多复变双解析函数中的一个非线性边值问题*

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摘 要:研究了二元复变双解析函数的一个非线性边值问题。首先给出了二元复变双解析函数的定义,讨论了二元双解析函数的 Cauchy 积分定理和 Cauchy 积分公式;其次给出了二元复变双解析函数的 Cauchy-Fredholm 型积分和 Plemelj 公式;最后,在此基础上提出了一个非线性边值问题,并将此边值问题转化为积分方程组问题,然后利用积分方程方法和 Schauder 不动点定理证明解的存在性,并获得解的积分表达式

$$W(z_1,z_2)=\frac{1}{(2\pi i)^2}\int_L\frac{\varphi_1(t_1,t_2)\,dt_1dt_2}{(t_1-z_1)(t_2-z_2)}-\frac{1}{(2\pi i)^2}\int_L\frac{\overline{t_1-z_1}\varphi_2(t_1,t_2)\,dt_1dt_2}{(t_1-z_1)(t_2-z_2)}-\frac{1}{(2\pi i)^2}\int_L\frac{\overline{t_2-z_2}\varphi_3(t_1,t_2)\,dt_1dt_2}{(t_1-z_1)(t_2-z_2)}+\frac{1}{(2\pi i)^2}\int_L\frac{\overline{(t_1-z_1)(t_2-z_2)}\varphi_4(t_1,t_2)\,dt_1dt_2}{(t_1-z_1)(t_2-z_2)}。$$

关键词:二元复变函数;双解析函数;非线性边值问题
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在文献[1]和[2]中讨论了多复变解析函数 Cauchy 型积分的边界性质,并获得了 Plemelj 公式。后来,文献[3]讨论了多复变广义解析函数的一个边值问题,文献[4]对多复变函数一阶拟线性椭圆型方程组的广义 Riemann-Hilbert 边值问题作了研究。文献[5-7]讨论了多复变函数在多圆柱型区域上的一些边值问题。特别是在文献[8]中,黄沙研究了二元复变解析函数于双圆柱域上的一个非线性边值问题,文献[9]研究了二元复变解析函数一个带位移的一个非线性边值问题,文献[10]又研究了二元复变广义解析函数的一个边值问题。另外,在文献[11-12]中研究了单复变函数的双解析函数性质和一些边值问题。本文在此基础上,利用与文献[13]相类似的方法,给出了二元复变双解析函数的定义和 Plemelj 公式,研究了二元复变双解析函数于双圆柱域上的非线性边值问题。

1 预备知识

设 C^2 空间中单位双圆柱域为 $D=D_1\times D_2$,设 D_1,D_2 的边界分别为 $L_1:|z_1|=1,L_2:|z_2|=1$,记 $L=L_1\times L_2$,用 $D_k^+,D_k^-(k=1,2)$ 分别表示 D_k 的内部和外部。设 $t=(t_1,t_2)\in L$,当点 $z=(z_1,z_2)$ 从 $D_1^\pm\times D_2^\pm$ 趋于 t 时,记函数 $\Phi(z)=\Phi(z_1,z_2)$ 的相应极限值为 $\Phi^{\pm\pm}(t_1,t_2)$。

设 $\varphi(t_1,t_2)$ 为 L 上的二元复变函数,若任给 $t,\tau\in L$ 有 $|\varphi(t)-\varphi(\tau)|\leqslant J_1|t-\tau|^\alpha$,其中 $|t-\tau|=(\sum_{k=1}^{2}|t_k-\tau_k|^2)^{\frac{1}{2}}$,$0<\alpha<1$,$J_1$ 是正常数,则称 $\varphi(t)=\varphi(t_1,t_2)$ 在 L 上满足 Hölder 条件,并记为 $\varphi(t)\in H(L,\alpha)$。

定义 $\varphi(t)$ 的范数为 $\|\varphi\|_\alpha=C(\varphi,L)+H(\varphi,L,\alpha)$,其中,$C(\varphi,L)=\max_{t\in L}|\varphi(t)|$,$H(\varphi,L,\alpha)=\sup_{t\neq\tau,t,\tau\in L}\frac{|\varphi(t)-\varphi(\tau)|}{|t-\tau|^\alpha}$,则 $H(L,\alpha)$ 构成 Banach 空间,并且

$$\|\varphi_1+\varphi_2\|_\alpha\leqslant\|\varphi_1\|_\alpha+\|\varphi_2\|_\alpha,\ \|\varphi_1\varphi_2\|_\alpha\leqslant J_2\|\varphi_1\|_\alpha\|\varphi_2\|_\alpha \tag{1}$$

其中 J_2 是正常数,$\varphi,\varphi_2\in H(L,\alpha)$。

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引理1 设函数 $h(z)$ 在 D 内解析，在 $\overline{D}$ 上连续，则 $\frac{1}{(2\pi \mathrm{i})^2}\int_L \frac{h(\tau_1,\tau_2)\mathrm{d}\tau_1\mathrm{d}\tau_2}{(\tau_1-z_1)(\tau_2-z_2)} = \begin{cases}0, & z\notin \overline{D},\\ h(z), & z\in D\end{cases}$

引理2[8] 设函数 $\varphi(\tau)$ 在 L 上连续，则 Cauchy 型积分

$$\Phi(z_1,z_2) = \frac{1}{(2\pi \mathrm{i})^2}\int_L \frac{\varphi(\tau_1,\tau_2)\mathrm{d}\tau_1\mathrm{d}\tau_2}{(\tau_1-z_1)(\tau_2-z_2)}(z_k\notin L_k) \tag{2}$$

是 z_1,z_2 的解析函数，并且 $\Phi(z_1,\infty)=\Phi(\infty,z_2)=\Phi(\infty,\infty)=0$。

2 二元双解析函数及其 Plemelj 公式

定义1 设二元复变函数 $W(z_1,z_2)\in C^2(D)$，$D=D_1\times D_2$。若 $W(z_1,z_2)$ 满足方程组 $\begin{cases}\frac{\partial^2 W}{\partial \overline{z_1}^2}=0\\ \frac{\partial^2 W}{\partial \overline{z_2}^2}=0\end{cases}$，则称 $W(z_1,z_2)$ 在 $D=D_1\times D_2$ 上为 z_1,z_2 的二元双解析函数。

