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Existence and Uniqueness of Positive Solution for Nonlinear Fractional Differential Equation Boundary Value Problem*

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Abstract: By using a fixed point theorem of a sum operator, the positive solution of nonlinear fractional differential equation boundary value problem: $\begin{cases} -D_{0+}^{\alpha}u(t)=f(t,u(t)),\ 0<t<1,\ 2<\alpha\leqslant 3 \\ u(0)=u'(0)=u'(1)=0 \end{cases}$ is studied, where D_{0+}^{α} is the standard Riemann-Liouville fractional derivative and $f(t,u(t))=g(t,u(t))+h(t,u(t))$ and $g,h:[0,1]\times[0,\infty)\to[0,\infty)$ are continuous and increasing with respect to the second argument. Its existence and uniqueness is proved, and an iterative scheme is constructed to approximate it. Finally, the example is given to illustrate the result.

Key words: fractional differential equation; boundary value problem; positive solution; existence and uniqueness; fixed point theorem of a sum operator

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1 Introduction

Fractional differential equations are used in mechanics, physics, chemistry, engineering, economics and biological sciences fields, etc.[1-9]. Recently, there are many papers discuss positive solutions for nonlinear fractional differential equation boundary value problem. Its existence and multiplicity is studied by using of Leray-Shauder theory, fixed-point theorems, etc.[10-14]. However, there are few papers consider its existence and uniqueness[15].

In this paper, we consider the existence and uniqueness of positive solution for nonlinear fractional differential equation boundary value problem:

$$-D_{0+}^{\alpha}u(t)=f(t,u(t)),0<t<1,2<\alpha\leqslant 3, \tag{1}$$

$$u(0)=u'(0)=u'(1)=0. \tag{2}$$

where $f(t,u(t))=g(t,u(t))+h(t,u(t))$,and D_{0+}^{α} is the standard Riemann-Liouville fractional derivative.

When $h(t,u(t))\equiv 0$, Zhao Y, Sun S and Han Z[10] investigated the positive solutions for the problem (1) and (2). Its existence is proved by means of the lower and upper solution method and fixed-point theorem. They

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present the following result.

Theorem 1[10] The fractional boundary value problem (1) and (2) has a positive solution u if the following condition is satisfied: (H_f) $f(t,u)\in C([0,1]\times[0,+\infty),\mathbf{R}^+)$ is nondecreasing with respect to u, $f(t,\rho(t))\not\equiv 0$ for $t\in(0,1)$ and there exists a positive constant $\mu<1$ such that $k^{\mu}f(t,u)\leqslant f(t,ku)$, $\forall\, 0\leqslant k\leqslant 1$, where $\rho(t)=\int_0^1 G(t,s)\mathrm{d}s$ and $G(t,s)=\begin{cases}\dfrac{t^{\alpha-1}(1-s)^{\alpha-2}-(t-s)^{\alpha-1}}{\Gamma(\alpha)}, & 0\leqslant s\leqslant t\leqslant 1\\ \dfrac{t^{\alpha-1}(1-s)^{\alpha-2}}{\Gamma(\alpha)}, & 0\leqslant t\leqslant s\leqslant 1\end{cases}$.

Theorem 2[10] Suppose $f(t,u)$ is continuous on $[0,1]\times[0,+\infty)$ and there exist constants $0<a<b<c$ such that the following assumptions hold: (B1) $f(t,u)<Ma$, for $(t,u)\in[0,1]\times[0,a]$; (B2) $f(t,u)\geqslant Nb$, for $(t,u)\in[1/4,3/4]\times[b,c]$; (B3) $f(t,u)\leqslant Mc$, for $(t,u)\in[0,1]\times[0,c]$.

Then the boundary value problem (1) and (2) has at least three positive solutions u_1, u_2, u_3 with

$$\max_{0\leqslant t\leqslant 1}|u_1(t)|<a,\ b<\min_{\frac14\leqslant t\leqslant\frac34}|u_2(t)|<\max_{0\leqslant t\leqslant 1}|u_2(t)|\leqslant c,\ a<\max_{0\leqslant t\leqslant 1}|u_3(t)|\leqslant c,\ \min_{\frac14\leqslant t\leqslant\frac34}|u_3(t)|<b.$$

In this study, our work is to extend and improve the main results of the paper [10]. By means of a fixed point theorem for a sum operator, we get the existence and uniqueness of positive solutions for problem (1) and (2). Meanwhile, an iterative scheme is constructed to approximate this unique solution.

