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Periodic Solution in a Discrete Multispecies Cooperation and Competition Predator-prey Model*

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Abstract: In this paper, a discrete multispecies cooperation and competition predator-prey model is investigated. By using coincidence degree theory and some analysis technique, a new criterion on the existence of positive periodic solution of difference equations in consideration is established. The paper ends with some interesting numerical simulations that illustrate our analytical predictions.

Key words: predator-prey model; periodic solution; discrete; coincidence degree

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It is well known that delayed differential equations have been widely used to model phenomena in economics, biology, medicine, ecology, and other fields. The investigation on delayed differential equations in population dynamics not only focus on the discussion of stability, attractivity, and persistence, but also involve many other dynamical behaviors such as periodic oscillatory, bifurcation and chaos[1-5]. In many applications, the nature of periodic oscillatory solutions are of great interest. Recently, Zhang and Luo[6] investigated the positive periodic solutions of a population model with delay and stage structure for the predator. Gyllenberg et al.[7] gave a theoretical study on limit cycles of a competitor-competitor-mutualist Lotka-Volterra model. Sen et al.[8] focused on the bifurcation behavior of a ratio-dependent prey-predator model with the Allee effect. Ding and Liu[9] analyzed the existence of positive periodic solution for ratio-dependent N-species predator-prey system. Xiong and Zhang[10] addressed the periodic phenomenon of a two-species competitive model with stage structure. For more research on the periodic behavior of predator-prey models, one can see [11-13].

Many researchers have argued that discrete time models are more suitable to characterize the dynamics of predator-prey models. In addition, discrete time models play an key roles in computer simulation. Thus, many researchers considered the complex behaviors of the discrete predator-prey systems governed by difference equations, see, for example [7,14-24].

In 2011, Zhou[5] investigated the global attractivity and periodic solution of the following discrete multispecies coorperation and competition predator-prey system

$$\begin{cases} x_i(k+1)=x_i(k)\exp\left[r_{1i}(k)\left(1-\dfrac{x_i(k)}{a_i(k)+\sum\limits_{l=1,l\neq i}^{n}b_{il}x_l(k)}-c_i(k)x_i(k)\right)\right]-\sum\limits_{l=1}^{m}d_{il}(k)y_l(k), \\ y_j(k+1)=y_j(k)\exp\left[r_{2j}(k)+\sum\limits_{l=1}^{n}e_{jl}x_l(k)-\sum\limits_{l=1}^{m}p_{jl}y_l(k)\right], \end{cases} \tag{1}$$

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where $i=1,2,\cdots,n;j=1,2,\cdots,m$. $x_i(k)$ is the density of prey species i at kth generation. $y_j(k)$ is the density of predator species j at kth generation. In detail, one can see [5].

The main object of this paper is to discuss the dynamical behavior of model (1). By means of the Mawhin's continuous theorem[8], we will consider the existence of positive periodic solutions for model (1).

The outline of the article is stated in the following: we shall derive some sufficient conditions for the existence of periodic behavior of system (1) by applying coincidence degree theory in Section 2. An example with its numerical simulations are given to illustrate the theoretical findings in Section 3.

1 Existence of positive periodic solutions

For the sake of simplification, we will list the notations which will be used in the later section.

$$I_\omega:=\{0,1,2,\cdots,\omega-1\},\bar{f}:=\frac{1}{\omega}\sum_{k=0}^{\omega-1}f(k),$$

where $f(k)$ is an ω-periodic sequence of real numbers for $k\in\mathbf{Z}$. Suppose that (H) $r_{1i},r_{2j},c_i,d_{il},p_{jl},e_{jl}:\mathbf{Z}\to\mathbf{R}^+$ are ω periodic, i. e.,

$$r_{1i}(k+\omega)=r_{1i}(k),r_{2j}(k+\omega)=r_{2j}(k),c_i(k+\omega)=c_i(k),$$
$$d_{il}(k+\omega)=d_{il}(k),p_{jl}(k+\omega)=p_{jl}(k),e_{jl}(k+\omega)=e_{jl}(k)$$

for any $k\in\mathbf{Z}$.

