DOI:10.11721/cqnuj20150214

定义在模糊数上的模糊值函数的连续及导数的新定义*

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摘要:本文首先给出了定义在模糊数上的模糊值函数以及模糊值函数的连续和可导的定义。在此基础上给出了模糊值函数的连续、截集和可导的新概念,利用模糊数的分解定理和序关系讨论了模糊值函数导数的性态,得出了求模糊值函数的导数的基本法则。所得结论拓展了模糊数学的基本概念,丰富了模糊值函数导数的基本理论。

关键词:模糊值函数;模糊值函数的截集;导数;连续
中图分类号:O159　**文献标志码:**A　**文章编号:**1672-6693(2015)02-0072-04

定义在实数上的模糊值函数的连续、导数,许多作者做了大量的研究,得出了相当的结论[1-9]。毕淑娟在文献[2]中利用区间值函数,借助扩张原理讨论了定义在实数上的模糊值函数的连续性;毕淑娟在文献[3]中讨论了定义在模糊数上的模糊值函数的极限;王鹏飞等人在文献[4]中讨论了定义在实数上的复模糊值函数的导数;殷凤等人在文献[5]中利用区间值函数,借助扩张原理讨论了定义在实数上的模糊值函数的一致连续性和一致导数;郭嗣琮在文献[6]中利用模糊结构元方法讨论了定义在实数上的模糊值函数的微分;洪勇在文献[7]中利用区间值函数,借助扩张原理讨论了定义在实数上的模糊值函数的微分;邵亚斌等人在文献[8]中讨论了定义在实数上的模糊值函数的可导性;但定义在模糊数上的模糊值函数的连续、导数鲜有作者涉足。本文在文献[3-9]基础上给出了定义在模糊数上的模糊值函数的连续和可导的概念,并讨论了定义在模糊数上的模糊值函数的连续、导数及其性质,得出了一些重要结论。定义在模糊数上的模糊值函数极限、连续、导数等微分理论在控制、规划、评价、优化、分类、识别等生活生产需求中有丰富的理论和实际应用价值。

1 模糊数知识

定义 1[1]　称$\tilde{u}:\mathbf{R}\to[0,1]$模糊数是指$\forall r\in[0,1]$,$[\tilde{u}]^r$为非空有界闭区间$[\tilde{u}_r^-,\tilde{u}_r^+]$,其中$[\tilde{u}]^r=\{x\in\mathbf{R}:\tilde{u}(x)\geqslant r\}$,$r\in(0,1]$,$[\tilde{u}]^0=\{x\in\mathbf{R}:\tilde{u}(x)>0\}$,所有模糊数的集记做$E^1$。

定理 1[1]　对$\forall\tilde{u},\tilde{v}\in E^1$和$\forall k\in\mathbf{R}$有以下运算法则:a)$([\tilde{u}+\tilde{v}]^r=[\tilde{u}_r^-+\tilde{v}_r^-,\tilde{u}_r^++\tilde{v}_r^+]$,$r\in[0,1]$;b)$[\tilde{u}-\tilde{v}]^r=[\tilde{u}_r^--\tilde{v}_r^+,\tilde{u}_r^+-\tilde{v}_r^-]$,$r\in[0,1]$;c)$[\tilde{u}\tilde{v}]^r=[\min\{\tilde{u}_r^-\tilde{v}_r^-,\tilde{u}_r^-\tilde{v}_r^+,\tilde{u}_r^+\tilde{v}_r^-,\tilde{u}_r^+\tilde{v}_r^+\},\max\{\tilde{u}_r^-\tilde{v}_r^-,\tilde{u}_r^-\tilde{v}_r^+,\tilde{u}_r^+\tilde{v}_r^-,\tilde{u}_r^+\tilde{v}_r^+\}]$,$r\in[0,1]$;d) $k[\tilde{u}]^r=k[\tilde{u}_r^-,\tilde{u}_r^+]=\begin{cases}[k\tilde{u}_r^-,k\tilde{u}_r^+],k\geqslant 0\\ [k\tilde{u}_r^+,k\tilde{u}_r^-],k<0\end{cases}$,$r\in[0,1]$。$E^1$赋之如上的加法和乘法运算将成为一个凸锥,该凸锥可等距同构地嵌入到一个 Banach 空间中去。