定理1 设 $W(z_1,z_2)$ 在 D 内为双解析函数，且 $W(z_1,z_2)$ 和 $\frac{\partial^2 W}{\partial \overline{z_1}}$，$\frac{\partial^2 W}{\partial \overline{z_2}}$，$\frac{\partial^2 W}{\partial \overline{z_1}\partial \overline{z_2}}$ 在 $\overline{D}$ 上连续，则 $\int_L\left[W(z_1,z_2)-\overline{z_1-a_1}\frac{\partial W}{\partial \overline{z_1}}-\overline{z_2-a_2}\frac{\partial W}{\partial \overline{z_2}}+\overline{(z_1-a_1)(z_2-a_2)}\frac{\partial^2 W}{\partial \overline{z_1}\partial \overline{z_2}}\right]\mathrm{d}z_1\mathrm{d}z_2=0$，其中 a 为二维复平面上的任一定点。

证明 由 $W(z_1,z_2)$ 在 $D=D_1\times D_2$ 内为双解析函数，即 $\begin{cases}\frac{\partial^2 W}{\partial \overline{z_1}^2}=0\\ \frac{\partial^2 W}{\partial \overline{z_2}^2}=0\end{cases}$。令 $\Phi(z_1,z_2)=W(z_1,z_2)-\overline{z_1-a_1}\frac{\partial W}{\partial \overline{z_1}}-\overline{z_2-a_2}\frac{\partial W}{\partial \overline{z_2}}+\overline{(z_1-a_1)(z_2-a_2)}\frac{\partial^2 W}{\partial \overline{z_1}\partial \overline{z_2}}$，于是有 $\frac{\partial \Phi}{\partial \overline{z_1}}=\frac{\partial W}{\partial \overline{z_1}}-\frac{\partial W}{\partial \overline{z_1}}-\overline{z_1-a_1}\frac{\partial^2 W}{\partial \overline{z_1}^2}-\overline{z_2-a_2}\frac{\partial^2 W}{\partial \overline{z_1}\partial \overline{z_2}}+\overline{z_2-a_2}\frac{\partial^2 W}{\partial \overline{z_1}\partial \overline{z_2}}+\overline{(z_1-a_1)(z_2-a_2)}\frac{\partial^3 W}{\partial \overline{z_1}^2\partial \overline{z_2}}=0$。同理可得 $\frac{\partial \Phi}{\partial \overline{z_2}}=0$，故 $\Phi(z_1,z_2)$ 在 $D=D_1\times D_2$ 内解析，又由已知 $\Phi(z_1,z_2)$ 在 $\overline{D}$ 上连续，所以结论成立。 证毕

由引理1和定理1有二元双解析函数的 Cauchy 积分公式。

定理2 设 $W(z_1,z_2)$ 在 D 内为双解析函数，且 $W(z_1,z_2)$ 和 $\frac{\partial W}{\partial \overline{z_1}}$，$\frac{\partial W}{\partial \overline{z_2}}$，$\frac{\partial^2 W}{\partial \overline{z_1}\partial \overline{z_2}}$ 在 $\overline{D}$ 上连续，则

$$\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{W(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}-\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\overline{t_1-z_1}\frac{\partial W}{\partial \overline{z_1}}\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}-\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\overline{t_2-z_2}\frac{\partial W}{\partial \overline{z_2}}\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}+$$

$$\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\overline{(t_1-z_1)(t_2-z_2)}\frac{\partial^2 W}{\partial \overline{z_1}\partial \overline{z_2}}\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}=\begin{cases}0, & (z_1,z_2)\notin \overline{D}\\ W(z_1,z_2), & (z_1,z_2)\in D\end{cases}$$

定义2 设 $\varphi_1(t_1,t_2)$，$\varphi_2(t_1,t_2)$，$\varphi_3(t_1,t_2)$，$\varphi_4(t_1,t_2)$ 为 $L=L_1\times L_2$ 上的绝对可积函数。称

$$W(z_1,z_2)=\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\varphi_1(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}-\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\overline{t_1-z_1}\varphi_2(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}-$$

$$\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\overline{t_2-z_2}\varphi_3(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}+\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\overline{(t_1-z_1)(t_2-z_2)}\varphi_4(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)} \tag{3}$$

为 Cauchy-Fredholm 型积分。其中 $\{\varphi_1(t_1,t_2),\varphi_2(t_1,t_2),\varphi_3(t_1,t_2),\varphi_4(t_1,t_2)\}$ 为核密度。

定理3 设 $\varphi_1(t_1,t_2)$，$\varphi_2(t_1,t_2)$，$\varphi_3(t_1,t_2)$，$\varphi_4(t_1,t_2)$ 为 L 上的连续函数，则 Cauchy-Fredholm 型积分(3)在 L 外是双解析函数，且 $W(z_1,\infty)$，$W(\infty,z_2)$，$W(\infty,\infty)$ 有界。

证明 由

$$W(z_1,z_2)=\frac{1}{(2\pi i)^2}\int_L\frac{\varphi_1\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}-\frac{1}{(2\pi i)^2}\int_L\frac{\overline{t_1-z_1}\varphi_2\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}-\frac{1}{(2\pi i)^2}\int_L\frac{\overline{t_2-z_2}\varphi_3\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}+\frac{1}{(2\pi i)^2}\int_L\frac{\overline{(t_1-z_1)(t_2-z_2)}\varphi_4\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}=\frac{1}{(2\pi i)^2}\int_L\frac{\varphi_1-[\bar t_1\varphi_2+\bar t_2\varphi_3-\overline{t_1t_2}\varphi_4]}{(t_1-z_1)(t_2-z_2)}\mathrm{d}t_1\mathrm{d}t_2+$$

$$\frac{\overline{z_1}}{(2\pi i)^2}\int_L\frac{\varphi_2-\bar t_2\varphi_4}{(t_1-z_1)(t_2-z_2)}\mathrm{d}t_1\mathrm{d}t_2+\frac{\overline{z_2}}{(2\pi i)^2}\int_L\frac{\varphi_3-\bar t_1\varphi_4}{(t_1-z_1)(t_2-z_2)}\mathrm{d}t_1\mathrm{d}t_2+\frac{\overline{z_1z_2}}{(2\pi i)^2}\int_L\frac{\varphi_4}{(t_1-z_1)(t_2-z_2)}\mathrm{d}t_1\mathrm{d}t_2。$$

