DOI:10.11721/cqnuj20250111

具有常数延迟的逐段连续系数的 Lasota-Wazewska 模型的伪概周期解*

宋　娜，夏正德
(中北大学 数学学院，太原 030051)

摘要:针对具有逐段连续系数的脉冲 Lasota-Wazewska 模型,研究模型的系数为逐段连续概周期的情况下,模型的解的性质变化。利用压缩不动点定理,证明逐段连续概周期解的存在性。研究表明,模型的解的连续性主要受到脉冲项的影响,而逐段概周期的性质同时受到系数的逐段概周期性与脉冲项的概周期性的影响。

关键词:伪概周期函数;逐段连续系数;脉冲方程
中图分类号:O175　　**文献标志码**:A　　**文章编号**:1672-6693(2025)01-0097-06

1976 年,Lasota-Wazewska 模型在文献[1]中被提出来,该模型的提出主要是为了研究红细胞的数量:

$$x'(t)=-\alpha x(t)+\beta \mathrm{e}^{-\gamma x(t-h)},$$

其中:$x(t)$表示循环系统中血红细胞的数量,α 表示单位时间内血红细胞被破坏的数量,$\beta>0$ 是常数表示对氧气的需求,$\gamma>0$ 描述肌体造血系统的兴奋度,h 是血液系统的延迟时间。由于模型受环境的影响,α,β,γ 受时间变化的影响模型变为:

$$x'(t)=-\alpha(t)x(t)+\beta(t)\mathrm{e}^{-\gamma(t)x(t-h)},$$

其中:$\alpha(t),\beta(t),\gamma(t)\in C(\mathbf{R})$。若环境变化中存在脉冲影响,脉冲 Lasota-Wazewska 模型进入学者们的视野:

$$\begin{cases} x'(t)=-\alpha(t)x(t)+\sum\limits_{j=1}^{m}\beta_j(t)\mathrm{e}^{-\gamma_j(t)x(t-h)}, t\neq t_k, \\ \Delta x(t_k)=x(t_k+0)-x(t_k-0)=\alpha_k x(t_k)+\nu_k, t=t_k, \end{cases} \tag{1}$$

其中:$\alpha(t),\beta(t),\gamma(t)$为连续函数,脉冲因素影响了模型的结构。针对具有连续系数的脉冲 Lasota-Wazewska 模型以及相关的推广模型,数学工作者们已经做了大量的工作[2-23]。进一步考虑,如果模型中的参数 $\alpha(t),\beta(t),\gamma(t)$为逐段连续函数,即环境中的脉冲因素影响了函数的连续性,例如受到疾病、药物刺激等的影响不仅会导致延迟的出现,还会导致 $\alpha(t),\beta(t),\gamma(t)$出现脉冲。疾病的发生会导致红细胞被破坏的数量突然的增加,肌体造血兴奋度突然变大。针对系数中脉冲导致模型的性质变化这一问题,文献[17-18]分别研究了具有逐段系数的 Lasota-Wazewska 模型的逐段概周期解与逐段伪概周期解;文献[24-25]分别研究了随机 Lasota-Wazewska 模型的均方概周期解与具有正负系数的 Lasota-Wazewska 模型的振荡性。本文针对具有逐段连续系数的脉冲 Lasota-Wazewska 模型(1),研究逐段连续系数的逐段概周期性对于模型的解概周期性的影响。

1　准备工作

在本文中令 $\mathbf{Z}$ 为整数全体,$T=\{t_i,i\in\mathbf{Z}\}$为实序列,使得 $\sigma=\inf\limits_{i\in\mathbf{Z}}(t_{i+1}-t_i)>0,\kappa=\sup\limits_{i\in\mathbf{Z}}(t_{i+1}-t_i)$。令有界逐段连续函数空间 $\mathrm{PC}_T(\mathbf{R},\mathbf{R}^+)=\{\varphi:\mathbf{R}\to\mathbf{R}^+\mid\varphi$ 为连续的,$t\neq t_i,i\in\mathbf{Z}$ 且 $\varphi(t_i)=\varphi^-(t_i),i\in\mathbf{Z}\}$。做出下面的假设:

(C1) $\alpha,\beta_j,\gamma_j\in\mathrm{PC}_T(\mathbf{R},\mathbf{R}^+),j=1,2,\cdots,m$ 是逐段伪概周期函数;