定义 2[1]　对$\forall\tilde{u},\tilde{v}\in E^1$ $\tilde{u}\leqslant\tilde{v}$当且仅当$\tilde{u}_r^-+\tilde{u}_r^+\leqslant\tilde{v}_r^-+\tilde{v}_r^+$($\forall r\in[0,1]$)称$\tilde{u}=\tilde{v}$当且仅当$\tilde{u}\leqslant\tilde{v}$,$\tilde{u}\geqslant\tilde{v}$同时成立。

设有模糊数集E^1,若存在对应关系$\tilde{f}$,使得$\forall\tilde{x}\in E^1$,有$\tilde{y}\in E^1$与之对应,则称$\tilde{f}$为定义在模糊集E^1上的模糊值函数,记为$\tilde{y}=\tilde{f}(\tilde{x})$[2]。

2 模糊值函数的连续定义

文献[3]中给出了定义在模糊数上的模糊值函数收敛性的定义,并讨论了一些性质,在此基础上给出定义在

* **收稿日期:**2013-10-29　**修回日期:**2014-12-31　**网络出版时间:**2015-01-22 11:30
资助项目:山西省教育科技开发项目(No. 20121111);忻州师范学院自然科学基金(No. QN201406)
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网络出版地址:http://www.cnki.net/kcms/detail/50.1165.N.20150122.1130.006.html

模糊数上模糊值函数连续的定义。

定义 3[3]　设有模糊值函数$\tilde{f}(\tilde{x})$，若对任意 $\varepsilon>0$，存在 $\delta>0$，当 $0<|\tilde{x}-\tilde{a}|<\delta$ 时，有 $|\tilde{f}(\tilde{x})-\tilde{b}|<\varepsilon$，则称$\tilde{f}(\tilde{x})$在$\tilde{x}=\tilde{a}$处的极限存在且为$\tilde{b}$，记作$\lim\limits_{\tilde{x}\to\tilde{a}}\tilde{f}(\tilde{x})=\tilde{b}$。

定义 4　设有模糊值函数$\tilde{f}(\tilde{x})$，若对任意 $\varepsilon>0$，存在 $\delta>0$，当$|\tilde{x}-\tilde{a}|<\delta$ 时，有$|\tilde{f}(\tilde{x})-\tilde{f}(\tilde{a})|<\varepsilon$，则称$\tilde{f}(\tilde{x})$在$\tilde{x}=\tilde{a}$处连续。

3 模糊值函数的连续性质

由模糊值函数极限和连续的定义知，模糊值函数$\tilde{f}(\tilde{x})$在$\tilde{x}=\tilde{a}$处连续必然在$\tilde{x}=\tilde{a}$处极限存在，下面利用模糊值函数极限的性质讨论模糊值函数连续的性质。

定理 2　$\tilde{f}(\tilde{x})$在$\tilde{x}=\tilde{a}$处连续的充要条件是若对 $\forall\varepsilon>0$，$\exists\delta>0$，$\forall\tilde{x}'$，$\tilde{x}''$有当$|\tilde{x}'-\tilde{a}|<\delta$，$|\tilde{x}''-\tilde{a}|<\delta$ 时，有$|\tilde{f}(\tilde{x}')-\tilde{f}(\tilde{x}'')|<\varepsilon$。（仿照文献[2]的定理 9 可证明）。

定理 3　设有模糊值函数$\tilde{f}(\tilde{x})$、$\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{a}$都连续，则有 $k\tilde{f}(\tilde{x})+l\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{a}$连续，且

$$\lim_{\tilde{x}\to\tilde{a}}(k\tilde{f}(\tilde{x})+l\tilde{g}(\tilde{x}))=k\tilde{f}(\tilde{a})+l\tilde{g}(\tilde{a})。$$