根据引理 2 可知

$$\frac{1}{(2\pi i)^2}\int_L\frac{\varphi_1-[\bar t_1\varphi_2+\bar t_2\varphi_3-\overline{t_1t_2}\varphi_4]}{(t_1-z_1)(t_2-z_2)}\mathrm{d}t_1\mathrm{d}t_2,\ \frac{1}{(2\pi i)^2}\int_L\frac{\varphi_2-\bar t_2\varphi_4}{(t_1-z_1)(t_2-z_2)}\mathrm{d}t_1\mathrm{d}t_2,\ \frac{1}{(2\pi i)^2}\int_L\frac{\varphi_3-\bar t_1\varphi_4}{(t_1-z_1)(t_2-z_2)}\mathrm{d}t_1\mathrm{d}t_2,$$

$\frac{1}{(2\pi i)^2}\int_L\frac{\varphi_4}{(t_1-z_1)(t_2-z_2)}\mathrm{d}t_1\mathrm{d}t_2$ 在 L 外是解析函数,从而可证 $W(z_1,z_2)$ 在 L 外是双解析函数。显然也有 $W(z_1,\infty)$,$W(\infty,z_2)$,$W(\infty,\infty)$ 有界。

引进奇异积分算子

$$S_1\varphi=\frac{1}{\pi i}\int_{L_1}\frac{\varphi(\tau_1,t_2)}{\tau_1-t_1}\mathrm{d}\tau_1,S_2\varphi=\frac{1}{\pi i}\int_{L_2}\frac{\varphi(\tau_1,t_2)}{\tau_2-t_2}\mathrm{d}\tau_2,S_3\varphi=\frac{1}{(\pi i)^2}\int_L\frac{\varphi(\tau_1,\tau_2)}{(\tau_1-t_1)(\tau_2-t_2)}\mathrm{d}\tau_1\mathrm{d}\tau_2,$$

$$P_1\varphi=\frac{1}{\pi i}\int_{L_1}\frac{\overline{\tau_1-t_1}\varphi(\tau_1,t_2)}{\tau_1-t_1}\mathrm{d}\tau_1,P_2\varphi=\frac{1}{\pi i}\int_{L_2}\frac{\overline{\tau_2-t_2}\varphi(t_1,\tau_2)}{\tau_2-t_2}\mathrm{d}\tau_2,$$

$$R_1\varphi=\frac{1}{(\pi i)^2}\int_L\frac{\overline{\tau_1-t_1}\varphi(\tau_1,\tau_2)}{(\tau_1-t_1)(\tau_2-t_2)}\mathrm{d}\tau_1\mathrm{d}\tau_2,R_2\varphi=\frac{1}{(\pi i)^2}\int_L\frac{\overline{\tau_2-t_2}\varphi(\tau_1,\tau_2)}{(\tau_1-t_1)(\tau_2-t_2)}\mathrm{d}\tau_1\mathrm{d}\tau_2,$$

$$R_3\varphi=\frac{1}{(\pi i)^2}\int_L\frac{\overline{(\tau_1-t_1)(\tau_2-t_2)}\varphi(\tau_1,\tau_2)}{(\tau_1-t_1)(\tau_2-t_2)}\mathrm{d}\tau_1\mathrm{d}\tau_2,t_k\in L_k,k=1,2。$$

证毕

引理 3[8] 设 Cauchy 型积分(2)式中 $\varphi(t)\in H(L,\alpha)$,则

$$\begin{cases}\Phi^{++}(t_1,t_2)=\frac{1}{4}[\varphi+S_1\varphi+S_2\varphi+S_3\varphi],\Phi^{+-}(t_1,t_2)=\frac{1}{4}[-\varphi-S_1\varphi+S_2\varphi+S_3\varphi]\\ \Phi^{-+}(t_1,t_2)=\frac{1}{4}[-\varphi+S_1\varphi-S_2\varphi+S_3\varphi],\Phi^{--}(t_1,t_2)=\frac{1}{4}[\varphi-S_1\varphi-S_2\varphi+S_3\varphi]\end{cases}\tag{4}$$

定理 4 设 Cauchy-Fredholm 型积分(3)式中 $\varphi_1(t_1,t_2)$,$\varphi_2(t_1,t_2)$,$\varphi_3(t_1,t_2)$,$\varphi_4(t_1,t_2)\in H(L,\alpha)$,则对于$(t_1,t_2)\in L$ 有下列 Plemelj 公式

$$\begin{cases}W^{++}(t_1,t_2)=\frac{1}{4}[\varphi_1+S_1\varphi_1+S_2\varphi_1+S_3\varphi_1-P_1\varphi_2-P_2\varphi_3-R_1\varphi_2-R_2\varphi_3+R_3\varphi_4]\\ W^{+-}(t_1,t_2)=\frac{1}{4}[-\varphi_1-S_1\varphi_1+S_2\varphi_1+S_3\varphi_1+P_1\varphi_2-P_2\varphi_3-R_1\varphi_2-R_2\varphi_3+R_3\varphi_4]\\ W^{-+}(t_1,t_2)=\frac{1}{4}[-\varphi_1+S_1\varphi_1-S_2\varphi_1+S_3\varphi_1-P_1\varphi_2+P_2\varphi_3-R_1\varphi_2-R_2\varphi_3+R_3\varphi_4]\\ W^{--}(t_1,t_2)=\frac{1}{4}[\varphi_1-S_1\varphi_1-S_2\varphi_1+S_3\varphi_1+P_1\varphi_2+P_2\varphi_3-R_1\varphi_2-R_2\varphi_3+R_3\varphi_4]\end{cases}\tag{5}$$

或 $W^{++}(t_1,t_2)-W^{+-}(t_1,t_2)-W^{-+}(t_1,t_2)+W^{--}(t_1,t_2)=\varphi_1$

$$\begin{cases}W_{\bar z_1}{}^{++}(t_1,t_2)=\frac{1}{4}[\varphi_2+S_1\varphi_2+S_2\varphi_2+S_3\varphi_2-P_2\varphi_4-R_2\varphi_4]\\ W_{\bar z_1}{}^{+-}(t_1,t_2)=\frac{1}{4}[-\varphi_2-S_1\varphi_2+S_2\varphi_2+S_3\varphi_2-P_2\varphi_4-R_2\varphi_4]\\ W_{\bar z_1}{}^{-+}(t_1,t_2)=\frac{1}{4}[-\varphi_2+S_1\varphi_2-S_2\varphi_2+S_3\varphi_2+P_2\varphi_4-R_2\varphi_4]\\ W_{\bar z_1}{}^{--}(t_1,t_2)=\frac{1}{4}[\varphi_2-S_1\varphi_2-S_2\varphi_2+S_3\varphi_2+P_2\varphi_4-R_2\varphi_4]\end{cases}\tag{6}$$