* **收稿日期**:2024-04-18　**修回日期**:2024-10-08　**网络出版时间**:2025-03-19T08:47
资助项目:国家自然科学基金青年科学基金项目(No. 42405061);山西省青年科技研究基金(No. 201901D211275);山西省基础研究计划资助项目(No. 20210302123031);山西研究生科研创新项目(No. 2023KY582)
第一作者简介:宋娜,女,副教授,博士,研究方向为概周期理论,E-mail:songna021@163.com;**通信作者简介**:夏正德,男,工程师,博士研究生,E-mail:qdxiazd@163.com
网络出版地址:https://link.cnki.net/urlid/50.1165.n.20250318.1109.002

(C2) 序列$\{\alpha_k\}$是伪概周期序列且$-1\leqslant\alpha_k\leqslant 0$;

(C3) 序列$\{t_k^j\}$,$k\in\mathbf{Z}$,$j=0,\pm1,\pm2,\cdots$是一致概周期序列;

(C4) 序列$\{v_k,k\in\mathbf{Z}\}$是伪概周期的;

(C5) 存在正常数 $K_2>K_1>0$ 使得:

$$K_1<\frac{(1-\mathrm{e}^{-\alpha^+\sigma})}{\alpha^+}\sum_{j=1}^{m}\beta_j^-\mathrm{e}^{-\gamma_j^+K_2}-\frac{\sup\limits_{k\in\mathbf{Z}}|v_k|}{1-\mathrm{e}^{-\alpha^-\kappa}},\sum_{j=1}^{m}\beta_j^+\leqslant\alpha^-\left(K_2-\frac{\sup\limits_{k\in\mathbf{Z}}|v_k|}{1-\mathrm{e}^{-\alpha^-\kappa}}\right)。$$

考虑线性系统:

$$\begin{cases}x'(t)=-\alpha(t)x(t),t\in\mathbf{R},t\neq t_k,\\ \Delta x(t_k)=\alpha_k x(t_k),k\in\mathbf{Z}。\end{cases}\tag{2}$$

得到下面的柯西矩阵(一维的情况):

$$W(t,s)=\begin{cases}\mathrm{e}^{-\int_s^t\alpha(\theta)\mathrm{d}\theta},t_{k-1}<s\leqslant t\leqslant t_k,\\ \prod\limits_{j=m}^{k}(1+\alpha_j)\mathrm{e}^{-\int_s^t\alpha(\theta)\mathrm{d}\theta},t_{m-1}<s\leqslant t_m\leqslant t_k<t\leqslant t_{k+1}。\end{cases}$$

系统(2)的解为 $x(t;t_0,x_0)=W(t,t_0)x_0$ 的形式,$t_0,x_0\in\mathbf{R}$。

令 $W^{ap}(t,s)$是 $W(t,s)$中的概周期函数表示的部分:

$$W^{ap}(t,s)=\begin{cases}\mathrm{e}^{-\int_s^t\alpha^{ap}(\theta)\mathrm{d}\theta},t_{k-1}<s\leqslant t\leqslant t_k,\\ \prod\limits_{j=m}^{k}(1+\alpha_j^{ap})\mathrm{e}^{-\int_s^t\alpha^{ap}(\theta)\mathrm{d}\theta},t_{m-1}<s\leqslant t_m\leqslant t_k<t\leqslant t_{k+1}。\end{cases}$$

其中:$\alpha^{ap}\in\mathrm{AP}_T(\mathbf{R})$,$\alpha_j^{ap}\in\mathrm{APS}(\mathbf{Z})$,$\mathrm{AP}_T(\mathbf{R})$是逐段连续概周期函数,$\mathrm{APS}(\mathbf{Z})$是概周期序列。

下面的引理可以从文献[17]的引理 1.7 得到。

引理 1[17] 令 $u\in\mathrm{AP}_T(\mathbf{R})$。对于 $\varepsilon>0$,存在 $\varepsilon_1\in(0,\varepsilon)$,相对稠的 $\Omega\subset\mathbf{R}$ 和 $Q\subset\mathbf{Z}$ 使得对于 $\tau\in\Omega$,$q\in Q$,$t\in\mathbf{R}$,其中$|t-t_i|>\varepsilon$,$i\in\mathbf{Z}$ 且 $k\in\mathbf{Z}$,下面的结论成立:

1) $|\rho(t+\tau)-\rho(t)|<\varepsilon$,其中 $\rho=\alpha^{ap},u,\beta_j^{ap},\gamma_j^{ap},\eta_j^{ap}\in\mathrm{AP}_T(\mathbf{R})$,$j=1,2,\cdots,m$;

2) $|\alpha_{k+q}^{ap}-\alpha_k^{ap}|<\varepsilon$,$|v_{k+q}^{ap}-v_k^{ap}|<\varepsilon$;

3) $|t_k^q-\tau|<\varepsilon_1$。

引理 2[18] 对于 $t\geqslant s\in\mathbf{R}$,有 $0<W(t,s)\leqslant\mathrm{e}^{-\alpha^-(t-s)}$,进一步,对于 $\varepsilon>0$,令 Ω 是如同引理 1 中的定义,存在常数 $\lambda>0$,使得对于 $\tau\in\Omega$,$|t-t_k|>\varepsilon$,$|s-t_k|>\varepsilon$,$k\in\mathbf{Z}$,有$|W^{ap}(t+\tau,s+\tau)-W^{ap}(t,s)|\leqslant\lambda\varepsilon\mathrm{e}^{-\alpha^-(t-s)/2}$。

引理 3[18] 对于 $t\geqslant s\in\mathbf{R}$,有 $\lim\limits_{r\to+\infty}\dfrac{1}{2r}\displaystyle\int_{-r}^{r}\int_{-\infty}^{t}|W(t,s)-W^{ap}(t,s)|\mathrm{d}s\mathrm{d}t=0$。

引理 4 令 $u(t)\geqslant0$,$t\in\mathbf{R}$,$u\in\mathrm{PAP}_T(\mathbf{R})$,$F(t)=\displaystyle\int_{-\infty}^{t}W(t,s)\sum_{j=1}^{m}\beta_j(s)\mathrm{e}^{-\gamma_j(s)u(s-h)}\mathrm{d}s$,其中 $\mathrm{PAP}_T(\mathbf{R})$是逐段连续伪概周期函数,则:

$$F(t)\in\mathrm{PAP}_T(\mathbf{R})。$$

证明 因为 $\beta_j,\gamma_j,\eta_j\in\mathrm{PAP}_T(\mathbf{R})$,$j=1,2,\cdots,m$,则可做如下的分解:

$$F(t):=F_1(t)+F(t)-F_1(t),$$

其中:

$$F_1(t)\triangleq\int_{-\infty}^{t}W^{ap}(t,s)\sum_{j=1}^{m}\beta_j^{ap}(s)\mathrm{e}^{(-\gamma_j^{ap}(s)u^{ap}(s-h))}\mathrm{d}s,\alpha^{ap},\beta_j^{ap},\gamma_j^{ap}\in\mathrm{AP}_T(\mathbf{R}),j=1,2,\cdots,m。$$

第 1 步,证明 $F_1(t)\in\mathrm{AP}_T(\mathbf{R})$。容易验证 $F_1(t)$是一致收敛的。对于任意的 $\varepsilon>0$,$u^{ap}\in\mathrm{AP}_T(\mathbf{R})$,存在 $\delta>0$ 使得对于 $i\in\mathbf{Z}$,$|t'-t''|<\delta$ 且 $t',t''\in(t_{i-1},t_i]$,那么有$\|u^{ap}(t')-u^{ap}(t'')\|<\varepsilon$。对于 $\varepsilon_1>0$,存在 $\Omega_1\in\mathbf{R}$ 使得对于 $\tau\in\Omega_1$,$|t-t_i|>\varepsilon_1$,$\|u^{ap}(t+\tau)-u^{ap}(t)\|<\varepsilon_1$。

令 $\varepsilon_2<\min\{\varepsilon_1,\delta,\varepsilon\}$,存在 $\Omega\in\mathbf{R}$ 使得当 $\tau\in\Omega$ 时,对于所有的

$$|t+\tau-h-t_i|>2\varepsilon_2,|t-t_i|>\varepsilon_2,|u^{ap}(t+\tau-h)-u^{ap}(t-h)|<\varepsilon_2,$$