2 Preliminaries and previous results

Definition 1[3] The integral $I_{0+}^{\alpha}f(x)=\dfrac{1}{\Gamma(\alpha)}\displaystyle\int_0^x\frac{f(t)}{(x-t)^{1-\alpha}}\mathrm{d}t$, $x>0$, where $\alpha>0$ and $\Gamma(\alpha)$ denotes the gamma function, is called the Riemann-Liouville fractional integral of orderα.

Definition 2[3] For a function $f(x)$ given in the interval $[0,\infty)$, the expression

$$D_{0+}^{\alpha}f(x)=\frac{1}{\Gamma(n-\alpha)}\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^n\int_0^x\frac{f(t)}{(x-t)^{\alpha-n+1}}\mathrm{d}t,$$

where $n=[\alpha]+1$, $[\alpha]$ denotes the integer part of number α, is called the Riemann-Liouville fractional derivative of order α.

Lemma 1[10] Let $y\in C[0,1]$ and $2<\alpha\leqslant 3$. The unique solution of problem

$$-D_{0+}^{\alpha}u(t)=y(t),\ 0<t<1, \tag{3}$$

$$u(0)=u'(0)=u'(1)=0, \tag{4}$$

is $u(t)=\int_0^1 G(t,s)y(s)\mathrm{d}s$, $t\in[0,1]$, where

$$G(t,s)=\begin{cases}\dfrac{t^{\alpha-1}(1-s)^{\alpha-2}-(t-s)^{\alpha-1}}{\Gamma(\alpha)}, & 0\leqslant s\leqslant t\leqslant 1\\ \dfrac{t^{\alpha-1}(1-s)^{\alpha-2}}{\Gamma(\alpha)}, & 0\leqslant t\leqslant s\leqslant 1\end{cases}。 \tag{5}$$

Here $G(t,s)$ is called the Green function.

Lemma 2 The Green function $G(t,s)$ in Lemma1 has the following property:

$$\frac{1}{\Gamma(\alpha)}t^{\alpha-1}(1-s)^{\alpha-2}s\leqslant G(t,s)\leqslant\frac{1}{\Gamma(\alpha)}t^{\alpha-1}(1-s)^{\alpha-2}\ \text{for}\ t,s\in(0,1). \tag{6}$$

Proof Evidently, the right inequality holds. So we only show that the left inequality.

If $0\leqslant s\leqslant t\leqslant 1$, then we have $0\leqslant t-s\leqslant t-ts=(1-s)t$, and thus $(t-s)^{\alpha-1}\leqslant(1-s)^{\alpha-1}t^{\alpha-1}$.

Hence $$G(t,s)=\frac{1}{\Gamma(\alpha)}[t^{\alpha-1}(1-s)^{\alpha-2}-(t-s)^{\alpha-1}]\geqslant\frac{1}{\Gamma(\alpha)}[t^{\alpha-1}(1-s)^{\alpha-2}-t^{\alpha-1}(1-s)^{\alpha-1}]=$$

$$\frac{1}{\Gamma(\alpha)}[(1-s)^{\alpha-2}-(1-s)^{\alpha-1}]t^{\alpha-1}=\frac{1}{\Gamma(\alpha)}t^{\alpha-1}(1-s)^{\alpha-2}s.$$

If $0\leqslant t\leqslant s\leqslant 1$, then we have $G(t,s)=\frac{1}{\Gamma(\alpha)}t^{\alpha-1}(1-s)^{\alpha-2}\geqslant\frac{1}{\Gamma(\alpha)}t^{\alpha-1}(1-s)^{\alpha-2}s$.

So the left inequality also holds.

A non-empty closed convex set $P\subset E$ is a cone if it meets: i) $x\in P,\lambda\geqslant 0\Rightarrow\lambda x\in P$; ii) $x\in P,-x\in P\Rightarrow x=\theta$. Suppose $(E,\|\cdot\|)$ is a order Banach space, if a cone $P\subset E$, i. e. $x\leqslant y$ if and only if $y-x\in P$. If $x\leqslant y$ and $x\neq y$, then we denote $x<y$. We denote the zero element of E by θ.

Putting $P^0=\{x\in P|x$ is an interior point of $P\}$, a cone P is said to be solid if P^0 is non-empty. If there is a positive constant $N>0$ such that, for all $x,y\in E,\theta\leqslant x\leqslant y$ implies $\|x\|\leqslant N\|y\|$, P is called normal; N is called the normality constant of P.

If $x\leqslant y$ implies $Ax\leqslant Ay$, we say that an operator $A:E\to E$ is increasing.