或 $W_{\overline{z_1}}^{++}(t_1,t_2) - W_{\overline{z_1}}^{+-}(t_1,t_2) - W_{\overline{z_1}}^{-+}(t_1,t_2) + W_{\overline{z_1}}^{--}(t_1,t_2) = \varphi_2$

$$\begin{cases} W_{\overline{z_2}}^{++}(t_1,t_2) = \frac{1}{4}[\varphi_3 + S_1\varphi_3 + S_2\varphi_3 + S_3\varphi_3 - P_1\varphi_4 - R_1\varphi_4] \\ W_{\overline{z_2}}^{+-}(t_1,t_2) = \frac{1}{4}[-\varphi_3 - S_1\varphi_3 + S_2\varphi_3 + S_3\varphi_3 + P_1\varphi_4 - R_1\varphi_4] \\ W_{\overline{z_2}}^{-+}(t_1,t_2) = \frac{1}{4}[-\varphi_3 + S_1\varphi_3 - S_2\varphi_3 + S_3\varphi_3 - P_1\varphi_4 - R_1\varphi_4] \\ W_{\overline{z_2}}^{--}(t_1,t_2) = \frac{1}{4}[\varphi_3 - S_1\varphi_3 - S_2\varphi_3 + S_3\varphi_3 + P_1\varphi_4 - R_1\varphi_4] \end{cases} \tag{7}$$

或 $W_{\overline{z_2}}^{++}(t_1,t_2) - W_{\overline{z_2}}^{+-}(t_1,t_2) - W_{\overline{z_2}}^{-+}(t_1,t_2) + W_{\overline{z_2}}^{--}(t_1,t_2) = \varphi_3$

$$\begin{cases} W_{\overline{z_1 z_2}}^{++}(t_1,t_2) = \frac{1}{4}[\varphi_4 + S_1\varphi_4 + S_2\varphi_4 + S_3\varphi_4] \\ W_{\overline{z_1 z_2}}^{+-}(t_1,t_2) = \frac{1}{4}[-\varphi_4 - S_1\varphi_4 + S_2\varphi_4 + S_3\varphi_4] \\ W_{\overline{z_1 z_2}}^{-+}(t_1,t_2) = \frac{1}{4}[-\varphi_4 + S_1\varphi_4 - S_2\varphi_4 + S_3\varphi_4] \\ W_{\overline{z_1 z_2}}^{--}(t_1,t_2) = \frac{1}{4}[\varphi_4 - S_1\varphi_4 - S_2\varphi_4 + S_3\varphi_4] \end{cases} \tag{8}$$

或 $W_{\overline{z_1 z_2}}^{++}(t_1,t_2) - W_{\overline{z_1 z_2}}^{+-}(t_1,t_2) - W_{\overline{z_1 z_2}}^{-+}(t_1,t_2) + W_{\overline{z_1 z_2}}^{--}(t_1,t_2) = \varphi_4$

证明 $W(z_1,z_2) = \frac{1}{(2\pi i)^2}\int_L \frac{\varphi_1 - [\bar t_1\varphi_2 + \bar t_2\varphi_3 - \overline{t_1 t_2}\varphi_4]}{(t_1 - z_1)(t_2 - z_2)}dt_1dt_2 + \frac{\overline{z_1}}{(2\pi i)^2}\int_L \frac{\varphi_2 - \bar t_2\varphi_4}{(t_1 - z_1)(t_2 - z_2)}dt_1dt_2 +$

$$\frac{\overline{z_2}}{(2\pi i)^2}\int_L \frac{\varphi_3 - \bar t_1\varphi_4}{(t_1 - z_1)(t_2 - z_2)}dt_1dt_2 + \frac{\overline{z_1 z_2}}{(2\pi i)^2}\int_L \frac{\varphi_4}{(t_1 - z_1)(t_2 - z_2)}dt_1dt_2$$

令 $k = \varphi_1 - [\bar t_1\varphi_2 + \bar t_2\varphi_3 - \overline{t_1 t_2}\varphi_4]$,$f = \varphi_2 - \bar t_2\varphi_4$,$h = \varphi_3 - \bar t_1\varphi_4$。由 $\varphi_1(t_1,t_2)$,$\varphi_2(t_1,t_2)$,$\varphi_3(t_1,t_2)$,$\varphi_4(t_1,t_2) \in H(L,\alpha)$,可证 $k,f,h \in H(L,\alpha)$。于是根据引理3,有(5)式成立。又

$$W_{\overline{z_1}}(z_1,z_2) = \frac{1}{(2\pi i)^2}\int_L \frac{\varphi_2 - \bar t_2\varphi_4}{(t_1 - z_2)(t_2 - z_2)}dt_1dt_2 + \frac{\overline{z_2}}{(2\pi i)^2}\int_L \frac{\varphi_4}{(t_1 - z_1)(t_2 - z_2)}dt_1dt_2,$$

$$W_{\overline{z_2}}(z_1,z_2) = \frac{1}{(2\pi i)^2}\int_L \frac{\varphi_3 - \bar t_1\varphi_4}{(t_1 - z_2)(t_2 - z_2)}dt_1dt_2 + \frac{\overline{z_1}}{(2\pi i)^2}\int_L \frac{\varphi_4}{(t_1 - z_1)(t_2 - z_2)}dt_1dt_2,$$

$$W_{\overline{z_1 z_2}}(z_1,z_2) = \frac{1}{(2\pi i)^2}\int_L \frac{\varphi_4}{(t_1 - z_1)(t_2 - z_2)}dt_1dt_2。$$ 同理可证(6)~(8)式成立。

3 问题的提出

设 $A_i(t)$,$B_i(t)$,$C_i(t)$,$D_i(t)$,$g_i(t)$ $(i = 1,2,3,4)$ 为 $L = L_1 \times L_2$ 上给定的二元复变函数,