Firstly, we will state a important lemma. Denote X,Y by two normed vector spaces, $L:Dom\,L\subset X\to Y$ stands for a linear mapping, $N:X\to Y$ is a continuous mapping. We call the mapping L a Fredholm mapping of index zero if $\dim Ker\,L=co\dim \mathrm{Im}\,L<+\infty$ is closed in Y. If L is a Fredholm mapping of index zero and there has continuous projectors $P:X\to X$ and $Q:Y\to Y$ which has the following peoperties $\mathrm{Im}\,P=Ker\,L$, $\mathrm{Im}\,L=Ker\,Q=\mathrm{Im}(I-Q)$, then $L|Dom\,L\cap Ker\,P:(I-P)X\to\mathrm{Im}\,L$ must be invertible. Denote K_P as the inverse for that map. If Ω is an open bounded subset of X, then N is L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I-Q)N:\bar{\Omega}\to X$ is compact. Because $\mathrm{Im}\,Q$ is isomorphic to $Ker\,L$, there has an isomorphism $J:\mathrm{Im}\,Q\to Ker\,L$.

Lemma 1[25] Suppose that L is a Fredholm mapping of index zero, N is L-compact on $\bar{\Omega}$. If the following conditions hold: (a) $\forall\lambda\in(0,1)$, every solution x of $Lx=\lambda Nx$ is such that $x\notin\partial\Omega$; (b) $QNx\neq0$, $\forall x\in Ker\,L\cap\partial\Omega$ and $\deg\{JQN,\Omega\cap Ker\,L,0\}\neq0$; Then the equation $Lx=Nx$ has at least one solution which lies in $Dom\,L\cap\bar{\Omega}$.

Lemma 2[15] Assume that $f:\mathbf{Z}\to\mathbf{R}$ is T periodic, namely, $f(k+T)=f(k)$, then for every $k_1,k_2\in I_T$ and any $k\in\mathbf{Z}$, one gets

$$f(k)\leqslant f(k_1)+\sum_{s=0}^{T-1}|f(s+1)-f(s)|,f(k)\geqslant f(k_2)-\sum_{s=0}^{T-1}|f(s+1)-f(s)|.$$

Define $l_{n+m}=\{y=\{y(k)\}:y(k)=(y_1(k),y_2(k),\cdots,y_{n+m}(k))^{\mathrm{T}}\in\mathbf{R}^{n+m},k\in\mathbf{Z}\}$. We denote the subspace of ω periodic sequences which have the usual supremum norm $\|\cdot\|$ as $l^\omega\subset l_{n+m}$, i. e., $\|y\|=|y_1(k)|+|y_2(k)|+\cdots+|y_{n+m}(k)|$ for any $y=\{y(k):k\in\mathbf{Z}\}\in l^\omega$. We can easily know that l^ω is a finite-dimensional Banach space.

Let

$$l_0^\omega=\{y=\{y(k)\}\in l^\omega:\sum_{k=0}^{\omega-1}y(k)=0\},\tag{2}$$

$$l_c^\omega=\{y=\{y(k)\}\in l^\omega:y(k)=h\in\mathbf{R}^{n+m},k\in\mathbf{Z}\},\tag{3}$$

then we know that both l_0^ω and l_c^ω are closed linear subspaces of l^ω and $l^\omega=l_0^\omega+l_c^\omega$, $\dim l_c^\omega=n+m$.

In the sequel, we will ready to establish our result.

Theorem 1 Let $S_j\,(j=1,2,\cdots,m)$ be defined by (16). Suppose that the condition (H) and $\bar{r}_{2j}>\overline{\sum_{l=1}^{m}p_{jl}(k)\exp(S_l)}$ are fulfilled, then model (1) posses at least an ω periodic solution.