证明　因为$\lim\limits_{\tilde{x}\to\tilde{a}}(k\tilde{f}(\tilde{x})+l\tilde{g}(\tilde{x}))=k\lim\limits_{\tilde{x}\to\tilde{a}}\tilde{f}(\tilde{x})+l\lim\limits_{\tilde{x}\to\tilde{a}}\tilde{g}(\tilde{x}))$，又$\lim\limits_{\tilde{x}\to\tilde{a}}\tilde{f}(\tilde{x})=\tilde{f}(\tilde{a})$，$\lim\limits_{\tilde{x}\to\tilde{a}}\tilde{g}(\tilde{x})=\tilde{g}(\tilde{a})$，所以$\lim\limits_{\tilde{x}\to\tilde{a}}(k\tilde{f}(\tilde{x})+l\tilde{g}(\tilde{x}))=k\tilde{f}(\tilde{a})+l\tilde{g}(\tilde{a})$。　证毕

定理 4　设有模糊值函数$\tilde{f}(\tilde{x})$、$\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{a}$都连续，则$\tilde{f}(\tilde{x})\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{a}$连续，且$\lim\limits_{\tilde{x}\to\tilde{a}}(\tilde{f}(\tilde{x})\tilde{g}(\tilde{x}))=\tilde{f}(\tilde{a})\tilde{g}(\tilde{a})$。

证明　因为$\lim\limits_{\tilde{x}\to\tilde{x}_0}\tilde{g}(\tilde{x})=\tilde{a}$，所以对任意 $\varepsilon_1>0$，存在 $\delta_1>0$，当$|\tilde{x}-\tilde{a}|<\delta_1$ 时，有$|\tilde{f}(\tilde{x})-\tilde{f}(\tilde{a})|<\varepsilon_1$，又存在一个$\tilde{M}$和 $\delta_3>0$，对任意的$|\tilde{x}-\tilde{a}|<\delta_3$，有$|\tilde{f}(\tilde{x})|<\tilde{M}$。又因为$\lim\limits_{\tilde{x}\to\tilde{x}_0}\tilde{g}(\tilde{x})=\tilde{a}$，所以对任意 $\varepsilon_2>0$，存在 $\delta_2>0$，当$|\tilde{x}-\tilde{a}|<\delta_2$ 时，有$|\tilde{g}(\tilde{x})-\tilde{g}(\tilde{a})|<\varepsilon_2$，所以对 $\forall\varepsilon>0$，只要 $\delta=\min(\delta_1,\delta_2,\delta_3)>0$，有$|\tilde{f}(\tilde{x})\tilde{g}(\tilde{x})-\tilde{f}(\tilde{a})\tilde{g}(\tilde{a})|=|(\tilde{f}(\tilde{x})-\tilde{f}(\tilde{a}))\tilde{g}(\tilde{x})+(\tilde{g}(\tilde{x})-\tilde{g}(\tilde{a}))\tilde{f}(\tilde{a})|<|\tilde{f}(\tilde{x})-\tilde{f}(\tilde{a})||\tilde{g}(\tilde{x})|+|\tilde{g}(\tilde{x})-\tilde{g}(\tilde{a})||\tilde{f}(\tilde{a})|<\tilde{M}\varepsilon_1+|\tilde{f}(\tilde{a})|\varepsilon_2<\varepsilon$，所以$\lim\limits_{\tilde{x}\to\tilde{a}}(\tilde{f}(\tilde{x})\tilde{g}(\tilde{x}))=\tilde{f}(\tilde{a})\tilde{g}(\tilde{a})$。　证毕