对于 $\varepsilon>0$,存在 $\eta>\sigma$ 使得 $\mathrm{e}^{-\eta\sigma^-}<\varepsilon$。令 $N=\left[\dfrac{\eta}{\sigma}\right]+1$。对于 $t\in\mathbf{R}$ 假设:

$$T\cap[t-\eta,t]=\{r_1,r_2,\cdots,r_p\},T\cap[t-h-\eta,t-h]=\{r'_1,r'_2,\cdots,r'_{p'}\},$$
$$(T-h)\cap[t-\eta,t]=\{\hat{r}'_1,\hat{r}'_2,\cdots,\hat{r}'_q\},(T-h)\cap[t-h-\eta,t-h]=\{\hat{r}'_1,\hat{r}'_2,\cdots,\hat{r}'_{q'}\},$$

其中：$j=1,2,\cdots,m,p,p',q,q'\leqslant N,T-h=\{t_k-h\}_{k\in\mathbf{Z},h\in\mathbf{R}}$。令：

$$A_t=\bigcup_{k=1}^{p}\left[r_k-\frac{\varepsilon}{8N},r_k+\frac{\varepsilon}{8N}\right]\bigcup_{k=1}^{p'}\left[r'_k-\frac{\varepsilon}{8N},r'_k+\frac{\varepsilon}{8N}\right]\bigcup_{k=1}^{q}\left[\hat{r}_k-\frac{\varepsilon}{8N},\hat{r}_k+\frac{\varepsilon}{8N}\right]\bigcup_{k=1}^{q'}\left[\hat{r}'_k-\frac{\varepsilon}{8N},\hat{r}'_k+\frac{\varepsilon}{8N}\right],$$

测度 $mA_t\leqslant\varepsilon$。对于$\frac{\varepsilon}{8N}$，存在 $\Omega\in\mathbf{R}$ 使得对于 $\tau\in\Omega$，有：

$$\|\varphi(s+\tau)-\varphi(s)\|<\frac{\varepsilon}{8N},\varphi=\alpha^{ap},\beta_j^{ap},\gamma_j^{ap},j=1,2,\cdots,m,$$

$$\|u^{ap}(s+\tau-h)-u^{ap}(s-h)\|<\frac{\varepsilon}{8N},\|W^{ap}(t+\tau,s+\tau)-W^{ap}(t,s)\|\leqslant\frac{\lambda\varepsilon}{8N},$$

其中：$|t-t_i|>\varepsilon,s\in\mathbf{R}\backslash A_t$。定义 $\varphi_j(s)=\mathrm{e}^{-\gamma_j^{ap}(s)u^{ap}(s-h)},j=1,2,\cdots,m$。由假设$-\gamma_j^{ap}>0$ 以及 $u^{ap}>0$，可得下式成立：

$$|\varphi_j(s+\tau)-\varphi_j(s)|=|\mathrm{e}^{-\gamma_j^{ap}(s)u^{aP}(s+\tau-h)}-\mathrm{e}^{-\gamma_j^{ap}(s)u^{ap}(s-h)}|\leqslant|\mathrm{e}^{-(\gamma_j^{ap}(s+\tau)u^{aP}(s+\tau-h)-\gamma_j^{ap}(s)u^{ap}(s-h))}-1|\leqslant$$
$$|\gamma_j^{ap}(s+\tau)u^{ap}(s+\tau-h)-\gamma_j^{ap}(s)u^{ap}(s-h)|,$$

注意到 $W^{ap}(t,s),\varphi_j(s)\in(0,1]$，那么对于 $\tau\in\Omega$，$|t-t_i|>\varepsilon,i\in\mathbf{Z},s\in[t-\eta,t]\backslash A_t$，令 $l(s)=\sum_{j=1}^{m}\beta_j^{ap}(s)\varphi_j(s)$，则：

$$\|l(s+\tau)-l(s)\|=\left\|\sum_{j=1}^{m}\beta_j^{ap}(s+\tau)\varphi_j(s+\tau)-\sum_{j=1}^{m}\beta_j^{ap}(s)\varphi_j(s)\right\|\leqslant\sum_{j=1}^{m}\|\beta_j^{ap}(s+\tau)-\beta_j^{ap}(s)\|+$$
$$\sum_{j=1}^{m}\|\beta_j^{ap}\|\,[\|u^{ap}\|\|\gamma_j^{ap}(s+\tau)-\gamma_j^{ap}(s)\|+\|\gamma_j^{ap}\|\|u^{ap}(s+\tau-h)-u^{ap}(s-h)\|]\leqslant$$
$$\left(\sum_{j=1}^{m}\|\beta_j^{ap}\|(\|u^{ap}\|+\|\gamma_j^{ap}\|)+1\right)\frac{\varepsilon}{8N}。$$