Proof First of all, we make the change of variables

$$x_i(t)=\exp(u_i(t)),y_j(t)=\exp(v_j(t)),i=1,2,\cdots,n;j=1,2,\cdots,m.$$

then (1) can be reformulated as

$$\begin{cases}u_i(k+1)-u_i(k)=f_i(u_i(k),v_j(k)),\\v_j(k+1)-v_j(k)=f_j(u_i(k),v_j(k)),\end{cases}\tag{4}$$

where

$$f_i(u_i(k),v_j(k))=r_{1i}(k)\left(1-\frac{\exp(u_i(k))}{a_i(k)+\sum_{l=1,l\neq i}^{n}b_{il}\exp(u_l(k))}-c_i(k)\exp(x_i(k))\right)-\sum_{l=1}^{m}d_{il}(k)\exp(v_l(k)),$$

$$f_j(u_i(k),v_j(k))=r_{2i}(k)+\sum_{l=1}^{n}e_{jl}\exp(u_l(k))-\sum_{l=1}^{m}p_{jl}\exp(v_l(k)).$$

Let $X=Y=l^{\omega}$,

$$(Lu)(k)=u(k+1)-u(k)=\begin{pmatrix}u_i(k+1)-u_i(k)\\ v_j(k+1)-v_j(k)\end{pmatrix},\tag{5}$$

$$(Nu)(k)=\begin{pmatrix}f_i(u_i(k),v_j(k))\\ f_j(u_i(k),v_j(k))\end{pmatrix},\tag{6}$$

where $u\in X,k\in\mathbf{Z}$. Then we can easily know that L is a bounded linear operator and

$$Ker\,L=l_c^{\omega},\mathrm{Im}\,L=l_0^{\omega},\ \dim Ker\,L=n+m=co\dim\mathrm{Im}\,L,$$

then it follows that L is a Fredholm mapping of index zero. Define

$$Py=\frac{1}{\omega}\sum_{s=0}^{\omega-1}y(s),y\in X,Qz=\frac{1}{\omega}\sum_{s=0}^{\omega-1}z(s),z\in Y.$$

It is not difficult to show that P and Q are continuous projectors which has the following properties:

$$\mathrm{Im}\,P=Ker\,L,\mathrm{Im}\,L=Ker\,Q=\mathrm{Im}\,(I-Q).$$

In addition, the generalized inverse (to L) K_p: $\mathrm{Im}\,L\to Ker\,P\cap Dom\,L$ exists and it can be denoted by

$$K_P(z)=\sum_{s=0}^{\omega-1}z(s)-\frac{1}{\omega}\sum_{s=0}^{\omega-1}(\omega-s)z(s).$$

Clearly, QN and $K_p(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, using the Ascoli-Arzela theorem, we can easily know that $\overline{K_P(I-Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega\subset X$. Moreover, $QN(\bar{\Omega})$ is bounded. Thus, N is L-compact on $\bar{\Omega}$ with any open bounded set $\Omega\subset X$.

In the sequel, we will seek an suitable open, bounded subset Ω. Considering the operator equation $Lu=\lambda Nu,\lambda\in(0,1)$, one has

$$\begin{cases}u_i(k+1)-u_i(k)=\lambda f_i(u_i(k),v_j(k)),\\ v_j(k+1)-v_j(k)=\lambda f_j(u_i(k),v_j(k)).\end{cases}\tag{7}$$

Assume that for a certain $\lambda\in(0,1)$, $u(k)=(u_i(k),v_j(k))^{\mathrm{T}}\in X$ is an arbitrary solution to system (7), in view of (7), we get

$$\begin{cases}\sum_{k=0}^{\omega-1}\left[\frac{r_{1i}(k)\exp(u_i(k))}{a_i(k)+\sum_{l=1,l\neq i}^{n}b_{il}\exp(u_l(k))}+c_i(k)r_{1i}(k)\exp(u_i(k))+\sum_{l=1}^{m}d_{il}(k)\exp(v_l(k))\right]=\bar{r}_{1i}\omega,\\ \sum_{k=0}^{\omega-1}\left[\sum_{l=1}^{m}p_{jl}(k)\exp(v_l(k))-\sum_{l=1}^{n}e_{jl}(k)\exp(u_l(k))\right]=\bar{r}_{2j}\omega.\end{cases}\tag{8}$$