定理 5　若模糊值函数$\tilde{f}(\tilde{x})$在$\tilde{x}=\tilde{a}$处连续，$\lim\limits_{\tilde{x}\to\tilde{x}_0}\tilde{g}(\tilde{x})=\tilde{a}$，则$\lim\limits_{\tilde{x}\to\tilde{x}_0}\tilde{f}[\tilde{g}(\tilde{x})]=\tilde{f}(\tilde{a})$。

证明　因为$\tilde{f}(\tilde{x})$在$\tilde{x}=\tilde{a}$处连续，所以对任意 $\varepsilon_1>0$，存在 $\delta_1>0$，当$|\tilde{x}-\tilde{a}|<\delta_1$ 时，有$|\tilde{f}(\tilde{x})-\tilde{f}(\tilde{a})|<\varepsilon_1$，又因为$\lim\limits_{\tilde{x}\to\tilde{x}_0}\tilde{g}(\tilde{x})=\tilde{a}$，对任意 $\varepsilon_2>0$，存在 $\delta_2>0$，当 $0<|\tilde{x}-\tilde{x}_0|<\delta_2$ 时，有$|\tilde{g}(\tilde{x})-\tilde{a}|<\varepsilon_2$，只要取 $\delta_1=\varepsilon_2$，则对任意 $\varepsilon_2>0$，存在 $\delta_2>0$，当 $0<|\tilde{x}-\tilde{x}_0|<\delta_2$ 时，有$|\tilde{f}(\tilde{g}(\tilde{x}))-\tilde{f}(\tilde{a})|<\varepsilon_2$，即$\lim\limits_{\tilde{x}\to\tilde{x}_0}\tilde{f}[\tilde{g}(\tilde{x})]=\tilde{f}(\tilde{a})$。　证毕

推论 1　若模糊值函数$\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{x}_0$ 处连续，$\tilde{f}(\tilde{x})$在$\tilde{a}=\tilde{g}(\tilde{x}_0)$处连续，则$\tilde{f}[\tilde{g}(\tilde{x})]$在$\tilde{x}=\tilde{x}_0$ 处连续。

根据上述讨论，两个模糊值函数$\tilde{f}(\tilde{x})$与$\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{x}_0$ 处都连续，则它们的线性组合在$\tilde{x}=\tilde{x}_0$ 处也连续，两个模糊值函数的积$\tilde{f}(\tilde{x})\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{x}_0$ 处也连续，$\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{x}_0$ 有极限$\tilde{a}$，$\tilde{f}(\tilde{x})$在$\tilde{x}=\tilde{a}$处连续，它的复合函数$\tilde{f}[\tilde{g}(\tilde{x})]$在$\tilde{x}=\tilde{x}_0$ 处也连续。

4 模糊值函数的导数定义

文献[4-8]中给出了定义在实数上的模糊值函数导数的定义，在此基础上给出定义在模糊数上模糊值函数导数的定义，模糊值函数$\tilde{f}(\tilde{x})$的截集的定义，及导数和截集的关系。

定义 5　设有模糊值函数$\tilde{f}(\tilde{x})$在点$\tilde{a}$的某个邻域内有定义，若对$\lim\limits_{x\to a}\dfrac{\tilde{f}(\tilde{x})-\tilde{f}(\tilde{a})}{\tilde{x}-\tilde{a}}$，则称模糊值函数$\tilde{f}(\tilde{x})$在

$\tilde{x}=\tilde{a}$处可导。

定义 6[4] 设有模糊值函数$\tilde{f}(\tilde{x})$在点 a 的邻域内有定义,若$\tilde{f}_r^-(\tilde{x})$,$\tilde{f}_r^+(\tilde{x})$在$\tilde{x}=\tilde{a}$处都可导,则称模糊值函数$\tilde{f}(\tilde{x})$在$\tilde{x}=\tilde{a}$处可导。

定义 7[4] 设模糊值函数$\tilde{f}(\tilde{x})$,对于$\forall r\in(0,1]$,模糊值函数$\tilde{f}(\tilde{x})$的截集为$\tilde{f}(\tilde{x})_r=\bigcup\limits_{r\in(0,1]} r[\tilde{f}_r^-(\tilde{x}),\tilde{f}_r^+(\tilde{x})]$。