$f_1(t,W^{++},W^{+-},W^{-+},W^{--})$,$f_2(t,W_{\overline{z_1}}^{++},W_{\overline{z_1}}^{+-},W_{\overline{z_1}}^{-+},W_{\overline{z_1}}^{--})$,$f_3(t,W_{\overline{z_2}}^{++},W_{\overline{z_2}}^{+-},W_{\overline{z_2}}^{-+},W_{\overline{z_2}}^{--})$,$f_4(t,W_{\overline{z_1 z_2}}^{++},W_{\overline{z_1 z_2}}^{+-},W_{\overline{z_1 z_2}}^{-+},W_{\overline{z_1 z_2}}^{--})$ 在 $L \times C \times C \times C \times C$ 上给定(C 为复平面),求在 $D_1^+ \times D_2^+$,$D_1^+ \times D_2^-$,$D_1^- \times D_2^+$,$D_1^- \times D_2^-$ 内双解析,在 $\overline{D}_1^+ \times \overline{D}_2^+$,$\overline{D}_1^+ \times \overline{D}_2^-$,$\overline{D}_1^- \times \overline{D}_2^+$,$\overline{D}_1^- \times \overline{D}_2^-$ 上连续的函数 $W(z_1,z_2)$,使它满足 $W^{+-}(z_1,\infty)$,$W^{-+}(\infty,z_2)$,$W^{--}(\infty,\infty)$ 有界,且适合下列非线性边界条件

$$A_1(t)W^{++} + B_1(t)W^{+-} + C_1(t)W^{-+} + D_1(t)W^{--} = g_1(t)f_1 \tag{9}$$

$$A_2(t)W_{\overline{z_1}}^{++} + B_2(t)W_{\overline{z_1}}^{+-} + C_2(t)W_{\overline{z_1}}^{-+} + D_2(t)W_{\overline{z_1}}^{--} = g_2(t)f_2 \tag{10}$$

$$A_3(t)W_{\overline{z_2}}^{++} + B_3(t)W_{\overline{z_2}}^{+-} + C_3(t)W_{\overline{z_2}}^{-+} + D_3(t)W_{\overline{z_2}}^{--} = g_3(t)f_3 \tag{11}$$

$$A_4(t)W_{\overline{z_1 z_2}}^{++} + B_4(t)W_{\overline{z_1 z_2}}^{+-} + C_4(t)W_{\overline{z_1 z_2}}^{-+} + D_4(t)W_{\overline{z_1 z_2}}^{--} = g_4(t)f_4 \tag{12}$$

其中 $t = (t_1,t_2) \in L$,称以上问题为问题 O。

假设 Cauchy-Fredholm 型积分(3)式为问题 O 的解,将(5),(6),(7),(8)式分别代入(9),(10),

(11)，(12) 式有

$$F_1\varphi_1 = \varphi_1 \tag{13}$$

$$F_2\varphi_2 = \varphi_2 \tag{14}$$

$$F_3\varphi_3 = \varphi_3 \tag{15}$$

$$F_4\varphi_4 = \varphi_4 \tag{16}$$

其中积分算子 $F_i(i = 1,2,3,4)$ 分别为

$$F_1\varphi_1 = (A_1 + B_1)(\varphi_1 + S_1\varphi_1 + S_2\varphi_1 + S_3\varphi_1) + (C_1 + D_1)(-\varphi_1 + S_1\varphi_1 - S_2\varphi_1 + S_3\varphi_3) + (B_1 + D_1)(2\varphi_1 - 2S_1\varphi_1) + (1 - 4B_1)\varphi_1 - 4g_1f_1 - (A_1 - B_1 + C_1 - D_1)P_1\varphi_2 - (A_1 + B_1 - C_1 - D_1)P_2\varphi_3 - (A_1 + B_1 + C_1 + D_1)(R_1\varphi_2 + R_2\varphi_3 - R_3\varphi_4)$$

$$F_2\varphi_2 = (A_2 + B_2)(\varphi_2 + S_1\varphi_2 + S_2\varphi_2 + S_3\varphi_2) + (C_2 + D_2)(-\varphi_2 + S_1\varphi_2 - S_2\varphi_2 + S_3\varphi_2) + (B_2 + D_2)(2\varphi_2 - 2S_1\varphi_2) + (1 - 4B_2)\varphi_2 - 4g_2f_2 - (A_2 + B_2 - C_2 - D_2)P_2\varphi_4 - (A_2 + B_2 + C_2 + D_2)R_2\varphi_4$$

$$F_3\varphi_3 = (A_3 + B_3)(\varphi_3 + S_1\varphi_3 + S_2\varphi_3 + S_3\varphi_3) + (C_3 + D_3)(-\varphi_3 + S_1\varphi_3 - S_2\varphi_3 + S_3\varphi_3) + (B_3 + D_3)(2\varphi_3 - 2S_1\varphi_3) + (1 - 4B_3)\varphi_3 - 4g_3f_3 - (A_3 - B_3 + C_3 - D_3)P_1\varphi_4 - (A_3 + B_3 + C_3 + D_3)R_1\varphi_4$$

$$F_4\varphi_4 = (A_4 + B_4)(\varphi_4 + S_1\varphi_4 + S_2\varphi_4 + S_3\varphi_4) + (C_4 + D_4)(-\varphi_4 + S_1\varphi_4 - S_2\varphi_4 + S_3\varphi_4) + (B_4 + D_4)(2\varphi_4 - 2S_1\varphi_4) + (1 - 4B_4)\varphi_4 - 4g_4f_4$$

于是问题 O 等价地转化为奇异积分方程组的问题。

4 问题 O 解的存在性和积分表示式

引理 4[8] 设 $\varphi(t) \in H(L,\alpha)$，则存在与 φ 无关的正常数 J_3 使得

$$\|\varphi \pm S_i\varphi\|_\alpha \leqslant J_3\|\varphi\|_\alpha,\ \|S_i\varphi\|_\alpha \leqslant J_3\|\varphi\|_\alpha(i = 1,2),\ \|S_2\varphi \pm S_3\varphi\|_\alpha \leqslant J_3\|\varphi\|_\alpha,$$
$$\|\varphi + S_1\varphi + S_2\varphi + S_3\varphi\|_\alpha \leqslant J_3\|\varphi\|_\alpha,\ \|-\varphi - S_1\varphi + S_2\varphi + S_3\varphi_\alpha\|_\alpha \leqslant J_3\|\varphi\|_\alpha,$$
$$\|-\varphi + S_1\varphi - S_2\varphi + S_3\varphi\|_\alpha \leqslant J_3\|\varphi\|_\alpha,\ \|\varphi - S_1\varphi - S_2\varphi + S_3\varphi\|_\alpha \leqslant J_3\|\varphi\|_\alpha。$$

推论 1 设 $\varphi(t) \in H(L,\alpha)$，则有 $P_k\varphi, R_i\varphi \in H(L,\alpha)$ 且 $\|P_k\varphi\|_\alpha \leqslant J_4\|\varphi\|_\alpha, k = 1,2$，$\|R_i\varphi\|_\alpha \leqslant J_4\|\varphi\|_\alpha, i = 1,2,3$。其中 J_4 是与 φ 无关的正常数。