It follows from (7) and (8) that

$$\sum_{k=0}^{\omega-1}|u_i(k+1)-u_i(k)|\leqslant 2\bar{r}_{1i}\omega,\tag{9}$$

$$\sum_{k=0}^{\omega-1}|v_j(k+1)-v_j(k)|\leqslant 2\bar{r}_{2j}\omega.\tag{10}$$

By the hypothesis $u=\{u(k)\}\in X$, there exist $\xi_i,\eta_i,\delta_j,\sigma_j\in I_{\omega}$ such that

$$u_i(\xi_i)=\min_{k\in I_{\omega}}\{u_i(k)\},u_i(\eta_i)=\max_{k\in I_{\omega}}\{u_i(k)\},v_j(\delta_j)=\min_{k\in I_{\omega}}\{v_j(k)\},v_j(\sigma_j)=\max_{k\in I_{\omega}}\{v_j(k)\},\tag{11}$$

where $i=1,2,\cdots,n;j=1,2,\cdots,m$. From the first equation of (8), we have

$$\sum_{k=0}^{\omega-1}c_i(k)r_{1i}(k)\exp(u_i(\xi_i))\leqslant\bar{r}_{1i}\omega,\sum_{k=0}^{\omega-1}\left[\sum_{l=1}^{m}d_{il}(k)\exp(v_l(\delta_l))\right]\leqslant\bar{r}_{1i}\omega$$

which leads to

$$u_i(\xi_i)\leqslant\ln\left[\frac{\bar{r}_{1i}}{\overline{c_ir_{1i}}}\right],v_l(\delta_l)\leqslant\ln\left[\frac{\bar{r}_{1i}}{\overline{\sum_{l=1}^{m}d_{il}}}\right].\tag{12}$$

where $i=1,2,\cdots,n;j=1,2,\cdots,m$. From the second equation of (8), we get

$$\sum_{k=0}^{\omega-1}\Big[\sum_{l=1}^{m}p_{jl}(k)\exp(v_l(\sigma_l))\Big]\geqslant \bar{r}_{2j}\omega .$$

Then

$$v_l(\sigma_l)\geqslant \ln\left[\frac{\bar{r}_{2j}}{\overline{\sum_{l=1}^{m}p_{jl}}}\right]. \tag{13}$$

In view of (12),(13) and Lemma 2, we get

$$v_j(k)\leqslant v_j(\delta_j)+\sum_{s=0}^{\omega-1}\mid v_i(k+1)-v_i(k)\mid\leqslant \ln\left[\frac{\bar{r}_{1i}}{\overline{\sum_{l=1}^{m}d_{il}}}\right]+2\,\bar{r}_{2j}\omega:=M_j, \tag{14}$$

$$v_j(k)\geqslant v_j(\sigma_j)-\sum_{s=0}^{\omega-1}\mid v_j(s+1)-v_j(s)\mid\geqslant \ln\left[\frac{\bar{r}_{2j}}{\overline{\sum_{l=1}^{m}p_{jl}}}\right]-2\,\bar{r}_{2j}\omega:=m_j, \tag{15}$$

Thus

$$\max_{k\in I_\omega}\{v_j(k)\}\leqslant \max\{|M_j|,|m_j|\}:=S_j. \tag{16}$$

From the second equation of (8), we have $\sum_{k=0}^{\omega-1}\Big[\sum_{l=1}^{m}p_{jl}(k)\exp(S_l)+\sum_{l=1}^{n}e_{jl}(k)\exp(u_l(\eta_l))\Big]\geqslant \bar{r}_{2j}\omega$, which leads to

$$u_l(\eta_l)\geqslant \ln\left[\frac{\bar{r}_{2j}-\overline{\sum_{l=1}^{m}p_{jl}(k)\exp(S_l)}}{\overline{\sum_{l=1}^{n}e_{jl}}}\right]. \tag{17}$$