定义 8[4] 设有模糊值函数$\tilde{f}(\tilde{x})$在 E^1 上有界,若$\tilde{f}_r^-(\tilde{x})$,$\tilde{f}_r^+(\tilde{x})$在 E^1 上都可导,且$\dfrac{\mathrm{d}\,\tilde{f}_r^-(\tilde{x})}{\mathrm{d}\,\tilde{x}}$关于 r 不减,$\dfrac{\mathrm{d}\,\tilde{f}_r^+(\tilde{x})}{\mathrm{d}\,\tilde{x}}$关于 r 不增,则称模糊值函数$\tilde{f}(\tilde{x})$在 E^1 上可导。并定义$\dfrac{\mathrm{d}\,\tilde{f}(\tilde{x})}{\mathrm{d}\,\tilde{x}}=\bigcup\limits_{r\in(0,1]} r\left[\dfrac{\mathrm{d}\,\tilde{f}_r^-(\tilde{x})}{\mathrm{d}\,\tilde{x}},\dfrac{\mathrm{d}\,\tilde{f}_r^+(\tilde{x})}{\mathrm{d}\,\tilde{x}}\right]$。

5 模糊值函数导数的性质

根据模糊值函数导数的定义,利用截集的性质及导数和截集的关系讨论定义在模糊数上的模糊值函数导数的性质。

定理 6 设模糊值函数$\tilde{f}(\tilde{x})$、$\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{a}$均可导,$\forall k,l\in\mathbf{R}$,则 $k\tilde{f}(\tilde{x})+l\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{a}$可导,且

$$\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(k\tilde{f}(\tilde{x})+l\tilde{g}(\tilde{x}))=k\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{f}(\tilde{x})+l\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{g}(\tilde{x})。$$

证明 因为$(\tilde{f}(\tilde{x})+\tilde{g}(\tilde{x}))_r=\tilde{f}_r(\tilde{x})+\tilde{g}_r(\tilde{x})$,所以$\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(\tilde{f}(\tilde{x})+\tilde{g}(\tilde{x}))=\bigcup\limits_{r\in(0,1]} r\left[\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(\tilde{f}(\tilde{x})+\tilde{g}(\tilde{x}))_r\right]=\bigcup\limits_{r\in(0,1]} r\left[\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(\tilde{f}_r(\tilde{x})+\tilde{g}_r(\tilde{x}))\right]$。又有当 $k\geqslant 0$ 时,因为

$$\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(k\tilde{f}(\tilde{x}))_r=\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}[k\tilde{f}_r^-(\tilde{x}),k\tilde{f}_r^+(\tilde{x})]=\left[k\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{f}_r^-(\tilde{x}),k\frac{\mathrm{d}}{\mathrm{d}x}\tilde{f}_r^+(\tilde{x})\right]=k\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{f}_r(\tilde{x}),$$

$$\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(l\tilde{f}(\tilde{x}))_r=\bigcup_{r\in(0,1]} r\left[\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}l\tilde{f}_r(\tilde{x})\right]=\bigcup_{r\in(0,1]} lr\left[\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{f}_r(\tilde{x})\right]=l\bigcup_{r\in(0,1]} r\left[\frac{\mathrm{d}}{\mathrm{d}x}\tilde{f}_r(\tilde{x})\right]=l\frac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{f}(\tilde{x}),$$

当 $k\leqslant 0$ 时,同理能证明上式成立,得证。 证毕

定理 7 设复模糊值函数$\tilde{\tilde{f}}(\tilde{x})=(\tilde{f}_1(\tilde{x}),\tilde{f}_2(\tilde{x}))$、$\tilde{\tilde{g}}(\tilde{x})=(\tilde{g}_1(\tilde{x}),\tilde{g}_2(\tilde{x}))$在$\tilde{x}=\tilde{a}$均可导,$\forall k,l\in z$,则 $k\tilde{\tilde{f}}(\tilde{x})+l\tilde{\tilde{g}}(\tilde{x})$在$\tilde{x}=\tilde{a}$可导,且$\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(k\tilde{\tilde{f}}(\tilde{x})+l\tilde{\tilde{g}}(\tilde{x}))=k\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{\tilde{f}}(\tilde{x})+l\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{\tilde{g}}(\tilde{x})$。