证明 由

$P_1\varphi = \frac{1}{\pi i}\int_{L_1}\frac{\overline{\tau_1 - t_1}\varphi(\tau_1,t_2)}{\tau_1 - t_1}d\tau_1 = \frac{1}{\pi i}\int_{L_1}\frac{\overline{\tau_1}\varphi(\tau_1,t_2)}{\tau_1 - t_1}d\tau_1 - \frac{\overline{t_1}}{\pi i}\int_{L_1}\frac{\varphi(\tau_1,t_2)}{\tau_1 - t_1}d\tau_1 = S_1(\overline{t_1}\varphi) - \overline{t_1}S_1\varphi$，又由 $\varphi(t) \in H(L,\alpha)$，可知 $\overline{t_1}\varphi(t) \in H(L,\alpha)$，于是根据引理 4 得 $S_1(\overline{t_1}\varphi), \overline{t_1}S_1\varphi \in H(L,\alpha)$，从而 $P_1\varphi \in H(L,\alpha)$。同理有 $P_2\varphi, R_i\varphi \in H(L,\alpha)$，并且可证 $\|P_k\varphi\|_\alpha \leqslant J_4\|\varphi\|_\alpha, k = 1,2$，$\|R_i\varphi\|_\alpha \leqslant J_4\|\varphi\|_\alpha, i = 1,2,3$。其中 J_4 是与 φ 无关的正常数。

定理 5 设问题 O 中的 $A_i(t), B_i(t), C_i(t), D_i(t), g_i(t) \in H(L,\alpha)(i = 1,2,3,4)$，且

$$|f_i(t,\Phi_1^{++},\Phi_1^{+-},\Phi_1^{-+},\Phi_1^{--}) - f(t',\Phi_2^{++},\Phi_2^{+-},\Phi_2^{-+},\Phi_2^{--},)| \leqslant J_5|t - t'|^\alpha + J_6|\Phi_1^{++} - \Phi_2^{++}| + \cdots + J_9|\Phi_1^{--} - \Phi_2^{--}| \tag{17}$$

其中 $\forall t,t' \in L, J_k(k = 5,\cdots,9)$ 为正常数，Φ 表 $W, W_{\bar z_1}, W_{\bar z_2}, W_{\bar z_1\bar z_2}$。又设 $f_i(0,0,0,0,0) = 0$，且 $\|A_i \pm B_i\|_\alpha < h$，$\|C_i \pm D_i\|_\alpha < h$，$\|D_i + B_i\|_\alpha < h$，$\|1 - 4B_i\|_\alpha < h$，$\|A_i \pm B_i \pm C_i \pm D_i\|_\alpha < h, 0 < h < 1, i = 1,2,3,4, 0 < u = J_2h(1 + 5J_4 + 3J_3) < 1$，$\|g_i\|_\alpha < \sigma$，则当 $0 < \sigma < \frac{M(1 - u)}{4J_2(J_{10} + J_{11}M)}$ 时，问题 O 可解，其表达式为(3) 式，J_{10}, J_{11}, M 为给定的正常数($\|\varphi_i\|_\alpha \leqslant M$)。

证明 记连续函数空间 $C(L)$ 的闭子集 $T = \{\varphi \mid \varphi \in H(L,\alpha), \|\varphi\|_\alpha \leqslant M\}$，由文献[8] 的定理 3 可知至少存在一个 $\varphi_4 \in T$ 适合奇异积分方程(16)。

由(1) 和(15) 式有

$$\|F_3\varphi_3\|_\alpha \leqslant J_2\|A_3 + B_3\|_\alpha\|\varphi_3 + S_1\varphi_3 + S_2\varphi_3 + S_3\varphi_3\|_\alpha + J_2\|C_3 + D_3\|_\alpha\|-\varphi_3 + S_1\varphi_3 - S_2\varphi_3 + S_3\varphi_3\|_\alpha + J_2\|B_3 + D_3\|_\alpha\|2\varphi_3 - 2S_1\varphi_3\|_\alpha + J_2\|1 - 4B\|_\alpha\|\varphi_3\|_\alpha +$$

$$4J_2\|g_3\|_\alpha\|f_3\|_\alpha+J_2\|A_3-B_3+C_3-D_3\|_\alpha\|P_1\varphi_4\|_\alpha+J_2\|A_3+B_3+C_3-D_3\|_\alpha\|R_1\varphi_4\|_\alpha \quad (18)$$

由(17)式,并运用与文献[8]相同的方法可得

$$\|f_3(t,W_{\bar{z}_2}^{++},W_{\bar{z}_2}^{+-},W_{\bar{z}_2}^{-+},W_{\bar{z}_2}^{--})\|_\alpha\leqslant J_{10}+J_{11}\|\varphi_3\|_\alpha \quad (19)$$

J_{10},J_{11} 为正常数。由此,当 $\varphi_3\in T$,根据题设,并利用(18),(19)式以及引理4和推论1有

$$\|F_3\varphi_3\|_\alpha\leqslant J_2h[3J_3+1]\|\varphi_3\|_\alpha+2J_2hJ_4\|\varphi_4\|_\alpha+4J_2\|g_3\|_\alpha\|f_3\|_\alpha\leqslant$$
$$J_2h[3J_3+2J_4+1]M+4J_2\sigma[J_{10}+J_{11}M]\leqslant uM+4J_2\sigma[J_{10}+J_{11}M]\leqslant M$$

故 F_3 是闭凸子集 T 到自身的映射。

下证 F_3 是 T 上的连续映射。对任意函数列 $\{\varphi_3^{(n)}\}\in T$,且 $\{\varphi_3^{(n)}\}$ 于 L 上一致收敛于 $\varphi_3(t)\in T$,$t\in L$,即对任意 $\varepsilon>0$,当 n 充分大时,有 $\|\varphi_3^{(n)}-\varphi_3\|_\alpha<\varepsilon$。而由文献[8]可得,对任意 $\varepsilon>0$,当 n 充分大时有

$$|S_1\varphi_3^{(n)}-S_1\varphi_3|<G_1\varepsilon,\ |S_2\varphi_3^{(n)}-S_2\varphi_3|<G_1\varepsilon,\ |S_3\varphi_3^{(n)}-S_3\varphi_3|<G_1\varepsilon \quad (20)$$

其中 G_1 为正常数。另外

$$|P_1\varphi_3^{(n)}-P_1\varphi_3|=\left|\frac{1}{\pi i}\int_{L_1}\frac{\overline{\tau_1-t_1}}{\tau_1-t_1}(\varphi_3^{(n)}-\varphi_3)d\tau_1\right|\leqslant\|\varphi_3^{(n)}-\varphi_3\|_\alpha\frac{1}{\pi}\int_{L_1}|d\tau_1|<2\varepsilon \quad (21)$$