For all $x,y\in E$, the notation $x\sim y$ means that there exist $\lambda>0$ and $\mu>0$ such that $\lambda x\leqslant y\leqslant\mu x$. Clearly $\sim$ is an equivalence relation. Given $w>\theta$ (i. e. $w\geqslant\theta$ and $w\neq\theta$), we denote the set $P_w=\{x\in E|x\sim w\}$ by P_w. It is easy to see that $P_w\subset P$ for $w\in P$.

Theorem 3[16] Let P be a normal cone in a real Banach space E, $A:P\to P$ be an increasing γ-concave operator and $B:P\to P$ be an increasing sub-homogeneous operator. Assume that:

(i) there is $w>\theta$ such that $Aw\in P_w$ and $Bw\in P_w$;

(ii) there exists a constant $\delta_0>0$ such that $Ax\geqslant\delta_0 Bx$, $\forall x\in P$.

Then operator equation $Ax+Bx=x$ has a unique solution $x*$ in P_w. Moreover, constructing successively the sequence $y_n=Ay_{n-1}+By_{n-1}, n=1,2,\cdots$ for any initial value $y_0\in P_w$, we have $y_n\to x^*$ as $n\to\infty$.

Remark 1 When B is a null operator, Theorem 3 also holds.

3 Main results

In this section, we apply Theorem 3 to investigate the problem (1) and (2), the new result on the existence and uniqueness of positive solution is obtained.

In this paper, we will work in the Banach space $C[0,1]=\{x:[0,1]\to\mathbf{R}$ is continuous$\}$ with the standard norm $\|x\|=\sup\{|x(t)|:t\in[0,1]\}$. Notice that this space can be endowed with a partial order given by $x,y\in C[0,1]$, $x\leqslant y\Leftrightarrow x(t)\leqslant y(t)$ for $t\in[0,1]$.

Let $P=\{x\in C[0,1]|x(t)\geqslant 0,t\in[0,1]\}$ be the standard cone. Evidently, $P$is a normal cone in $C[0,1]$ and the normality constant is 1.

Theorem 4 Assume that

(H1) $g,h:[0,1]\times[0,\infty)\to[0,\infty)$ are continuous and increasing with respect to the second argument, $h(t,0)\not\equiv 0$;

(H2) there exists a constant $\gamma\in(0,1)$ such that $g(t,\lambda x)\geqslant\lambda^\gamma g(t,x)$, $\forall t\in[0,1],\lambda\in(0,1),x\in[0,\infty)$ and $h(t,\mu x)\geqslant\mu h(t,x)$ for $\mu\in(0,1),t\in[0,1],x\in[0,\infty)$;

(H3) there exists a constant $\delta_0>0$ such that $g(t,x)\geqslant\delta_0 h(t,x),t\in[0,1],x\geqslant 0$.

Then problem (1) and (2) has a unique positive solution u^* in P_w, where $w(t)=t^{\alpha-1}$, $t\in[0,1]$. Moreover, for any initial value $u_0\in P_w$, constructing successively the iterative scheme

$$u_{n+1}(t)=\int_0^1 G(t,s)f(s,u_n(s))\mathrm{d}s, n=0,1,2,\cdots,$$

we have $u_n(t)\to u^*(t)$ as $n\to\infty$, where $G(t,s)$ is given as (5).

Proof To begin with, from Lemma 1, the problem (1) and (2) has an integral formulation given by

$$u(t)=\int_0^1 G(t,s)f(s,u(s))\mathrm{d}s=\int_0^1 G(t,s)[g(s,u(s))+h(s,u(s))]\mathrm{d}s,$$

where $G(t,s)$ is given as in Lemma 1.

Define two operators $A:P\to E$ and $B:P\to E$ by

$$Au(t)=\int_0^1 G(t,s)g(s,u(s))\mathrm{d}s, Bu(t)=\int_0^1 G(t,s)h(s,u(s))\mathrm{d}s.$$

It is easy to prove that u is the solution of problem (1) and (2) if and only if $u=Au+Bu$.

By assumption (H1) and Lemma 2, we know that $A:P\to P$ and $B:P\to P$. In the sequel we check that A,B satisfy all assumptions of Theorem 3.

Firstly, we prove that A,B are two increasing operators.

In fact, from (H1) and Lemma 2, for $u,v\in P$ with $u\geqslant v$, we know that $u(t)\geqslant v(t), t\in[0,1]$ and obtain $Au(t)=\int_0^1 G(t,s)g(s,u(s))\mathrm{d}s\geqslant\int_0^1 G(t,s)g(s,v(s))\mathrm{d}s=Av(t)$. That is, $Au\geqslant Av$. Similarly, $Bu\geqslant Bv$.