进一步可得：

$$\|W^{ap}(t+\tau,s+\tau)l(s+\tau)-W^{ap}(t,s)l(s)\|\leqslant$$
$$\sum_{j=1}^{m}\|\beta_j^{ap}\|\|W^{ap}(t+\tau,s+\tau)-W^{ap}(t,s)\|+\left(\sum_{j=1}^{m}\|\beta_j^{ap}\|(\|u^{ap}\|+\|\gamma_j^{ap}\|)+1\right)\leqslant$$
$$\left(\sum_{j=1}^{m}\|\beta_j^{ap}\|(\lambda+\|u^{ap}\|+\|\gamma_j^{ap}\|)+1\right)\frac{\varepsilon}{8N},$$

所以有：

$$\int_{t-\eta}^{t}\|W^{ap}(t+\tau,s+\tau)l(s+\tau)-W^{ap}(t,s)l(s)\|\mathrm{d}s\leqslant$$
$$\int_{[t-\eta,t]\cap A_t}\sum_{j=1}^{m}\|\beta_j^{ap}\|\mathrm{d}s+\int_{[t-\eta,t]\backslash A_t}\left(\sum_{j=1}^{m}\|\beta_j^{ap}\|(\lambda+\|u^{ap}\|+\|\gamma_j^{ap}\|)+1\right)\frac{\varepsilon}{8N}\mathrm{d}s\leqslant$$
$$\left[\sum_{j=1}^{m}\|\beta_j^{ap}\|+\frac{\left(\sum_{j=1}^{m}\|\beta_j^{ap}\|(\lambda+\|u^{ap}\|+\|\gamma_j^{ap}\|)+1\right)\eta}{8N}\right]\varepsilon\triangleq M\varepsilon。$$

因此，对于 $t\in\Omega$，$|t-t_i|>\varepsilon$，有：

$$\|F_1(t+\tau)-F_1(t)\|\leqslant\int_{-\infty}^{t}\left\|W^{ap}(t+\tau,s+\tau)\sum_{j=1}^{m}\beta_j^{ap}(s+\tau)\varphi_j(s+\tau)-W^{ap}(t,s)\sum_{j=1}^{m}\beta_j^{ap}(s)\varphi_j(s)\right\|\mathrm{d}s\leqslant$$
$$\int_{-\infty}^{t-\eta}\sum_{j=1}^{m}\|\beta_j^{ap}\|\mathrm{e}^{-\alpha^-(t-s)}\mathrm{d}s+M\varepsilon\leqslant\left(\sum_{j=1}^{m}\|\beta_j^{ap}\|+M\right)\varepsilon。$$

也就是说 $F_1(t)\in\mathrm{AP}_T(\mathbf{R})$。

第 2 步，记 PPAP_T^0 是函数的逐段遍历空间，$\mathrm{PPAP}_T^0(\mathbf{R})=\left\{f\in\mathrm{PC}_T(\mathbf{R}):\lim_{s\to\infty}\frac{1}{2s}\int_{-s}^{s}\|f(t)\|\mathrm{d}t=0\right\}$。证明

$F(t)-F_1(t)\in \mathrm{PPAP}_T^0(\mathbf{R})$。当

$$\alpha,\beta_j,\gamma_j\in \mathrm{PPAP}_T(\mathbf{R}),j=1,2,\cdots,m\alpha^{ap},\beta_j^{ap},\gamma_j^{ap}\in \mathrm{AP}_T(\mathbf{R}),j=1,2,\cdots,m,$$

$$|F(t)-F_1(t)|=\left|\int_{-\infty}^t W(t,s)\sum_{j=1}^m\beta_j(s)\mathrm{e}^{-\gamma_j(s)u(s-h)}\mathrm{d}s-\int_{-\infty}^t W^{ap}(t,s)\sum_{j=1}^m\beta_j^{ap}(s)\mathrm{e}^{-\gamma_j^{aP}(s)u^{aP}(s-h)}\mathrm{d}s\right|\leqslant$$

$$\int_{-\infty}^t|W(t,s)-W^{aP}(t,s)|\sum_{1=1}^m\|\beta_j\|\mathrm{d}s+\int_{-\infty}^t\mathrm{e}^{-a^-(t-s)}\sum_{1=1}^m|\beta_j(s)-\beta_j^m(s)|\mathrm{d}s+$$

$$\int_{-\infty}^t\mathrm{e}^{-a^-(t-s)}\sum_{j=1}^m\|\beta_j\|[M_u|\gamma_j(s)-\gamma_j^{ap}(s)|+\|\gamma_j\||u(s-h)-u^{ap}(s-h)|]\mathrm{d}s,$$