In view of (12), (17) and Lemma 2, we get

$$u_i(k)\leqslant u_i(\xi_i)+\sum_{s=0}^{\omega-1}\mid u_i(s+1)-u_i(s)\mid\leqslant \ln\left[\frac{\bar{r}_{1j}}{\overline{c_i r_{1i}}}\right]+2\,\bar{r}_{1i}\omega:=N_i, \tag{18}$$

$$u_i(k)\geqslant u_i(\eta_i)-\sum_{s=0}^{\omega-1}\mid u_i(s+1)-u_i(s)\mid\geqslant \ln\left[\frac{\bar{r}_{2j}-\overline{\sum_{l=1}^{m}p_{jl}(k)\exp(S_l)}}{\overline{\sum_{l=1}^{n}e_{jl}}}\right]-2\,\bar{r}_{1i}\omega:=n_i. \tag{19}$$

Thus

$$\max_{k\in I_\omega}\{u_i(k)\}\leqslant \max\{|N_i|,|n_i|\}:=T_i. \tag{20}$$

Obviously, $M_j,m_j,N_i,n_i,S_j,T_i\,(i=1,2,\cdots,n;j=1,2,\cdots,m)$ do not depend on $\lambda\in(0,1)$. Choose $M=\sum_{j=1}^{m}S_j+\sum_{i=1}^{m}T_i+M_0$, where M_0 is large enough to make the following $\max\{|u_1^*|,|u_2^*|,\cdots,|u_n^*|,|v_1^*|,|v_2^*|,\cdots,|v_m^*|\}<M_0$ hold, where $(u_1^*,u_2^*,\cdots,u_n^*,v_1^*,v_2^*,\cdots,v_m^*)^{\mathrm T}$ is the unique solution of the following equation

$$\begin{cases}\bar{r}_{11}-\bar{r}_{11}\,\bar{c}_1\exp(u_1(k))=0,\\ \bar{r}_{12}-\bar{r}_{12}\,\bar{c}_2\exp(u_2(k))=0,\\ \qquad\cdots,\\ \bar{r}_{1n}-\bar{r}_{1n}\,\bar{c}_n\exp(u_n(k))=0,\\ \bar{r}_{21}-\overline{\sum_{l=1}^{m}p_{1l}}\exp(v_l(k))=0,\\ \bar{r}_{22}-\overline{\sum_{l=1}^{m}p_{2l}}\exp(v_l(k))=0,\\ \qquad\cdots,\\ \bar{r}_{2m}-\overline{\sum_{l=1}^{m}p_{ml}}\exp(v_l(k))=0.\end{cases} \tag{21}$$

Now we have proved that any solution $u=\{u(k)\}=\{(u_1(k),u_2(k),u_2(k))^{\mathrm{T}}\}$ of (7) in X satisfies $\|u\|<M, k\in\mathbf{Z}$.

Let $\Omega:=\{u=\{u(k)\}\in X: \|u\|<M\}$, then it is easy to see that Ω is an open, bounded set in X and verifies requirement (a) of Lemma 1. When $u\in\partial\Omega\cap Ker\ L$, $u=\{(u_1(k),u_2(k),\cdots,u_n(k),v_1(k),v_2(k),\cdots,v_m(k))^{\mathrm{T}}\}$ is a constant vector in $\mathbf{R}^{n+m}$ with $\|u\|=|u_1|+|u_2|+\cdots+|u_m|+|v_1|+|v_2|+\cdots+|v_n|=M$. Then

$$QNu=\begin{bmatrix}\bar{f}_{11}\\ \bar{f}_{12}\\ \cdots\\ \bar{f}_{1n}\\ \bar{f}_{21}\\ \bar{f}_{22}\\ \cdots\\ \bar{f}_{2m}\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}, \tag{22}$$

where
$$\bar{f}_{1i}=\bar{r}_{1i}-\frac{1}{\omega}\sum_{i=0}^{\omega}\frac{r_{1i}(k)\exp(u_i(k))}{a_i(k)+\overline{\sum_{l=1,l\neq i}^{n}b_{il}(k)\exp(u_l(k))}}-\overline{r_{1i}c_i}\exp(u_i(k)-\overline{\sum_{l=1}^{m}d_{il}(k)\exp(v_l(k))}),$$

$$\bar{f}_{2j}=\bar{r}_{2j}+\overline{\sum_{l=1}^{n}e_{jl}(k)\exp(u_l(k))}-\overline{\sum_{l=1}^{m}p_{jl}(k)\exp(v_l(k))}.$$