记 $\Psi_3^{(n)}(\tau_1,\tau_2)=\varphi_3^{(n)}(\tau_1,\tau_2)-\varphi_3^{(n)}(t_1,\tau_2)-\varphi_3^{(n)}(\tau_1,t_2)+\varphi_3^{(n)}(t_1,t_2)$,$\Psi_3(\tau_1,\tau_2)=\varphi_3(\tau_1,\tau_2)-\varphi_3(t_1,\tau_2)-\varphi_3(\tau_1,t_2)+\varphi_3(t_1,t_2)$。则有

$$|\Psi_3^{(n)}(\tau_1,\tau_2)|\leqslant2\|\varphi_3^{(n)}\|_\alpha|\tau_1-t_1|^\alpha,\ |\Psi_3^{(n)}(\tau_1,\tau_2)|\leqslant2\|\varphi_3^{(n)}\|_\alpha|\tau_2-t_2|^\alpha;$$
$$|\Psi_3(\tau_1,\tau_2)|\leqslant2\|\varphi_3\|_\alpha|\tau_1-t_1|^\alpha,\ |\Psi_3(\tau_1,\tau_2)|\leqslant2\|\varphi_3\|_\alpha|\tau_2-t_2|^\alpha \quad (22)$$

记 $R_1\varphi_3^{(n)}(t)-R_1\varphi_3(t)=I_1(L)+\cdots+I_5(L)$,$t=(t_1,t_2)$,$L=L_1\times L_2$,其中

$$I_1(L)=\frac{1}{(\pi i)^2}\int_L\frac{\overline{\tau_1-t_1}\Psi_3^{(n)}(\tau_1,\tau_2)}{(\tau_1-t_1)(\tau_2-t_2)}d\tau_1d\tau_2,\ I_2(L)=\frac{-1}{(\pi i)^2}\int_L\frac{\overline{\tau_1-t_1}\Psi_3(\tau_1,\tau_2)}{(\tau_1-t_1)(\tau_2-t_2)}d\tau_1d\tau_2,$$

$$I_3(L)=\frac{1}{(\pi i)^2}\int_L\frac{\overline{\tau_1-t_1}[\varphi_3^{(n)}(t_1,\tau_2)-\varphi_3(t_1,\tau_2)]}{(\tau_1-t_1)(\tau_2-t_2)}d\tau_1d\tau_2,\ I_4(L)=\frac{1}{(\pi i)^2}\int_L\frac{\overline{\tau_1-t_1}[\varphi_3^{(n)}(\tau_1,t_2)-\varphi_3(\tau_1,t_2)]}{(\tau_1-t_1)(\tau_2-t_2)}d\tau_1d\tau_2,$$

$$I_5(L)=\frac{-1}{(\pi i)^2}\int_L\frac{\overline{\tau_1-t_1}[\varphi_3^{(n)}(t_1,t_2)-\varphi_3(t_1,t_2)]}{(\tau_1-t_1)(\tau_2-t_2)}d\tau_1d\tau_2。$$

以 $t=(t_1,t_2)\in L=L_1\times L_2$ 作 t 的 3δ 领域,其中 $\delta=(\delta_1^2+\delta_2^2)^{\frac{1}{2}}$。记 $L_i=L_{i1}+L_{i2}$,$i=1,2$。L_{i1} 为在 $o(t_i,3\delta_i)$ 内部的部分,L_{i2} 为在 $o(t_i,3\delta_i)$ 外部的部分,则有

$$I_i(L)=I_i(L_1\times L_{21})+I_i(L_1\times L_{22}),i=1,2$$

记 $|\tau_1-t_1|=\rho_1$,$|\tau_2-t_2|=\rho_2$,则 $|d\tau_1|=d\rho_1$,$|d\tau_2|=d\rho_2$,所以

$$|I_1(L_1\times L_{21})|\leqslant\frac{1}{\pi^2}\int_{L_1\times L_{21}}\left|\frac{\Psi_3^{(n)}(\tau_1,\tau_2)}{\tau_2-t_2}\right||d\tau_1||d\tau_2|\leqslant\frac{4\|\varphi_3^{(n)}\|_\alpha}{\pi}\int_0^{3\delta_2}|\tau_2-t_2|^{\alpha-1}|d\tau_2|\leqslant\frac{4\|\varphi_3^{(n)}\|_\alpha}{\pi\alpha}(3\delta_2)^\alpha\leqslant$$

$G_2\delta^\alpha$,其中 $G_2=\frac{4}{\pi\alpha}3^\alpha\|\varphi_3^{(n)}\|_\alpha$。同理 $|I_2(L_1\times L_{21})|\leqslant G_3\delta^\alpha$,其中 $G_3=\frac{4}{\pi\alpha}3^\alpha\|\varphi_3\|_\alpha$。

令 $E(\tau_1,\tau_2)=\Psi_3^{(n)}(\tau_1,\tau_2)-\Psi_3(\tau_1,\tau_2)=[\varphi_3^{(n)}(\tau_1,\tau_2)-\varphi_3(\tau_1,\tau_2)]-[\varphi_3^{(n)}(t_1,\tau_2)-\varphi_3(t_1,\tau_2)]-$
$[\varphi_3^{(n)}(\tau_1,t_2)-\varphi_3(\tau_1,t_2)]+[\varphi_3^{(n)}(t_1,t_2)-\varphi_3(t_1,t_2)]$

类似于(22)有

$$|E(\tau_1,\tau_2)|\leqslant2\|\varphi_3^{(n)}-\varphi_3\|_\alpha|\tau_1-t_2|^\alpha,\ |E(\tau_1,\tau_2)|\leqslant2\|\varphi_3^{(n)}-\varphi_3\|_\alpha|\tau_2-t_2|^\alpha$$

$$|I_1(L_1\times L_{22})+I_2(L_1\times L_{22})|=\left|\frac{1}{(\pi i)^2}\int_{L_1\times L_{22}}\frac{\overline{\tau_1-t_1}[\Psi_3^{(n)}(\tau_1,\tau_2)-\Psi_3(\tau_1,\tau_2)]}{(\tau_1-t_1)(\tau_2-t_2)}d\tau_1d\tau_2\right|\leqslant$$
$$\frac{1}{\pi^2}\int_{L_1\times L_{22}}\left|\frac{E(\tau_1,\tau_2)}{\tau_2-t_2}\right||d\tau_1||d\tau_2|\leqslant\frac{4}{\pi}\|\varphi_3^{(n)}-\varphi_3\|_\alpha\int_{3\delta_2}^{A_2}|\tau_2-t_2|^{\alpha-1}|d\tau_2|\leqslant$$
$$\frac{4}{\pi\alpha}\|\varphi_3^{(n)}-\varphi_3\|_\alpha A_2^\alpha<G_4\varepsilon$$