Next we show that A is a γ-concave operator and B is a sub-homogeneous operator.

In fact, for any $\lambda\in(0,1)$ and $u\in P$, from (H2) we know that

$$A(\lambda u)(t)=\int_0^1 G(t,s)g(s,\lambda u(s))\mathrm{d}s\geqslant\lambda^\gamma\int_0^1 G(t,s)g(s,u(s))\mathrm{d}s=\lambda^\gamma Au(t).$$

That is, $A(\lambda u)\geqslant\lambda^\gamma Au$ for $\lambda\in(0,1), u\in P$. So the operator A is a γ-concave operator. Also, for any $\mu\in(0,1)$ and $u\in P$, by (H2) we obtain

$$B(\mu u)(t)=\int_0^1 G(t,s)h(s,\mu u(s))\mathrm{d}s\geqslant\mu\int_0^1 G(t,s)h(s,u(s))\mathrm{d}s=\mu Bu(t),$$

That is, $B(\mu u)\geqslant\mu Bu$ for $\mu\in(0,1), u\in P$. So the operator B is a sub-homogeneous operator.

Now we show that $Aw\in P_w$ and $Bw\in P_w$, where $w(t)=t^{\alpha-1}$.

By (H1) and Lemma 2,

$$Aw(t)=\int_0^1 G(t,s)g(s,w(s))\mathrm{d}s\leqslant\frac{1}{\Gamma(\alpha)}w(t)\int_0^1(1-s)^{\alpha-2}g(s,1)\mathrm{d}s,$$

$$Aw(t)=\int_0^1 G(t,s)g(s,w(s))\mathrm{d}s\geqslant\frac{1}{\Gamma(\alpha)}w(t)\int_0^1 s(1-s)^{\alpha-2}g(s,0)\mathrm{d}s.$$

From (H1) and (H3), we have $g(s,1)\geqslant g(s,0)\geqslant\delta_0 h(s,0)\geqslant0$.

Since $h(t,0)\not\equiv0$, we can get $\int_0^1 g(s,1)\mathrm{d}s\geqslant\int_0^1 g(s,0)\mathrm{d}s\geqslant\delta_0\int_0^1 h(s,0)\mathrm{d}s>0$, and in consequence,

$$l_1:=\frac{1}{\Gamma(\alpha)}\int_0^1 s(1-s)^{\alpha-2}g(s,0)\mathrm{d}s>0, l_2:=\frac{1}{\Gamma(\alpha)}\int_0^1(1-s)^{\alpha-2}g(s,1)\mathrm{d}s>0.$$

So $l_1 w(t)\leqslant Aw(t)\leqslant l_2 w(t)$, $t\in[0,1]$; and hence we have $Aw\in P_w$. Similarly,

$$\frac{1}{\Gamma(\alpha)}w(t)\int_0^1 s(1-s)^{\alpha-2}h(s,0)\mathrm{d}s\leqslant Bw(t)\leqslant\frac{1}{\Gamma(\alpha)}w(t)\int_0^1(1-s)^{\alpha-2}h(s,1)\mathrm{d}s,$$

From $h(t,0)\not\equiv0$, we easily prove $Bw\in P_w$. Hence the condition (i) of Theorem 3 is satisfied.

For $u\in P$, by (H3), $Au(t)=\int_0^1 G(t,s)g(s,u(s))\mathrm{d}s\geqslant\delta_0\int_0^1 G(t,s)h(s,u(s))\mathrm{d}s=\delta_0 Bu(t)$. Then we get $Au\geqslant\delta_0 Bu, u\in P$.

Finally, by means of Theorem 3, the operator equation $Au+Bu=u$ has a unique positive solution u^* in P_w. Moreover, constructing successively the iterative scheme $u_n=Au_{n-1}+Bu_{n-1}$, $n=1,2,\cdots$ for any initial value $u_0\in P_w$, we have $u_n\to u^*$ as $n\to\infty$. That is, problem (1) and (2) has a unique positive solution u^* in P_w. For any initial value $u_0\in P_w$, constructing successively the iterative scheme

$$u_{n+1}(t)=\int_0^1 G(t,s)f(s,u_n(s))\mathrm{d}s, n=0,1,2,\cdots,$$

we have $u_n\to u^*$ as $n\to\infty$.

Corollary 1　When $h(t,u(t))\equiv0$, assume that

(H4) $g:[0,1]\times[0,\infty)\to[0,\infty)$ is continuous and increasing with respect to the second argument, $g(t,0)\not\equiv0$;

(H5) there exists a constant $\gamma\in(0,1)$ such that $g(t,\lambda x)\geqslant\lambda^\gamma g(t,x)$, $\forall t\in[0,1], \lambda\in(0,1), x\in[0,\infty)$.