Now let us consider homotopic $\varphi(u_1,u_2,\cdots,u_n,v_1,v_2,\cdots,v_m,\mu)=\mu QNu+(1-\mu)Gu, \mu\in[0,1]$, where

$$Gu=\begin{bmatrix}\bar{r}_{11}-\bar{r}_{11}\ \bar{c}_1\exp(u_1(k))\\ \bar{r}_{12}-\bar{r}_{12}\ \bar{c}_2\exp(u_2(k))\\ \cdots\\ \bar{r}_{1n}-\bar{r}_{1n}\ \bar{c}_n\exp(u_n(k))\\ \bar{r}_{21}-\overline{\sum_{l=1}^{m}p_{1l}\exp(v_l(k))}\\ \bar{r}_{22}-\overline{\sum_{l=1}^{m}p_{2l}\exp(v_l(k))}\\ \cdots\\ \bar{r}_{2m}-\overline{\sum_{l=1}^{m}p_{ml}\exp(v_l(k))}\end{bmatrix}.$$

Choosing J as the identity mapping, then we obtain

$$\deg\{JQN(u_1,u_2,\cdots,u_n,v_1,v_2,\cdots,v_n)^{\mathrm{T}};\Omega\cap Ker\ L;0\}=\deg\{QN(u_1,u_2,\cdots,u_n,v_1,v_2,\cdots,v_m)^{\mathrm{T}};\Omega\cap Ker\ L;0\}=$$
$$\deg\{\varphi(u_1,u_2,\cdots,u_n,v_1,v_2,\cdots,v_n,1);\Omega\cap Ker\ L;0\}=\deg\{\varphi(u_1,u_2,\cdots,u_n,v_1,v_2,\cdots,v_m,0);\Omega\cap Ker\ L;0\}=$$
$$\operatorname{sign}\left\{\bar{r}_{11}\ \bar{r}_{12}\cdots\bar{r}_{1n}\sum_{j=1}^{m}\left[\overline{\sum_{l=1}^{m}p_{1l}}\right]\exp\left(\sum_{i=1}^{n}u_i^*+\sum_{j=1}^{n}v_j^*\right)\right\}=1\neq 0.$$

By now, we have proved that all the conditions of Lemma 1 are fulfilled, then we can conclude that $Lu=Nu$ has at least one solution in $Dom\ L\cap\bar{\Omega}$. Namely, (4) has at least one ω periodic solution in $Dom\ L\cap\bar{\Omega}$, say $u^*=\{u^*(k)\}=\{(u_1^*(k),u_2^*(k),\cdots,u_n^*(k),v_1^*(k),v_2^*(k),\cdots,v_m^*(k))^{\mathrm{T}}\}$, then it follows that

$$(x_1^*(k),x_2^*(k),\cdots,x_n^*(k),y_1^*(k),y_2^*(k),\cdots,y_m^*(k))^{\mathrm{T}}=$$
$$(\exp(u_1^*(k)),\exp(u_2^*(k)),\cdots,\exp(u_n^*(k)),\exp(v_1^*(k)),\exp(v_2^*(k)),\cdots,\exp(v_m^*(k)))^{\mathrm{T}},$$

is a positive ω periodic solution of system (1). We complete the proof of Theorem 1.