其中 A_2 为 D_2 的直径,$G_4=\frac{4}{\pi\alpha}A_2^\alpha$。

$$|I_3(L)| = \left|\frac{1}{(\pi i)^2}\int_L \frac{\overline{\tau_1 - t_1}[\varphi_3^{(n)}(t_1,\tau_2) - \varphi_3(t_1,\tau_2)]}{(\tau_1 - t_1)(\tau_2 - t_2)}d\tau_1 d\tau_2\right| =$$

$$\left|\frac{1}{\pi i}\int_{L_1}\frac{\overline{\tau_1 - t_1}}{\tau_1 - t_1}d\tau_1\right|\left|\frac{1}{\pi i}\int_{L_2}\frac{\varphi_3^{(n)}(t_1,\tau_2) - \varphi_3(t_1,\tau_2)}{\tau_2 - t_2}d\tau_2\right| = 2|S_2\varphi_3^{(n)} - S_2\varphi_3| < 2G_1\varepsilon$$

同理可得$|I_4(L)| = |P_1\varphi_3^{(n)} - P_1\varphi_3| < 2\varepsilon$,$|I_5(L)| < 2\varepsilon$。于是有

$$|R_1\varphi_3^{(n)}(t) - R_1\varphi_3(t)| = |I_1(L) + \cdots + I_5(L)| < (G_2 + G_3)\delta^\alpha + (2G_1 + G_4 + 4)\varepsilon$$

所以对于任意的$\varepsilon > 0$,可取δ充分小,当n充分大时,对$\forall t = (t_1,t_2) \in L$,有

$$|R_1\varphi_3^{(n)}(t) - R_1\varphi_3(t)| < G_5\varepsilon(G_5 \text{ 为正常数}) \tag{23}$$

综合(15),(17),(20),(21)和(23)式,必存在正常数G,使得$|F_3\varphi_3^{(n)} - F_3\varphi_3| < G\varepsilon$,即$F_3$是$T$到自身的连续映射。依据Arzela-Ascoli定理知,T是连续函数空间$C(L)$中的紧集。因此连续映射F_3是$C(L)$中闭凸集T到自身的映射,并且$F_3(T)$也是$C(L)$中的紧集。再由Schauder不动点定理有,至少存在一个$\varphi_3(t) \in H(L,\alpha)$适合奇异积分方程(15)。

同理可证,至少存在一个$\varphi_2(t)$,$\varphi_1(t) \in T \subset H(L,\alpha)$分别适合奇异积分方程(14)和(13)。于是问题$O$至少存在一个解$W(z_1,z_2)$,并且以(3)式为解的积分表达式。

参考文献:

[1] 陆启铿,钟同德. ПРивалов 定理的推广[J]. 数学学报,1957,7:144-165.

[2] 钟同德. 多复变函数哥西型积分的边界性质[J]. 数学学报,1965,15:227-241.

[3] TUTSCHKE W. Boundary Value Problems for Generalized Analytic Functions of Several Complex Variables[J]. Ann Polon Math,1981,38:227-238.

[4] LI M Z. Generalized Riemann-Hilbert Problems for a System First Order Quasilinear Elliptic Equations of Several Complex Variables[J]. Complex Variables,1987,7:383-394.

[5] 杨丕文. 多复变函数在广义多圆柱区域上的黎曼边值问题[J]. 四川师范大学学报(自然科学版),1991,14(2):117-123.

[6] 杨丕文. 多圆柱区域边界上的多复变函数是区域内多元解析函数边界值的充要条件[J]. 四川师范大学学报(自然科学版),1991,14(4):32-39.

[7] 王明华. 二元复变函数在圆柱域上的 Hilbert 边值问题[J]. 西南师范大学学报(自然科学版),1998,23(4):385-393.

[8] 黄沙. 多复变解析函数的一个非线性边值问题[J]. 数学物理学报,1997,17(4):382-388.

[9] 姚益民,赵晓东,曾纯一. 多复变解析函数中一个带位移的非线性边值问题[J]. 四川师范大学学报(自然科学版),2007,30(4):439-442.

[10] 杨贺菊,黄沙,乔玉英. 多复变广义解析函数的一个非线性边值问题[J]. 自然科学进展,2002,12(5):466-433.

[11] 赵桢. 双解析函数与复调和函数以及它们的基本边值问题[J]. 北京师范大学学报(自然科学版),1995,31(2):175-179.

[12] 王明华. 双解析函数的性质及其 Hilbert 边值问题[J]. 北京师范大学学报(自然科学版),1995,34(1):13-20.

[13] 李觉友. 非齐次 Moisil-Theodorsco 方程组的 Riemann 边值问题[J]. 重庆师范大学学报(自然科学版),2007,24(4):26-29.

A Nonlinear Boundary Value Problem in Bianalytic Analytic Functions of Several Complex Variable

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Abstract: In this paper, we study a nonlinear boundary value problem in bianalytic functions of double complex variables. Firstly, we give the definition of bianalytic functions of double complex variables and discuss the Cauchy integral theorem and the Cauchy integral formula in the bianalytic functions of double complex variables. Secondly, we give the Cauchy-Fredholm type integral and the Plemelj formula in bianalytic functions of double complex variables . Finally, we pose a nonlinear boundary value problem in the questions mentioned above, Moreover, we transform the boundary value problem into an integral equation system problem. Applying the method of integral equations and Schauder fixed-point theorem, we prove existence of solution and get integral representations of solution:

$$W(z_1,z_2)=\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\varphi_1(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}-\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\overline{t_1-z_1}\varphi_2(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}-\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\overline{t_2-z_2}\varphi_3(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}+\frac{1}{(2\pi \mathrm{i})^2}\int_L\frac{\overline{(t_1-z_1)(t_2-z_2)}\varphi_4(t_1,t_2)\,\mathrm{d}t_1\mathrm{d}t_2}{(t_1-z_1)(t_2-z_2)}.$$

Key words: double complex variables function; Bianalytic functions; Nonlinear boundary value problem

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