其中:$M_u=\sup\limits_{t\in\mathbf{R}}\|u(t)\|$。

通过引理 1～引理 3,对于 $\varepsilon>0$,存在 r_0 使得对于所有的 $r>r_0$,有:

$$\frac{1}{2r}\int_{-r}^r\int_{-\infty}^t|W(t,s)-W^{ap}(t,s)|\mathrm{d}s\mathrm{d}t\leqslant\frac{\varepsilon}{4\sum\limits_{j=1}^m\|\beta_j\|},\frac{1}{2r}\int_{-r}^r|\beta_j(s)-\beta_j^{ap}(s)|\mathrm{d}s\leqslant\frac{a^-\varepsilon}{4m},$$

$$\frac{1}{2r}\int_{-r}^r|\gamma_j(s)-\gamma_j^{ap}(s)|\mathrm{d}s\leqslant\frac{a^-\varepsilon}{4M_u\sum\limits_{j=1}^m|\beta_j|},\frac{1}{2r}\int_{-r}^r|u(s-h)-u^{ap}(s-h)|\mathrm{d}s\leqslant\frac{a^-\varepsilon}{4\sum\limits_{j=1}^m\|\beta_j\|\|\gamma_j\|}。$$

利用控制收敛定理可得:

$$\frac{1}{2r}\int_{-r}^r|F(t)-F_1(t)|\mathrm{d}t\leqslant\frac{\varepsilon}{4}+\left\{\frac{\alpha^-\varepsilon}{4}+\alpha^-\left[\frac{M_u\sum\limits_{j=1}^m\|\beta_j\|\varepsilon}{4M_u\sum\limits_{j=1}^m\|\beta_j\|}+\frac{\sum\limits_{j=1}^m\|\beta_j\|\|\gamma_j\|\varepsilon}{4\sum\limits_{j=1}^m\|\beta_j\|\|\gamma_j\|}\right]\right\}\int_{-\infty}^t\mathrm{e}^{-a^-(t-s)}\mathrm{d}s\mathrm{d}t=\varepsilon,$$

也即 $F\in \mathrm{PPAP}_T(\mathbf{R})$。 证毕

2 主要结论

定理 1 假设 $K>0,\sum\limits_{j=1}^m\beta_j^+\gamma_j^+<\alpha^-$。方程(1)有唯一的正解 $u\in \mathrm{PAP}_T(\mathbf{R})$使得 $0<u(t)\leqslant K,t\in\mathbf{R}$。

证明 令 $K_1\leqslant K_2$ 为正常数,使得 $\mathcal{B}=\{u\in \mathrm{PAP}_T(\mathbf{R}):K_1\leqslant u(t)\leqslant K_2,t\in\mathbf{R}\}$。对于 $u\in\mathcal{B}$,定义:

$$\Gamma u(t)=\int_{-\infty}^t W(t,s)\sum_{j=1}^m\beta_j(s)\mathrm{e}^{-\gamma_j(s)u(s-h)}\mathrm{d}s+\sum_{t_k<t}W(t,t_k)v_k=\Gamma_1u(t)+\Gamma_2u(t)。$$

第 1 步,通过引理 2 和假设可以得到:

$$|\Gamma u(t)|\leqslant\int_{-\infty}^t\mathrm{e}^{-a^-(t-s)}\sum_{j-1}^m\beta_j^+\mathrm{d}s+\sum_{t_k<t}\mathrm{e}^{-a^-(t-t_k)}v_k\leqslant\frac{\sum\limits_{j=1}^m\beta_j^+}{\alpha^-}+\frac{\sup\limits_{k\in\mathbf{Z}}v_k}{1-\mathrm{e}^{-\sigma a^-}}\leqslant K_2。$$