Then problem

$$\begin{cases}-D_{0+}^{\alpha}u(t)=f(t,u(t)),0<t<1,2<\alpha\leqslant 3\\ u(0)=u'(0)=u'(1)=0\end{cases}$$

has a unique positive solution u^* in P_w, where $w(t)=t^{\alpha-1},t\in[0,1]$. Moreover, for any initial value $u_0\in P_w$, constructing successively the iterative scheme

$$u_{n+1}(t)=\int_0^1 G(t,s)f(s,u_n(s))\mathrm{d}s,n=0,1,2,\cdots,$$

we have $u_n(t)\to u^*(t)$ as $n\to\infty$, where $G(t,s)$ is given as (5).

Remark 2 By Remark 1 and Theorem 4, Corollary 1 is obvious. The unique positive solution is not considered in Theorem 1 and Theorem 2; Corollary 1 gives the existence and uniqueness. Moreover, the unique positive solution u^* we obtain satisfies: (i) there exist $\lambda>\mu>0$ such that $\mu t^{\alpha-1}\leqslant u^*\leqslant\lambda t^{\alpha-1},t\in[0,1]$, (ii) we can take any initial value in P_w and then construct an iterative scheme which can approximate the unique solution.

4 Example

We present one example to illustrate Theorem4.

Example 1 Consider the following problem:

$$\begin{cases}-D_{0+}^{\frac{5}{2}}u(t)=u^{\frac{1}{5}}(t)+\arctan u(t)+t^2+t+\dfrac{\pi}{2}\\ u(0)=u'(0)=u'(1)=0\end{cases}. \tag{7}$$

In this example, we have $\alpha=\dfrac{5}{2}$. Let $g(t,u)=u^{\frac{1}{5}}(t)+t+\dfrac{\pi}{2},h(t,u)=\arctan u(t)+t^2,\gamma=\dfrac{1}{5}$. Obviously, $g,h:[0,1]\times[0,\infty)\to[0,\infty)$ are continuous and increasing with respect to the second argument, $h(t,0)=t^2\not\equiv 0$. Besides, for $t\in[0,1],\lambda\in(0,1),x\in[0,\infty)$, we have

$$g(t,\lambda u)=\lambda^{\frac{1}{5}}u^{\frac{1}{5}}(t)+t+\frac{\pi}{2}\geqslant\lambda^{\frac{1}{5}}u^{\frac{1}{5}}(t)+\lambda^{\frac{1}{5}}\left(t+\frac{\pi}{2}\right)=\lambda^{\frac{1}{5}}\left(u^{\frac{1}{5}}(t)+t+\frac{\pi}{2}\right)=\lambda^{\gamma}g(t,u);$$

and for $t\in[0,1],\mu\in(0,1),x\in[0,\infty)$, we have $\arctan(\mu u)\geqslant\mu\arctan u$, and thus $h(t,\mu u)\geqslant\mu h(t,u)$.

Moreover, if we take $\delta_0\in(0,1]$, then we obtain

$$g(t,u)=u^{\frac{1}{5}}(t)+t+\frac{\pi}{2}\geqslant t+\frac{\pi}{2}\geqslant t^2+\arctan u\geqslant\delta_0(t^2+\arctan u)=\delta_0 h(t,u).$$

Hence all the conditions of Theorem 4 are satisfied. An application of Theorem 4 implies that problem (7) has a unique positive solution in P_w, where $w(t)=t^{\frac{3}{2}}$, $t\in[0,1]$.

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非线性分数阶微分方程边值问题正解的存在性与唯一性

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摘要：利用和算子的不动点定理，研究了非线性分数阶微分方程边值问题：$\begin{cases}-D_{0+}^{\alpha}u(t)=f(t,u(t)),0<t<1,2<\alpha\leqslant 3\\ u(0)=u'(0)=u'(1)=0\end{cases}$的正解，其中$D_{0+}^{\alpha}$是标准的Riemann-Liouville分数阶微分，$f(t,u(t))=g(t,u(t))+h(t,u(t))$和$g,h:[0,1]\times[0,\infty)\to[0,\infty)$都是连续函数且$g(t,u),h(t,u)$关于$u$是单调递增。证明了其解存在唯一性，同时构造一迭代序列去逼近它。最后，举例应用了所得结果。

关键词：分数阶微分方程；边值问题；正解；存在唯一性；和算子的不动点定理

（责任编辑　黄　颖）