]\to g(\xi)$ converges uniformly to a non-constant meromorphic function $g(\xi)$ in $\mathbf{C}$ with respect to the spherical metric. Moreover, $g(\xi)$ is of order at most 2, and $g^{\#}(\xi)\leqslant g^{\#}(0)=A+1$ for all $\xi\in\mathbf{C}$.

In the following, we claim that i) $g(\xi)=0\Leftrightarrow g'(\xi)=\varphi(z_0)$, where $\varphi(z_0)=\frac{b(z_0)-a_0(z_0)a'(z_0)-a_1(z_0)a(z_0)}{a_0(z_0)}$; ii) $g'(\xi)\neq\lambda$. where $\lambda=\frac{c(z_0)-a_0(z_0)a'(z_0)-a_1(z_0)a(z_0)}{a_0(z_0)}\neq 0$ is a constant.

Next, we prove the claim as follows.

We assume that $g(\xi_0)=0$. Then by Hurwitz's theorem, there exist ξ_n, $\xi_n\to\xi_0$, such that

$$g_n(\xi_n)=\rho_n^{-1}[f_n(z_n+\rho_n\xi_n)-a(z_n+\rho_n\xi_n)]=0. \tag{4}$$

Thus $L[f_n](z_n+\rho_n\xi_n)=b(z_n+\rho_n\xi_n)$.

Since
$$\frac{L[f_n](z_n+\rho_n\xi_n)}{a_0(z_n+\rho_n\xi_n)}=f'_n(z_n+\rho_n\xi_n)+\frac{a_1(z_n+\rho_n\xi_n)}{a_0(z_n+\rho_n\xi_n)}f_n(z_n+\rho_n\xi_n)=$$
$$f'_n(z_n+\rho_n\xi_n)+\frac{a_1(z_n+\rho_n\xi_n)}{a_0(z_n+\rho_n\xi_n)}[\rho_n g_n(\xi_n)+a(z_n+\rho_n\xi_n)]\to g'(\xi_0)+a'(z_0)+\frac{a_1(z_0)}{a_0(z_0)}a(z_0). \tag{5}$$

Then, from (5), we have

$$g'(\xi_0)=\lim_{n\to\infty}\frac{L[f_n](z_n+\rho_n\xi_n)}{a_0(z_n+\rho_n\xi_n)}-a'(z_0)-\frac{a_1(z_0)}{a_0(z_0)}a(z_0)=\lim_{n\to\infty}\frac{b(z_n+\rho_n\xi_n)}{a_0(z_n+\rho_n\xi_n)}-a'(z_0)-\frac{a_1(z_0)}{a_0(z_0)}a(z_0)=$$
$$\frac{b(z_0)-a_0(z_0)a'(z_0)-a_1(z_0)a(z_0)}{a_0(z_0)}=\varphi(z_0). \tag{6}$$

This is $g(\xi)=0\Rightarrow g'(\xi)=\varphi(z_0)$.

Suppose now that $g'(\xi)=\varphi(z_0)$. We claim that $g'(\xi)\not\equiv\varphi(z_0)$. Indeed, If $g'(\xi)\equiv\varphi(z_0)$, we have $g(\xi)=\varphi(z_0)\xi+C$, where C is a constant. By a simple calculation, we obtain $g^{\#}(0)\leqslant|\varphi(z_0)|\leqslant A<A+1$, which contradict that $g^{\#}(0)=A+1$.

Since $g'(\xi)=\varphi(z_0)$ but $g'(\xi)\not\equiv\varphi(z_0)$ and $\frac{L[f_n](z_n+\rho_n\xi)-b(z_n+\rho_n\xi)}{a_0(z_n+\rho_n\xi)}\Rightarrow g'(\xi)-\varphi(z_0)$ on some neighborhood of the point ξ_0, $\xi_n\to\xi_0$, such that $L[f_n(z_n+\rho_n\xi_n)]=b(z_n+\rho_n\xi_n)$. Thus, $f_n(z_n+\rho_n\xi_n)=a(z_n+\rho_n\xi_n)$. It is easy to deduce that $g(\xi_0)=0$. Thus, we prove i).

Now, we prove ii). From (5), there exist ξ_n, $\xi_n \to \xi$, we get

$$\frac{L[f_n](z_n+\rho_n\xi_n)-c(z_n+\rho_n\xi_n)}{a_0(z_n+\rho_n\xi_n)}=f_n'(z_n+\rho_n\xi_n)+\frac{a_1(z_n+\rho_n\xi_n)}{a_0(z_n+\rho_n\xi_n)}[\rho_n g_n(\xi_n)+a(z_n+\rho_n\xi_n)]-\frac{c(z_n+\rho_n\xi_n)}{a_0(z_n+\rho_n\xi_n)}\to$$

$$g'(\xi)+a'(z_0)+\frac{a_1(z_0)}{a_0(z_0)}a(z_0)-\frac{c(z_0)}{a_0(z_0)}=g'(\xi)-\frac{c(z_0)-a_0(z_0)a'(z_0)-a_1(z_0)a(z_0)}{a_0(z_0)}=g'(\xi)-\lambda. \qquad (7)$$

Suppose $g'(\xi_0)=\lambda$, We claim that $g'(\xi)\not\equiv\lambda$. Indeed, If $g'(\xi)\equiv\lambda$, we have $g(\xi)=\lambda\xi+C$, where C is a constant. By a simple calculation, we obtain $g^{\#}(0)\leqslant|\lambda|\leqslant A<A+1$, which contradict that $g^{\#}(0)=A+1$.

Thus, by Hurwitz's theorem and (7), there exists a sequence $\xi_n\to\xi_0$, such that $L[f_n(z_n+\rho_n\xi_n)]-c(z_n+\rho_n\xi_n)=0$.

From $L[f](z)=c(z)\Rightarrow f(z)=d(z)$, we have $f_n(z_n+\rho_n\xi_n)=d(z_n+\rho_n\xi_n)$. From (4) and $a(z)\neq d(z)$, we deduce

$$g(\xi_0)=\lim_{n\to\infty}\rho_n^{-1}[f_n(z_n+\rho_n\xi_n)-a(z_n+\rho_n\xi_n)]=\lim_{n\to\infty}\rho_n^{-1}[d(z_n+\rho_n\xi_n)-a(z_n+\rho_n\xi_n)]=\infty \qquad (8)$$

a contradiction. Thus, ii) is proved.

By i), ii) and Lemma 7, we can obtain $\mathscr{F}$ is normal in D.

Theorem 1 is proved completely.

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分担全纯函数的亚纯函数的正规族

吴 春

(重庆师范大学 数学学院，重庆 401331)

摘要：主要研究了亚纯函数分担全纯函数的正规族问题，证明了：如果 $\mathscr{F}$ 是区域 D 上的亚纯函数族，且满足 $L[f]=a_0f'+a_1f(a_0\neq 0)$，$a,b,c,d$ 为 D 上的 4 个全纯函数。如果对任意的 $f\in\mathscr{F}$，满足 $a(z)\neq d(z)$，$b(z)+a_1(z)a(z)+a_0(z)a'(z)\neq 2c(z)$，$c(z)-a_0(z)a'(z)-a_1(z)a(z)\neq 0$，$f(z)=a(z)\Leftrightarrow L[f](z)=b(z)$ 且 $L[f](z)=c(z)\Rightarrow f(z)=d(z)$，则 $\mathscr{F}$ 在 D 正规。

关键词：亚纯函数；正规族；分担函数

（责任编辑　黄　颖）