证明 因为$\tilde{\tilde{f}}(\tilde{x})_r=(\tilde{f}_1(\tilde{x}),\tilde{f}_2(\tilde{x}))_r=(\tilde{f}_1(\tilde{x})_r,\tilde{f}_2(\tilde{x})_r)$,$(\tilde{f}(\tilde{x})+\tilde{g}(\tilde{x}))_r=\tilde{f}(\tilde{x})_r+\tilde{g}(\tilde{x})_r$,且$\forall\tilde{\tilde{f}}(\tilde{x})=(\tilde{f}_1(\tilde{x}),\tilde{f}_2(\tilde{x}))\in F$,$c=c_1+ic_2$,有 $c\tilde{\tilde{f}}(\tilde{x})=(c_1\tilde{f}_1(\tilde{x}),c_2\tilde{f}_2(\tilde{x}))$,所以$\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(\tilde{\tilde{f}}(\tilde{x})+\tilde{\tilde{g}}(\tilde{x}))=\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(\tilde{f}_1(\tilde{x})+\tilde{g}_1(\tilde{x}),\tilde{f}_2(\tilde{x})+\tilde{g}_2(\tilde{x}))=\bigcup\limits_{r\in(0,1]} r\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(\tilde{f}_1(\tilde{x})+\tilde{g}_1(\tilde{x}),\tilde{f}_2(\tilde{x})+\tilde{g}_2(\tilde{x}))_r=\bigcup\limits_{r\in(0,1]} r\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(\tilde{f}_1(\tilde{x})_r+\tilde{g}_1(\tilde{x})_r,\tilde{f}_2(\tilde{x})_r+\tilde{g}_2(\tilde{x})_r)=\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{\tilde{f}}(\tilde{x})+\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{\tilde{g}}(\tilde{x})$。因为$\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}k\tilde{f}(\tilde{x})_r=\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}[k\tilde{f}_r^-(\tilde{x}),k\tilde{f}_r^+(\tilde{x})]=\left[k\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{f}_r^-(\tilde{x}),k\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{f}_r^+(\tilde{x})\right]=k\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{f}_r^-(\tilde{x})$,$(k\in\mathbf{R})$成立,所以当 $k\in z$ 时,$\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(k\tilde{\tilde{f}}(\tilde{x}))=\bigcup\limits_{r\in(0,1]} r\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(k_1\tilde{f}_1(\tilde{x}),k_2\tilde{f}_2(\tilde{x}))_r=\left(\bigcup\limits_{r\in(0,1]} r\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(k_1\tilde{f}_1(\tilde{x})\right)_r,\bigcup\limits_{r\in(0,1]} r\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(k_2\tilde{f}_2(\tilde{x}))_r)=k\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}(\tilde{f}_1(\tilde{x}),\tilde{f}_2(\tilde{x}))=k\dfrac{\mathrm{d}}{\mathrm{d}\,\tilde{x}}\tilde{\tilde{f}}(\tilde{x})$。 证毕

定理 8 若模糊值函数$\tilde{g}(\tilde{x})$在$\tilde{x}=\tilde{x}_0$处可导,$\tilde{f}(\tilde{x})$在$\tilde{a}=\tilde{g}(\tilde{x}_0)$处可导,则$\tilde{f}[\tilde{g}(\tilde{x})]$在$\tilde{x}=\tilde{x}_0$处可导,且导数为$\dfrac{\mathrm{d}}{\mathrm{d}x}\tilde{f}[\tilde{g}(\tilde{x}_0)]=\dfrac{\mathrm{d}}{\mathrm{d}x}\tilde{f}[\tilde{g}(\tilde{x}_0)]\dfrac{\mathrm{d}}{\mathrm{d}x}\tilde{g}(\tilde{x}_0)$。