如果 $t\in(t_n,t_{n+1}]$,对于 $n\in\mathbf{Z}$,利用 W 的定义,有:

$$\int_{-\infty}^t W(t,s)\mathrm{d}s=\left(\int_{t_n}^t+\sum_{k=-\infty}^n\int_{t_{k1}}^{t_k}\right)W(t,s)\mathrm{d}s\geqslant\int_{t_n}^t\mathrm{e}^{-a^+(t-s)}\mathrm{d}s+\sum_{k=-\infty}^n\int_{t_{k1}}^{t_k}M\mathrm{e}^{-a^+(t-s)}\mathrm{d}s=$$

$$\frac{1-\mathrm{e}^{-a^+(1-t_n)}}{\alpha^+}+\sum_{k=-\infty}^n M\frac{\mathrm{e}^{-a^+(t-t_k]}(1-\mathrm{e}^{-a^+(t_k-t_{k-1})})}{\alpha^+}\geqslant\frac{1-\mathrm{e}^{-a^+\sigma}}{\alpha^+}\left(1+\frac{1}{(1-(1+\overline{\alpha})\mathrm{e}^{-a^+\kappa})}\right)\geqslant\frac{1-\mathrm{e}^{-a^+\sigma}}{\alpha^+},$$

其中:$M=\prod\limits_{j=k}^n(1+a_j),\overline{\alpha}=\inf\limits_{j\in\mathbf{Z}}a_j$。由假设(C5)可得:

$$\Gamma u(t)\geqslant\sum_{j=1}^m\beta_j^-\mathrm{e}^{-\gamma_j^+K_2}\int_{-\infty}^t W(t,s)\mathrm{d}s-\sum_{t_k<t}\mathrm{e}^{-a^-(t-t_k)}\sup_{k\in\mathbf{Z}}v_k\geqslant\frac{1-\mathrm{e}^{-a^-\sigma}}{\alpha^+}\sum_{j=1}^m\beta_j^-\mathrm{e}^{-\gamma_j^+K_2}-\frac{\sup\limits_{k\in\mathbf{Z}}v_k}{1-\mathrm{e}^{-\sigma a^-}}\triangleq K_1。$$

可以得到 $K_1\leqslant\Gamma u(t)\leqslant K_2$。

第 2 步,证明 $\Gamma u\in\mathcal{B}$,则需要证明 $\Gamma u(t)\in \mathrm{PAP}_T(\mathbf{R})$。通过引理 4,可以得到 $\Gamma_1u(t)\in \mathrm{PPAP}_T(\mathbf{R})$,下面只需要证明 $\Gamma_2u(t)\in \mathrm{PAP}_T(\mathbf{R})$。

事实上,对于任意的 $\varepsilon>0$,令 $\tau\in\Omega,q\in Q$。对于 $t\in\mathbf{R},|t-t_i|>\varepsilon$,且 $t_k\in T,t_k<t$,如果对于某些 $p\geqslant k$,

$[t_k,t]\cap T=\{t_k,t_{k+1},\cdots,t_p\}$那么 $p-k\leqslant(t-t_k)/\sigma$。由引理 1 容易得到：

$$[t_{k+q},t+\tau]\cap T=\{t_{k+q},t_{k+q+1},\cdots,t_{p+q}\}$$

且

$$|\alpha^{ap}(\theta+\tau)-\alpha^{ap}(\theta)|<\varepsilon,|\alpha_{i+q}^{ap}-\alpha_i^{ap}|<\varepsilon,|v_{i+q}^{ap}-v_i^{ap}|<\varepsilon,|t_{k+q}-t_k-\tau|<\varepsilon。$$

对于 $\theta\in\mathbf{R}$，$|\theta-t_j|>\varepsilon$，$i,j\in\mathbf{Z}$，定义 $\Gamma_{21}u(t)=\sum\limits_{t_k<t}W^{ap}(t,t_k)v_k^{ap}$，分解 $\Gamma_2u(t)$可得：

$$\Gamma_2u(t)=\Gamma_{21}u(t)+(\Gamma_2u(t)-\Gamma_{21}u(t))。$$

利用文献[17]中引理 3.1，容易看出：

$$|\Gamma_{21}u(t+\tau)-\Gamma_{21}u(t)|=\Big|\sum_{t_k<t}(W^{ap}(t+\tau,t_{k+q})v_{k+q}^{ap}-W^{ap}(t,t_k)v_k^{ap})\Big|\leqslant\varepsilon。$$