2 An example and it computer simulations

To illustrate the theoretical predictions, we give an example with its numerical simulations. Let us consider the following discrete system:

$$\begin{cases} x_1(k+1)=x_1(k)\exp\left[\left(1-\frac{\cos k\pi}{4}\right)\left(1-\frac{x_1(k)}{0.4+\frac{\sin k\pi}{6}+(0.6-\frac{\sin k\pi}{4})x_1(k)}-\right.\right. \\ \left.\left.\left(\frac{0.2-\cos k\pi}{5}\right)x_1(k)-\left(\frac{0.6+\cos k\pi}{3}\right)y_1(k)\right)\right], \\ y_1(k+1)=y_1(k)\exp\left[\frac{0.8-\cos k\pi}{6}+\left(\frac{0.6-\cos k\pi}{6}\right)x_1(k)-\left(\frac{0.5-\cos k\pi}{10}\right)y_1(k)\right]. \end{cases} \tag{23}$$

Here $r_{11}(k)=1-\frac{\cos k\pi}{4}$, $a_1(k)=0.4+\frac{\sin k\pi}{6}$, $b_{11}(k)=0.6-\frac{\sin k\pi}{4}$, $c_1(k)=\frac{0.2-\cos k\pi}{5}$, $d_{11}(k)=\frac{0.6+\cos k\pi}{3}$, $r_{21}(k)=\frac{0.8-\cos k\pi}{6}$, $e_{11}(k)=0.6-\frac{0.6-\cos k\pi}{6}$, $p_{11}(k)=\frac{0.5-\cos k\pi}{10}$ are all 2-periodic functions, and it is easy to see that all the conditions of Theorem 1 are satisfied. Thus system (23) has at least a positive two-periodic solution which is shown in Fig. 1.

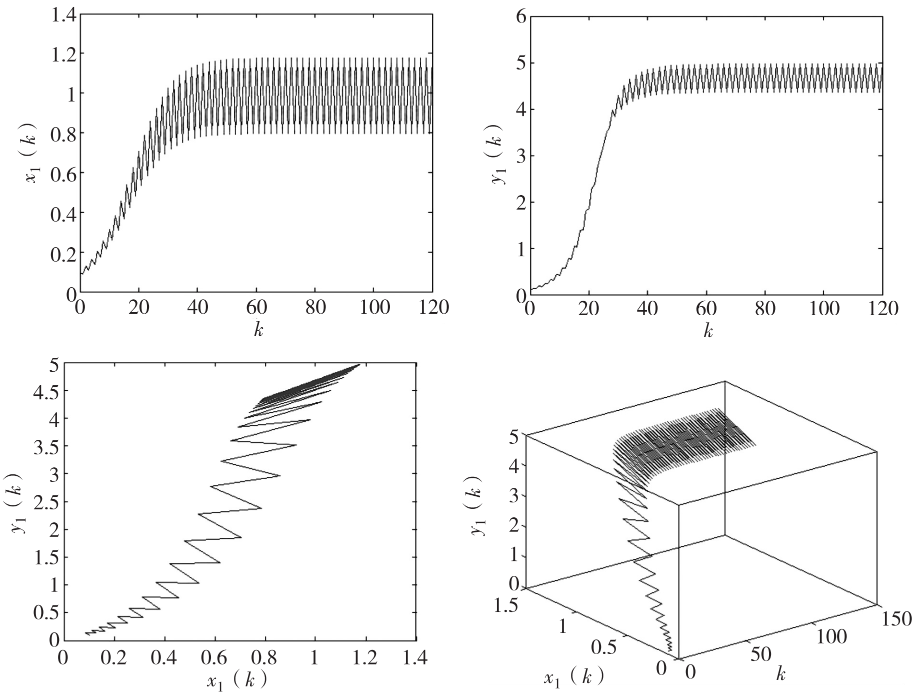

Fig. 1 The state space of $t-x_1-y_1$ and the plane of $t-x_1$, $t-y_1$, x_1-y_1

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一类离散多种群合作与竞争的捕食者与食饵模型的周期解

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摘要：研究了一类离散多种群合作与竞争的捕食者与食饵模型。运用迭合度理论和一些先验估计，得到了保证所研究的差分方程存在正周期解的易于检验的充分条件，最后给出了有趣的数值模拟验证了所得分析结果的正确性。

关键词：捕食模型；周期解；离散；迭合度

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