证明 因为$\lim\limits_{x\to\tilde{x}_0}\dfrac{\tilde{g}(\tilde{x})-\tilde{g}(\tilde{x}_0)}{\tilde{x}-\tilde{x}_0}=\tilde{g}'(\tilde{x}_0)$,$\lim\limits_{x\to a}\dfrac{\tilde{f}(\tilde{x})-\tilde{f}(\tilde{a})}{\tilde{x}-\tilde{a}}=\tilde{f}'(\tilde{a})$,所以$\dfrac{\mathrm{d}}{\mathrm{d}x}\tilde{f}[\tilde{g}(\tilde{x}_0)]=\lim\limits_{x\to\tilde{x}_0}\dfrac{\tilde{f}(\tilde{g}(\tilde{x}))-\tilde{f}(\tilde{g}(\tilde{x}_0))}{\tilde{x}-\tilde{x}_0}=\lim\limits_{x\to\tilde{x}_0}\dfrac{\tilde{f}(\tilde{g}(\tilde{x}))-\tilde{f}(\tilde{g}(\tilde{x}_0))}{\tilde{g}(\tilde{x})-\tilde{g}(\tilde{x}_0)}\cdot\dfrac{\tilde{g}(\tilde{x})-\tilde{g}(\tilde{x}_0)}{\tilde{x}-\tilde{x}_0}=\lim\limits_{x\to\tilde{x}_0}\dfrac{\tilde{f}(\tilde{g}(\tilde{x}))-\tilde{f}(\tilde{g}(\tilde{x}_0))}{\tilde{g}(\tilde{x})-\tilde{g}(\tilde{x}_0)}\lim\limits_{x\to\tilde{x}_0}\dfrac{\tilde{g}(\tilde{x})-\tilde{g}(\tilde{x}_0)}{\tilde{x}-\tilde{x}_0}=\tilde{f}'(\tilde{a})\tilde{g}'(\tilde{x}_0)=$

$\frac{\mathrm{d}}{\mathrm{d}x}\widetilde{f}[\widetilde{g}(\widetilde{x}_0)]\frac{\mathrm{d}}{\mathrm{d}x}\widetilde{g}(\widetilde{x}_0)$。　证毕

根据上述讨论，两个模糊值函数$\widetilde{g}(\widetilde{x})$与$\widetilde{f}(\widetilde{x})$在$\widetilde{x}=\widetilde{x}_0$处都可导，则它们的线性组合在$\widetilde{x}=\widetilde{x}_0$处也可导，且导数等于它们导数的线性组合；$\widetilde{g}(\widetilde{x})$在$\widetilde{x}=\widetilde{x}_0$处可导，$\widetilde{f}(\widetilde{x})$在$\widetilde{a}=\widetilde{g}(\widetilde{x}_0)$处可导，复合函数$\widetilde{f}[\widetilde{g}(\widetilde{x})]$在$\widetilde{x}=\widetilde{x}_0$也可导，导数等于内函数导数乘外函数导数。

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New Definition of the Differential and Continuity of Fuzzy-Valued Function

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Abstract：In this paper，the definition on the fuzzy number compound fuzzy-valued function and the complex fuzzy value function of the definition of continuous and derivative are given. Based on which it define are given a cut sets，derivative and analytics of fuzzy-valued function，and the differential of fuzzy-valued function are discussed by using resolution theorem of fuzzy function and order relation of fuzzy function，the basic law of fuzzy-valued function differential are proved，The conclusions extend the basic concepts of fuzzy mathematics，and enrich the basic theory of complex fuzzy-valued function series.

Key words：complex fuzzy-valued function；cut sets of fuzzy-valued function；differential；continuity

(责任编辑　方　兴)