下面证明 $\Gamma_2u(t)-\Gamma_{21}u(t)\in\mathrm{PAP}_T^0(\mathbf{R})$。从 $\Gamma_2u(t)$以及 $\Gamma_{21}u(t)$的定义，可以看出：

$$|\Gamma_2u(t)-\Gamma_{21}u(t)|\leqslant\Big|\sum_{t_k<t}(W(t,t_k)v_k-W^{ap}(t,t_k)v_k^{ap})\Big|\leqslant$$

$$\sum_{t_k<t}|W(t,t_k)-W^{ap}(t,t_k)||v_k|+\sum_{t_k<t}W^{ap}(t,t_k)|v_k-v_k^{ap}|\leqslant$$

$$M_v\sum_{t_k<t}\sum_{t_k<t_i<t}|\alpha_i^{\mathrm{e}}|\mathrm{e}^{-\alpha^-(t-t_i)}+\sum_{t_k<t}|W^{ap}(t,t_k)||v_k-v_k^{ap}|\leqslant M_v\frac{1}{1-\mathrm{e}^{-\alpha^-\sigma}}\sum_{t_i<t}|\alpha_i^{\mathrm{e}}|\mathrm{e}^{-\alpha^-(t-t_i)}+\sum_{t_k<t}\mathrm{e}^{-\alpha^-(t-t_k)}|v_k^{\mathrm{e}}|,$$

其中：$M_v=\sup\limits_{k\in\mathbf{Z}}|v_k|$。

设 $\mathrm{PAPS}_0(\mathbf{Z})$是遍历序列，$\mathrm{PAPS}_0(\mathbf{Z})=\left\{x_n\in l^\infty(\mathbf{Z}):\lim\limits_{n\to\infty}\frac{1}{2n}\sum\limits_{k=-n}^{n}|x_n|=0\right\}$。因为 $\alpha_i^{\mathrm{e}},v_i^{\mathrm{e}}\in\mathrm{PAPS}_0(\mathbf{Z})$可以得到：

$$\sum_{t_i<t}|\alpha_i^{\mathrm{e}}|\mathrm{e}^{-\alpha^-(t-t_i)}\in\mathrm{PAP}_T^0(\mathbf{R}),\sum_{t_i<t}|v_i^{\mathrm{e}}|\mathrm{e}^{-\alpha^-(t-t_i)}\in\mathrm{PAP}_T^0(\mathbf{R}),$$

容易证明 $\lim\limits_{r\to\infty}\frac{1}{2r}\int_{-r}^{r}|\Gamma_2u(t)-\Gamma_{21}u(t)|\mathrm{d}t=0$，这就意味着 $\Gamma(u)\in\mathrm{PAP}_T(\mathbf{R})$。

第 3 步，对于 $u,v\in\mathcal{B}$，有 $|\Gamma u(t)-\Gamma v(t)|\leqslant\frac{\sum\limits_{j=1}^{m}\beta_j^+\gamma_j^+}{\alpha^-}|u-v|$。利用条件 $\sum\limits_{j=1}^{m}\beta_j^+\gamma_j^+<\alpha^-$，可以看出 Γ 有唯一的不动点 $u\in\mathcal{B}$。即 u 是模型(1)满足条件 $0<u\leqslant K_2$ 的伪概周期解。　　证毕

注　从引理 4 和定理 1 的证明可以看出，系数的逐段连续性质不会影响函数的 $F(t)$的连续性。解的逐段连续性主要是由求和部分的逐段连续影响的。系数的逐段概周期性与脉冲部分的概周期性共同影响了解的逐段概周期性质。对于逐段概周期系数的方程 $x'(t)=-\alpha(t)x(t)+\sum\limits_{j=1}^{m}\beta_j(t)\mathrm{e}^{-\gamma_j(t)x(t-h)}$ 的解是连续的逐段概周期的。

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Pseudo Almost Periodic Solution of the Lasota-Wazewska Model with a Piecewise Continuous Coefficient of Constant Delay

SONG Na, XIA Zhengde

(Department of Mathematics, North University of China, Taiyuan 030051, China)

Abstract: For a impulsive Lasota-Wazewska model with a piecewise continuous coefficient, it studies the change of the properties of the model's solution when the coefficients of the model are piecewise continuous almost period. Using fixed point theorem, it is proved that the existence of the piecewise almost periodic solutions. Studies have shown that the continuity of the solution of the model is mainly affected by the impulsive term and the piecewise almost period is affected by both the piecewise almost periodicity of the coefficients and the almost periodicity of the impulsive term.

Keywords: pseudo almost periodic; piecewise continuous coefficient; impulsive model

(责任编辑 黄 颖)